COURSE FILE INDEX

VidyaJyothi Institute of Technology

(An Autonomous Institution)
(Accredited by NAAC, Approved by AICTE New Delhi & Permanently Affiliated to JNTUH)
Aziz Nagar Gate, C.B. Post, Hyderabad-500 075

Department of Electronics and Communication Engineering

(Accredited by NBA)

REGULATION : R18

BATCH : 2018-2022

ACADEMIC YEAR : 2020-2021

PROGRAM : B.Tech

YEAR/SEM : III Year / I Sem

COURSE NAME : CONTROL SYSTEMS ENGINEERING

COURSE CODE : A25415

K.l.lokesh

Course Coordinator : **Lakshmi Lokesh Karna**

H.O.D

Name Of The Faculty : **Lakshmi Lokesh Karna**

Designation : **Assistant Professor**

Name Of The Faculty : **K. Pavani**

Designation : **Assistant Professor**

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Vision and Mission

Vision:

- To develop into a reputed Institution at National and International level in Engineering, Technology and Management by generation and dissemination of knowledge through intellectual, cultural and ethical efforts with human values
- To foster Scientific Temper in promoting the World class professional and technical expertise

Mission:

- To create state of art infrastructural facilities for optimization of knowledge acquisition
- To nurture the students holistically and make them competent to excel in the global scenario
- To promote R&D and Consultancy through strong Industry Institute Interaction to address the societal problems

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Department of Electronics and Communications Engineering

Vision of the Department

The Electronics & Communication Engineering department intends to be a leader in creating the high quality engineers in the field of electronics and associated technologies to cater to national and global technological needs promoting the human prosperity and well being.

Mission of the Department

M1: Providing an infrastructural and conducive environment to the students, faculty and researchers for attaining domain knowledge and expertise in electronics & communication engineering.

M2: Enable the students to develop into outstanding professionals with high ethical standards capable of creating, developing and managing global engineering enterprises.

M3: Inculcate the spirit of lifelong learning by interacting with outside world and strengthen professional, communication skills.

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POs, PEOs &PSOs

Program Outcomes (PO's)

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization for the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for public health and safety, and cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools, including prediction and modeling to complex engineering activities, with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal, and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication:** Communicate effectively on complex engineering activities with the engineering community and with the society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Department of Electronics and Communication Engineering

Program Educational Objectives (PEOs)

PEO1: To impart the student's solid foundation in basic sciences, and Electronics & Communication Engineering with an attitude to pursue continuing education by meeting industry requirements (Continuing Education)
PEO2: To prepare engineering graduates proficient and competent in application domains: Communication, Signal Processing, Embedded Systems and Solid-state electronics (Excellence in Career)
PEO 3: To develop the students with professional skills to function as members of multi-Disciplinary teams in engineering and to achieve leadership role with innovative skills (Multi-Disciplinary Engineering and Leadership)
PEO4: To prepare engineering graduates engaged in lifelong learning with professional honesty and integrity together with an appreciation of social responsibility (Contribution to Society).

Program Specific Outcomes (PSO's)

PSO1: To impart knowledge in the field of Electronics & Communication Engineering by training the students in contemporary technologies which meet the needs of industry.

PSO2: To confide information on thrust areas of semiconductor technologies for students to pursue research in their field of interest.

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Department of Electronics and Communication Engineering

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ACADEMIC CALENDAR FOR II B.Tech & I SEMESTER FOR THE YEAR 2020-21

VIDYA JYOTHI INSTITUTE OF TECHNOLOGY (AUTONOMOUS)
(Accredited by NAAC & NBA, Approved By A I C T E., New Delhi, Permanently Affiliated to JNTU, Hyderabad)
(Aziz Nagar, C.B.Post, Hyderabad -500075)

ACADEMIC CALENDAR FOR II, III & IV B.Tech I SEMESTER FOR THE YEAR 2020-21

FIRST SEMESTER		Commencement of Class work : **13.07.2020**	
	FROM	**TO**	**DURATION**
I Spell of Instruction (Online)	13.07.2020	19.09.2020	10 Weeks
I Mid Examinations	21.09.2020	26.09.2020	1 Week
II Spell of Instructions (Online)	28.09.2020	16.10.2020	3 Weeks
Dussehra Holidays	17.10.2020	25.10.2020	9 Days
II Spell of Instructions Continuation (Online/Offline)	26.10.2020	14.11.2020	3 Weeks
II Mid Examinations	16.11.2020	21.11.2020	1 Week
Practical Examinations	23.11.2020	28.11.2020	1 Week
III Mid Examinations	01.12.2020	03.12.2020	3 Days
End Semester Examinations	04.12.2020	19.12.2020	2 Weeks

COE **DEAN,Exams.** **DIRECTOR**

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Department of Electronics and Communication Engineering

(Accredited by NBA)

CONTROL SYSTEMS

(B.Tech. Electronics and Communication Engineering)

III B.Tech I semester

L	T	P	C
3	0	0	3

COURSE OUTCOMES:

At the end of the Course, the student will be able to:

1. Understand the modeling of linear-time-invariant systems using transfer function.
2. Analyse system response and evaluate error dynamics in time domain.
3. Understand the concept of stability and its assessment for linear-time invariant systems.
4. Design simple feedback controllers.
5. Infer the general concept of state variable, state space and analyse the stability of linear Time discrete systems

UNIT I

Introduction to control problem:

Industrial Control examples. Mathematical models of physical systems. Control hardware and their models. Transfer function models of linear time-invariant systems. Feedback Control: Open-Loop and Closed-loop systems. Benefits of Feedback. Block diagram algebra.

UNIT II:

Time response analysis of standard test signals:

Time response of first and second order systems for standard test inputs. Application of initial and final value theorem. Design specifications for second-order systems based on the time-response. Concept of Stability. Routh-Hurwitz Criteria. Relative Stability analysis. Root-Locus technique. Construction of Root-loci.

UNIT III:

Frequency-response analysis:

Relationship between time and frequency response, Polar plots, Bode plots. Nyquist stability criterion. Relative stability using Nyquist criterion – gain and phase margin. Closed-loop frequency response.

UNIT IV:

Introduction to controller design:

Stability, steady-state accuracy, transient accuracy, disturbance rejection, insensitivity and robustness of control systems. Root-loci method of feedback controller design. Design specifications in frequency-domain. Frequency-domain methods of design. Application of Proportional, Integral and Derivative Controllers, Lead and Lag compensation in designs. Analog and Digital implementation of controllers.

UNIT V:

State variable analysis and concepts of state variables:

State space model. Diagonalization of State Matrix. Solution of state equations. Eigen values and Stability Analysis. Concept of controllability and observability. Pole-placement by state feedback.

Discrete-time systems. Difference Equations. State-space models of linear discrete-time systems. Stability of linear discrete-time systems.

TEXT BOOKS:

1. Nagrath I J & Gopal M, Control Systems Engineering, New Age International, 2009.
2. Kuo B. C, Automatic Control Systems, John Wiley, 2003.

REFERENCE BOOKS:

1. Nagoorkani A, Control Systems Engineering, CBS PUB & DIST, 2020.
2. Jagan N.C, Control Systems, BS Publications, 2014.
3. Katsuhiko Ogata, Modern Control Engineering, Prentice Hall of India, 1998.

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Course outcomes

At the end of the Course, the student will be able to:

C301.1	Understand the modeling of linear-time-invariant systems using transfer function.
C301.2	Analyse system response and evaluate error dynamics in time domain.
C301.3	Understand the concept of stability and its assessment for linear-time invariant systems.
C301.4	Design simple feedback controllers.
C301.5	Infer the general concept of state variable, state space and analyse the stability of linear Time discrete systems.

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Articulation matrix of Course outcomes with PO's

A15415.CONTROL SYSTEMS ENGINEERING

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2
CO1	3	3	3	3	2	2	1	-	-	2	2	3	3	-
CO2	3	3	3	3	2	2	1	-	-	1	2	3	3	-
CO3	3	3	3	3	2	2	1	-	-	2	2	3	3	-
CO4	3	3	3	3	2	2	1	-	-	-	2	3	3	-
CO5	3	3	3	3	2	2	1	-	-	-	2	3	3	-
Average	3	3	3	3	2	2	1	-	-	1.7	2	3	3	-

HoD-ECE

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TIME TABLE

Branch: ECE **Section - A**

Day/Hours	9.00 AM to 9.55 AM	9.55 AM to 10.50 AM	10.50 AM to 11.45 PM	11.45 AM to 12.30 PM	12.30 PM to 1.25 PM	1.25 PM to 2.20 PM	2.20 PM to 3.15 PM	3.15 PM to 4.10 PM
	1	2	3		4	5	6	7
MON		CSE	CSE	LUNCH BREAK				
TUE								
WED								
THU		CSE						CSE
FRI						CSE		
SAT	CSE						CSE	

Section – B

Day/Hours	9.00 am to 9.55 am	9.55 am to 10.50 am	10.50 am to 11.45 pm	11.45 am to 12.30 pm	12.30 pm to 1.25 pm	1.25 pm to 2.20 pm	2.20 pm to 3.15 pm	3.15 pm to 4.10 pm
	1	2	3		4	5	6	7
MON				LUNCH BREAK				
TUE	CSE						CSE	CSE
WED								
THU					CSE			
FRI		CSE				CSE		
SAT						CSE		CSE

Department of Electronics and Communication Engineering

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Day/Hours	9.00 AM to 9.55 AM	9.55 AM to 10.50 AM	10.50 AM to 11.45 PM	11.45 AM to 12.30 PM	12.30 PM to 1.25 PM	1.25 PM to 2.20 PM	2.20 PM to 3.15 PM	3.15 PM to 4.10 PM
	1	2	3		4	5	6	7
MON				LUNCH BREAK				
TUE	CSE				CSE			CSE
WED								
THU	CSE					CSE		
FRI							CSE	
SAT			CSE					CSE

Section - D

Day/Hours	9.00 AM to 9.55 AM	9.55 AM to 10.50 AM	10.50 AM to 11.45 PM	11.45 AM to 12.30 PM	12.30 PM to 1.25 PM	1.25 PM to 2.20 PM	2.20 PM to 3.15 PM	3.15 PM to 4.10 PM
	1	2	3		4	5	6	7
MON				LUNCH BREAK	CSE			
TUE							CSE	CSE
WED	CSE							
THU								
FRI			CSE				CSE	
SAT			CSE					

HOD-ECE

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Lesson Plan

Name of the Faculty: K.L.LOKESH **Branch: ECE**

A.Y: 2020-21 **Section: A**

	TOPICS	**LECTURE HOURS/TEXT BOOKS**		
	Unit I : Introduction & Transfer Functions			
1.1	Introduction to Control Systems - System concept - open and closed loop	2	15	**Control Systems** – Nagoor kani **Control Systems** – A Anand Kumar.PHI. **Automatic Control Systems** – B.C.KUO. John Wiley & Sons
1.2	Differential equations and transfer functions - Introduction to Laplace transform	1		
1.3	Mechanical translational systems – Basic elements, Free Body Diagram and Transfer Function - Problems	2		
1.4	Mechanical rotational systems - Basic elements, Free Body Diagram and Transfer Function - Problems	2		
1.5	Electrical Analogous system – Force-Voltage, Force-Current, Torque-Voltage and Torque-Current analogy	1		
1.6	Block diagram representation of systems and reduction methods	1		
1.7	Determination of Closed loop transfer function using Block diagram Reduction Technique	3		
1.8	Terminologies of Signal Flow Graph and Mason's Gain Formula	1		
1.9	Determination of Closed loop transfer function using Mason's gain Formula - Problems	2		
	Unit II : Time Response Analysis			
2.1	Test signals - time response of first order systems	1	16	**Control Systems** – Nagoor kani **Control Systems** – A Anand Kumar.PHI. **Controls Systems Engineering** , S.Palani.TMH
2.2	Time response of second order systems- Different Damping conditions	1		
2.3	Time domain specifications – Rise time, Peak time, Peak Overshoot and Settling time	3		
2.4	Types and order of systems and Introduction to Steady State Error	1		
2.5	Determination of Steady State Error using Static Error Constant – Positional, Velocity and Acceleration Error Constant	2		
2.6	Determination of Steady State Error using Generalized Error Coefficient	1		

2.7	Routh-Hurwitz Criterion – Problems for different cases	3		
2.8	Root locus techniques, construction of root-loci	3		
	Unit III : Frequency Response Analysis			
3.1	Introduction to frequency response – Frequency Domain Specifications.	2	12	**Control Systems** – Nagoor kani **Control Systems** – A Anand Kumar.PHI. **Controls Systems Engineering** , S.Palani.TMH
3.2	Stability analysis using Bode plots - Determination of Gain and Phase Margin	3		
3.3	Determination of Gain and Phase Margin using Bode Plots	3		
3.4	Polar Plots, polar plots – Determination of Gain and Phase Margin	2		
3.5	Nyquist Stability Criteria	2		
	Unit IV : Introduction to controller design			
4.1	Stability ,steady state, transient accuracy robustness of control systems	2	11	**Control Systems** – Nagoor kani **Control Systems** – A Anand Kumar.PHI. **Controls Systems Engineering** .Palani.TMH
4.2	Root loci method of feedback controller design	2		
4.3	Design specifications in frequency domain	2		
4.4	Applications of proportional ,integral and derivative controllers	2		
4.5	Lead ,lag compensator in design	3		
	Unit V : State variable analysis and concepts of state variables:			
5.1	State space representation of Continuous time systems – State equations-	2	11	**Control Systems** – Nagoor kani **Control Systems** – A Anand Kumar.PHI.
5.2	Determination of State Models from Block Diagrams	2		
5.3	Diagonalization	1		
5.4	Solving Time Invariant State Equations	2		
5.5	STM & it's Properties	1		
5.6	Concept of Controllability and Observability	3		
	Total no of classes required		65	

Signature of Faculty

Signature of HOD

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Course Delivery Plan

Course Schedule

Distribution of Hours in Unit – Wise

Unit	Topic	Chapters		Total No. of Hours
		Book1	Book2	
I	Introduction to Control Systems &Transfer Function Representation	Control systems theory and applications- S.K. Bhattacharya, Pearson.	Control systems engineering-by I. J .Nagarath and M. Gopal	15
II	Time Response Analysis & Stability analysis in S-Domain	Control systems engineering-by I. J .Nagarath and M. Gopal	Control systems theory and applications- S.K. Bhattacharya	16
III	Frequency Response Analysis- Stability analysis-Frequency Domain- Classical Control Design Techniques	Control systems theory and applications- S.K. Bhattacharya	Control systems engineering-by I. J .Nagarath and M. Gopal	12
IV	**Unit IV : Introduction to controller design**	Control systems theory and applications- S.K. Bhattacharya	Control systems engineering-by I. J .Nagarath and M. Gopal	11
V	**Unit V : State variable analysis and concepts of state variables:**	Control systems engineering-by I. J .Nagarath and M. Gopal	Control systems theory and applications- S.K. Bhattacharya	11

Total contact classes for Syllabus coverage	65
Assignment Tests : 02 (Before Mid1 & Mid2 Examinations)	

Number of hours / lectures available in Semester / Year: 72

The number of topics in each unit are not the same – because of the variation, all the units have an unequal distribution of hours.

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Date of Unit completion & Remarks

Unit – I
Date: 07/08/20
Remarks: As per the plan the unit is completed
Unit – II
Date: 21/09/20
Remarks: As per the plan the unit is completed
Unit – III
Date: 04/12/2020
Remarks: As per the plan the unit is completed
Unit – IV
Date: 29/01/21
Remarks: As per the plan the unit is completed
Unit – V
Date: 16/2/2021
Remarks: As per the plan the unit is completed

HOD-ECE

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Lecture Plan

S. No.	Topic	Expected Date of Completion	Actual Date of Completion	Teaching Learning Process
UNIT I				
1	Introduction to systems, syllabus review	17/7/2020	17/7/2020	PPT
2	Concepts of control systems, open & closed loop control systems-examples	22/7/2020	22/7/2020	PPT
3	Classification of control systems	24/7/2020	24/7/2020	PPT
4	Block diagram of control systems,	27/7/2020	27/7/2020	Mind map
5	Feedback characteristics & effects of feedback on system	29/7/2020	29/7/2020	PPT
6	Impulse Response & T. F. of a Control System, Mathematical Models- D. Es	30/7/2020	30/7/2020	PPT
7	Electrical Systems –Transfer Function	31/7/2020	31/7/2020	PPT
8	Mechanical Translational & Rotational Systems	3/8/202	3/8/202	PPT
9	Block diagram of Algebra	5/8/2020	5/8/2020	PPT
10	Block diagram reduction technique	6/8/2020	6/8/2020	PPT

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	Phase Margin			
3	Determination of Gain and Phase Margin using Bode Plots	30/9/2020	30/9/2020	PPT
4	Polar Plots, polar plots – Determination of Gain and Phase Margin	5/10/2020	5/10/2020	PPT
5	Nyquist Stability Criteria	24/11/2020	24/11/2020	Think pair share
6	Introduction to frequency response – Frequency Domain Specifications.	25/11/2020	25/11/2020	PPT
7	Stability analysis using Bode plots - Determination of Gain and Phase Margin	2/12/2020	2/12/2020	PPT
8	Determination of Gain and Phase Margin using Bode Plots	4/12/2020	4/12/2020	PPT
UNIT 4				
1	Stability ,steady state, transient accuracy robustness of control systems	29/12/2020	29/12/2020	PPT
2	Root loci method of feedback controller design	30/12/2020 &4/1/2021	30/12/2020 &4/1/2021	PPT
3	Design specifications in frequency domain	5,8,11/1/2021	5,8,11/1/2021	PPT
4	Applications of proportional ,integral and derivative controllers	12,18,19/1/2021	12,18,19/1/2021	PPT
5	Lead ,lag compensator in design	22,25,29/1/2021	22,25,29/1/2021	PPT
UNIT-V				
1	Concepts of state, State variables and state model	22/2/2021	22/2/2021	Chalk and Board
2	Realization of state models from	5/2/2021	5/2/2021	Chalk and

	signal flow graph & block diagrams			Board
3	Diagonalisation	6/2/2021	6/2/2021	Chalk and Board
4	Solving the time invariant state equation	9/2/2021	9/2/2021	Chalk and Board
5	State transition Matrix & it's properties	11/2/2021	11/2/2021	Chalk and Board
6	Transfer Function From State Models	13/2/2021	13/2/2021	Chalk and Board
7	Concepts of Controllability & Observability	16/2/2021	16/2/2021	Chalk and Board

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Student Roll list

Section A

SUBJECT		
S. NO.	ROLL NO.	NAME OF THE STUDENT
1	18911A0401	AKULA NAGENDER
2	18911A0402	ALURI SHRAVANI
3	18911A0403	AMMANNAGARI SRIVANI
4	18911A0404	ANJURI DURGA RAM PRASAD
5	18911A0405	ANNAMANENI DRUVI
6	18911A0406	ARPULA PRAKASH
7	18911A0407	BAIRAPOGU RANJITH KUMAR
8	18911A0408	BALA SHRAVYA DEVARAPU
9	18911A0409	BOGGULA NARENDER REDDY
10	18911A0410	CHAPPARAPU ANJANI SRINITHA
11	18911A0411	CHINTABATHINA RAHUL
12	18911A0412	DAMMALAPATI NIVED KUMAR
13	18911A0413	DUDYALA VINOD KUMAR
14	18911A0414	ETACHETTU VENKATESH GOUD
15	18911A0415	GADE KARTHIK
16	18911A0416	GOLLAMUDI SAI LIKHITHA
17	18911A0417	GONNURU AKHILA SAI
18	18911A0418	JAMPANI AMARESH
19	18911A0419	JAVVAJI VISHNU KUMAR
20	18911A0420	K KOTESWARA RAO
21	18911A0421	K RADHAKRISHNA
22	18911A0422	KAMMARI RAVI TEJA
23	18911A0423	KANKATA ROHAN SETH
24	18911A0424	KANNAM VENKATA SAI
25	18911A0425	KOTHAKAPU SHASHIVARDHAN REDDY
26	18911A0426	KURUVA NIHARIKA
27	18911A0427	KYATHAM SANDEEP YADAV
28	18911A0428	LAKNAPURAM PRANEETH KUMAR REDDY
29	18911A0429	M RAKESH

30	18911A0430	MACHA VINAY KUMAR
31	18911A0431	MADHAM SHETTY MEGHANA
32	18911A0432	MADIHA FAREHA FAIYYAZ
33	18911A0433	MADURAI HIRANMEIH
34	18911A0434	MAHAJAN SHARANMAI
35	18911A0435	MALAPATI GREESHMANTH REDDY
36	18911A0436	MANGIPUDI ROHIT
37	18911A0437	MARIYALA BHARADWAJ
38	18911A0438	MASIPEDDI VIKRAM RAO
39	18911A0439	MENDE SRAVAN KUMAR YADAV
40	18911A0440	MOHD ZEESHAN KAREEM
41	18911A0441	MUNAGALA VARUN
42	18911A0442	NAGABHAIRAVA SAKETH
43	18911A0443	NAKKA RANJITH KUMAR
44	18911A0444	NIKHIL TEJA MANDAN
45	18911A0445	P AKHILESH REDDY
46	18911A0446	P SAI SUDHEER CHARY
47	18911A0447	PAKANATI SHARATH KUMAR
48	18911A0448	PAPAGARI SANGEETHA
49	18911A0449	RAGHUPATHI PRIYANKA
50	18911A0450	RAVULAKOLA LAVANYA
51	18911A0451	S ABHISHEK GOUD
52	18911A0452	SAYAM SHRAVIKA
53	18911A0453	SINDHUJA GALLA
54	18911A0454	SUGALI HEMANTH KUMAR NAIK
55	18911A0455	TALLURI PARVASH CHOUDHARY
56	18911A0456	TANKALA LEELA PRAKASH
57	18911A0457	TANNIRU BHARATH KUMAR
58	18911A0458	TATTIKOTA EASWARI LAKSHMI SAI EAPSITA
59	18911A0459	UPPULA AJAY
60	18911A0460	VINAYAKA SANJANA
61	19915A0401	AVUSULA NIKHIL KUMAR
62	19915A0402	AYELA NIVEDITHA
63	19915A0403	BANDI RESHMA
64	19915A0404	CHILUKURI SPANDANA

KY

65	19915A0405	DASARI CHAITHANYA
66	19915A0406	DHARAMKAR SHASHANK
67	17911A04D5	CHERUKU OM PRAKASH (Re-Admn dt 17/12/2018)
68	**17911A0447**	**RACHAMALLA NIKITHA (Re-Admn dt 16/07/2020)**
69	**16911A04A6**	**RUNVAL ARAVIND (Re-Admn dt 20/07/2020)**

Section B

S. NO.	ROLL NO.	NAME OF THE STUDENT
1	18911A0461	A ABHINAVA SRINIVAS
2	18911A0462	A DEEKSHITHA
3	18911A0463	AENUGU DEEKSHITHA
4	18911A0464	AKULA SIRISHA
5	18911A0465	ALLIBILLI BHARGAVI
6	18911A0467	ARDHA PAVAN KALYAN REDDY
7	18911A0468	BADA SAI KALYAN
8	18911A0469	BANTU RISHITHA
9	18911A0470	CHAGARLAMUDI ANVESH
10	18911A0471	CHALLA DINESH YADAV
11	18911A0472	CHERUPALLI NAVYA
12	18911A0473	CHEVELLA ASHWINI
13	18911A0474	CHITUKULURI HARSHITHA REDDY
14	18911A0475	DUGYALA MANOJ KUMAR
15	18911A0476	GANTI SAI DIVYA
16	18911A0478	GOPI KEERTHANA
17	18911A0479	JAMBULA GAYATHRI
18	18911A0481	JELLA RAVI TEJA
19	18911A0482	KANAPURAM KRISHNAVENI
20	18911A0483	KINDODDI NANI
21	18911A0484	KUMMARI RAKESH KUMAR
22	18911A0485	KUPPAM BHARGAV
23	18911A0486	MADDELA SHIVANI GOUD
24	18911A0487	MALREDDY SRICHARAN REDDY
25	18911A0488	MANCHU SUNATH
26	18911A0489	MASAGAARI SAI KUMAR REDDY
27	18911A0490	MATTA DIVYA EVANGILIN

KY

28	18911A0491	MD NISSAR SAMEER
29	18911A0492	METHIRAJULA KARTHIKEYANI
30	18911A0494	MOHAMMED EBAD ROOSHAN
31	18911A0496	MORAMPUDI SANDEEP
32	18911A0497	MUNGI MANISH REDDY
33	18911A0498	MUSHNAM KALYAN
34	18911A0499	MUTHYALA KAVYA
35	18911A04A0	MUTUKULLOJU LIKITHA
36	18911A04A1	NIMISHAKAVI VAMSHI KRISHNA
37	18911A04A2	P VARUN TEJA
38	18911A04A3	PETTEM ARCHANA
39	18911A04A4	PULIGARI SAI CHARAN REDDY
40	18911A04A5	PUTTY KRANTHI
41	18911A04A6	RUPAVATH NIKHIL
42	18911A04A7	SAMA MADHUSUDHAN REDDY
43	18911A04A8	SANDRA SANDEEP KUMAR
44	18911A04A9	SEELAM NIHARIKA
45	18911A04B0	SUDHA RAJIVMANIKANTA
46	18911A04B1	TADESENA PRAVALIKA
47	18911A04B2	TANAJI AKHILA
48	18911A04B3	TATIKONDA ANVITHA
49	18911A04B4	THAKUR AKANSHA
50	18911A04B5	VANKADOTH ANIL
51	18911A04B6	VEERAVENI KRANTHI KIRAN
52	18911A04B7	VEMULA HARSHITH
53	18911A04B8	VUGRANAM CHAKRAVARTHY ARCHANA
54	18911A04B9	YALALLA SAINATH REDDY
55	18911A04C0	YELKA DINESH REDDY
56	19915A0407	DURGAPPAGARI VAISHNAVI
57	19915A0408	GAJJALA KIRAN KUMAR YADAV
58	19915A0409	GANGULA SAI KUMAR REDDY
59	19915A0410	GORTHA SRIJA
60	19915A0411	JAKKULA DHANRAJU
61	19915A0412	K BHARGAVI
62	17911A04A1	PULAPALLI AKHIL (Re-Admn dt 18/06/2019)

KY

63	17911A04A6	SANDIRI SAI TEJA (Re-Admn dt 25/06/2019)
64	**17911A04P3**	**TANKARI MANI VARDHAN (Re-Admn dt 16/07/2020)**
65	**16911A0484**	**MALOTHU GOPIBABU (Re-Admn dt 17/07/2020)**
66	**16911A0485**	**MALUGARI PRAVEEN REDDY (Re-Admn dt 16/07/2020)**
67	**17911A0460**	**YESHWANTH REDDY K (Re-Admn dt 19/07/2020)**
68	**17911A0497**	**ODURU BHARGAV KUMAR REDDY (Re-Admn dt 20/07/2020)**
69	**17911A0490**	**KUNDURU AJITH KUMAR (Re-Admn dt 21/07/2020)**

Section C

SUBJECT		
S. NO.	**ROLL NO.**	**NAME OF THE STUDENT**
1	18911A04C1	ACHARY BHASWANTH
2	18911A04C2	ADLA RAJASRI
3	18911A04C3	AEDIRA VARSHINI
4	18911A04C5	ANJALI GARG
5	18911A04C6	BANDAMEEDI SRIKANTH
6	18911A04C7	BHAIRI DHARANI
7	18911A04C8	BODDU SAI CCHARVI REDDI
8	18911A04C9	BODDU SANDEEP
9	18911A04D0	BYAGARI RAKESH
10	18911A04D1	CHITHANOORI MANIKESHWAR
11	18911A04D2	DASARI DINESH
12	18911A04D3	DINGARI SRIHARSHINI
13	18911A04D4	EDULAPALLY KOUSHIK
14	18911A04D5	FURQUAN AHMED DANISH
15	18911A04D6	G VIJAY
16	18911A04D7	GADDAMIDI SAI PRASHANTH REDDY
17	18911A04D8	GOLLA CHANDI PRIYA
18	18911A04D9	GOLLA SRIKANTH
19	18911A04E0	GUNDEPUDI PRUDHVIRAJ
20	18911A04E1	J SUNITHA
21	18911A04E2	JERRIPOTHU VAMSHI BABU
22	18911A04E3	JIGIDARLA BHAVANI
23	18911A04E4	K SAI TEJA
24	18911A04E5	K SREE SAI CHAITANYA

25	18911A04E6	KAMBALA VENKATA MAHESH
26	18911A04E8	KAVALA BHANU TEJA V S N SAI
27	18911A04E9	KORUKONDA MANILA
28	18911A04F0	KOSGI NANDINI
29	18911A04F1	KOTHAKAPU DEEKSHITH REDDY
30	18911A04F2	MADHURI KODIYALAM
31	18911A04F3	MALREDDY SHIVANAND REDDY
32	18911A04F4	MANDE RAVI TEJA
33	18911A04F5	MANDUMULA AKHILA
34	18911A04F6	MAREPALLY RAKESH
35	18911A04F7	MEKALA NAVYA
36	18911A04F8	MOHAMMED FAHAD MEHRAJ
37	18911A04F9	MOHD ASLAM MOHIUDDIN
38	18911A04G0	NAYAKAM AISHWARYA
39	18911A04G1	NEHA K
40	18911A04G2	NUKA RUSHMITHA
41	18911A04G3	NUNE SAKETH
42	18911A04G4	P DEEKSHITHA
43	18911A04G5	P VARUN KUMAR
44	18911A04G6	PATLOLLA CHANDRA SHEKAR REDDY
45	18911A04G7	POTHIREDDYPALLI VENKATA SRI HARSHA
46	18911A04G8	RAVIKANTI SRAVYA
47	18911A04G9	REPAKA BHOOMIKA
48	18911A04H0	SAICHARAN PARASHARAM
49	18911A04H1	SALIBINDLA VINEETH REDDY
50	18911A04H2	SALOLLA VIVEKANANDA
51	18911A04H3	SANDEPAGU VIJAY KUMAR
52	18911A04H4	SANGEM LOKESH
53	18911A04H5	THOTA ROHITH
54	18911A04H6	THUMMALABAVI SANKSHAY REDDY
55	18911A04H7	U.GOPI PRIYA
56	18911A04H8	YEDDANAPUDI VARUN
57	18911A04H9	YERRA RUCHITHA
58	18911A04J0	YERRAMSETTI MANIPRIYA
59	19915A0413	KORRA VEERABHADRU

KU

60	19915A0414	MADDI SHIVA PRIYA
61	19915A0415	MADUGULA SRIVANI
62	19915A0416	MAHIMA SINGH
63	19915A0417	MANASA VEENA D
64	19915A0418	MEDI MOHAN SAI
65	19915A0421	NEELI SRAVANI SANDHYA
66	17911A04L1	KASHAPOGU RAHUL SMITH (Re. Admt. 18/12/2019)
67	**17911A0489**	**KOTHAKAPU VAMSHIDHAR REDDY (Re-Admn dt 16/07/2020)**
68	**17911A0476**	**GANGISHETTY SAI MANISH CHANDRA (Re-Admn dt 16/07/2020)**
69	**17911A04P2**	**SULUGE SRINATH (Re-Admn dt 16/07/2020)**

Section D

S. NO.	ROLL NO.	NAME OF THE STUDENT
1	18911A04J1	A HARSHITA
2	18911A04J2	A VISHNU VARDHAN REDDY
3	18911A04J3	AVVALDAR VEENITH
4	18911A04J4	BADAVATH KALYAN
5	18911A04J5	BALA DUSHYANTH
6	18911A04J6	BANDULA SHARMILA
7	18911A04J7	BOORA SAI CHARAN
8	18911A04J8	BUKYA RAHUL
9	18911A04J9	CHALUVADI SAKETH
10	18911A04K0	CHAMANTHI AKSHITH
11	18911A04K1	CHANDINI TIRUPATI MAMIDWAR
12	18911A04K2	CHORAGUDI YAMINI SHRIYA
13	18911A04K3	CPJ KEERTHANA
14	18911A04K5	DUDYALA ANAND KUMAR
15	18911A04K6	EEDIGI PRATHYUSHA
16	18911A04K7	G USHA RAKSHITHA
17	18911A04K8	GOWLIKAR ABHISHEK
18	18911A04K9	GUGULOTH PREM CHAND
19	18911A04L0	GUNDLA AKARSHA
20	18911A04L1	GUNNE NIKHIL
21	18911A04L2	GUNTALA SHIRISHA
22	18911A04L3	GURRAM PUSHPA LATHA

23	18911A04L4	INALA NIKHIL SREEVEN
24	18911A04L5	JARA MADHUSUDHAN REDDY
25	18911A04L6	JARUPLA MANOJ KUMAR
26	18911A04L7	K V N P S R DATTATREYA
27	18911A04L8	KALAL ABHILASH
28	18911A04M0	KOTAGALLA REVANTH
29	18911A04M1	KUNDETI SAI PRATYUSHA
30	18911A04M2	MACHARLA POSHETTY VISHNU VARDHAN
31	18911A04M3	MADABHUSHI TIRUMALA ADARSH RAGHAVAN
32	18911A04M4	MANNE AKSHAYA
33	18911A04M5	MANPATI RAVALI
34	18911A04M6	MARADANI PRAVALLIKA
35	18911A04M7	MOHAMMAD ABDUL AHAD
36	18911A04M8	NANNURI KOWSALYA CHOWDARY
37	18911A04M9	PANJALA SAI KUMAR
38	18911A04N0	PATAKOTA VENKATA RAMI REDDY
39	18911A04N1	PEDIREDLA MEGHANA
40	18911A04N2	POLAMURI MANIMALA
41	18911A04N3	POTHULA VENKATA SAI KRITHIK
42	18911A04N4	PUPPALA SAI SRUTHI
43	18911A04N5	RAMACHANDROJU KALYAN KUMAR
44	18911A04N6	RENTALA AKHILA
45	18911A04N7	SADA MAHENDER
46	18911A04N8	SAMA VINEETH REDDY
47	18911A04N9	SHAIK ABDUL REHMAN AHMED
48	18911A04P0	SHETTUKADI SOWMYA
49	18911A04P1	SIMHARAJU SRIKAR KARTHIKEYA
50	18911A04P2	SIRIKONDA SAIRAM
51	18911A04P3	SIRIPIREDDY LOKESH REDDY
52	18911A04P4	TALARI VAMSHI
53	18911A04P5	THOTA AKHIL
54	18911A04P6	THOTA SRI RATNANJALI
55	18911A04P7	U THIRUPATHI
56	18911A04P8	V CHINMAYEE SRICHANDANA
57	18911A04P9	V VARSHA

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58	18911A04Q0	YADAVALI SHANTI
59	19915A0419	MOHAMMAD ABDUL TARANNUM TABASSUM
60	19915A0420	MOTHAE KIRAN
61	19915A0422	PAKALA RAJINI
62	19915A0423	TEDDU LAXMI
63	19915A0424	U VISHWANTH
64	**17911A04H2**	**SINGAM RAO KARTHIK YADAV (Re-Admn dt 16/07/2020)**
65	**17911A04H4**	**TANGIRALA DINESH (Re-Admn dt 16/07/2020)**
66	**18915A0410**	**SARVIGARI UPENDER REDDY (Re-Admn dt 20/07/2020)**
67	**17911A04F9**	**MASIPEDDI ADITYA RAO (Re-Admn dt 16/07/2020)**
68	**18915A0427**	**NITHIN STEEPHEN PATHI (Re-Admn dt 16/07/2020)**
69	**18915A0413**	**MUNJAGALLA SOUSALYA (Re-Admn dt 27/07/2020)**

HOD-ECE

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Blooms Taxonomy Direct

Level 1	**Remembering**	Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers.
Level 2	**Understanding**	Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas.
Level 3	**Applying**	Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.
Level 4	**Analyzing**	Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations.
Level 5	**Evaluating**	Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria.
Level 6	**Creating**	Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.

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Unit Wise Assignments (With different Levels of thinking – Blooms Taxonomy and Course Outcomes)

Assignementt – I		**PO**	**B L**	**C O**
1	Differentiate Open loop control System with Closed loop control System?	1,5,12	2	1
2	Compare Block Diagram Reduction technique and Signal flow graph approach?	1,3,5,12	4	1
3	Describe about the translational and rotational mechanical systems each with one example?	1,5,12	1	1,2
4	Derive the step response of second order under damped system?	1,3,5,12	4	3
Assigment-2				
1	Derive the time response of First order system for different test inputs?	1,5,12	4	3
2	Discuss the limitations of RH criterion?	1,3,5,12	2	4
3	Determine Phase Margin and Gain margin from a BODE diagram?	1,5,12	4	4
4	Obtain the transfer function from a State model?	1,3,5,12	4	2

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(Aziz Nagar, C.B.Post, Hyderabad -500075)

III B.Tech I Semester II Mid Examination, February -2021

Subject Name: CONTROL SYSTEMS
Time: 90 Minutes **BRANCH: ECE**
Max Marks: 20

Bloom's Level:

Remember	L1
Understand	L2
Apply	L3
Analyze	L4
Evaluate	L5
Create	L6

Q.No.	PART-A	BL	CO	PO	Marks
ANSWER ALL THE QUESTIONS	**(2Q x 3M = 6M)**				
1	What is Bode plot, List the advantages of Bode plot?	L1	CO4	1,2,3	2M
2	Explain the importance of controllability and observability of the control system model in the design of the control system	L2	C05	1,2,3	2M
3.a)	Define Gain Margin and phase margin?	L1	CO4	1,2,3	1M
b)	Draw the block model of state equation	L1	CO5	1,2,3	1M
	PART-B				
ANSWER ALL THE QUESTIONS					
4. i	Draw the Nyquist plot for the system whose open loop transfer function is G(S) H(S) =K / S (S+2) (S+10).	L4	CO3	1,2,3	4M
	(OR)				
4. ii	The open loop transfer function of a unity feedback system is G(S) = 1/ S (1+S) (1+2S). Sketch the Polar plot and determine the Gain margin and Phase margin.	L4	CO3	1,2,3	4M
5 i..	Obtain the transfer function of lag and lead networks .locate their poles and zeros and write their advantages.	L3	CO4	1,2,3	5M
	(OR)				
ii	Plot the Bode diagram for the following transfer function and obtain the gain and phase cross over frequencies G(S) =KS^2 / (1+0.2S) (1+0.02S).Determine the value of K for a gain cross over frequency of 20 rad/sec.	L4	CO4	1,2,3	5M
6. i)	Discuss the advantage of state space techniques over the transfer function techniques of analyzing the control system	L3	CO5	1,2,3	5M
	(OR)				
ii)	Given the transfer function of a system, determine a state variable representation for the system Y(S) / U(S) = 10(S+4)/S(S+1) (S+3)	L3	CO5	1,2,3	5M

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VJIT(A)

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(Aziz Nagar, C.B.Post, Hyderabad -500075)

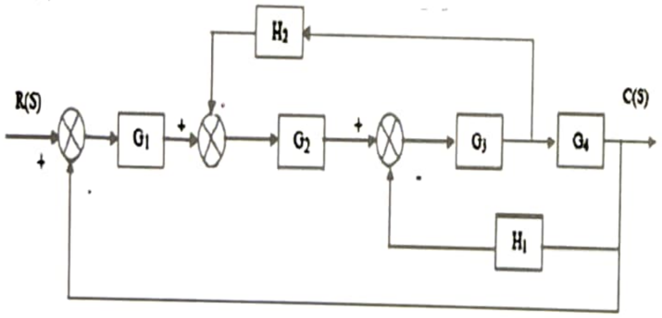
III B.Tech I Semester II Mid Examination, November -2020

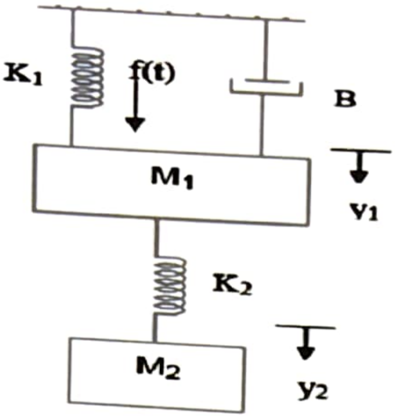
Subject Name: CONTROL SYSTEMS
Time: 90 Minutes

BRANCH: ECE
Max Marks: 20

Bloom's Level:

Remember	L1
Understand	L2
Apply	L3
Analyze	L4
Evaluate	L5
Create	L6

Q.No.	PART-A	BL	CO	PO	Marks
ANSWER ALL THE QUESTIONS	**(2Q x 3M = 6M)**				
1	Compare open loop and closed loop systems	L2	CO 1	1,2,3	2M
2	Define steady state error constants and obtain the expression for them.	L1	CO 2	1,2,3	2M
3.a)	Give the mason's gain formula.		CO 1	1,2,3	1M
b)	Define settling time.		CO 2	1,2,3	1M
	PART-B				
ANSWER ALL THE QUESTIONS	**(2Q x 7M=14M)**				
4. i	Determine the overall transfer function C(S)/R(S) for the system shown in fig. 	L3	CO 1	1,2,3	5M
	(OR)				

4. ii	Determine the transfer function $Y_2(s)/F(s))$ for the system shown in fig.  	L3	CO 1	1,2,3	5M
5 i.a).	Test the stability of the system with transfer function **$C(S)/R(S)=10/S^5+2S^4+3S^3+6S^2+5S+3$**	L4	CO 2	1,2,3	3M
b)	Obtain time domain specifications for the given system with unity feedback and **$G(s)=1/S(S+1)$**	L4	CO 2	1,2,3	2M
	(OR)				
ii	Sketch root locus diagram for the given unity feedback system with **$G(S)= k / S(S+1)(S+2)$.**	L4	CO 2	1,2,3	5M
6. i)	Plot the Bode diagram for the following transfer function and obtain the gain and phase cross over frequencies. $G(S) = 10/ S (1+0.4S) (1+0.1S)$	L4	CO 3	1,2,3	4M
ii)	The open loop transfer function of a unity feedback system is $G(S) = 1/ S (1+S) (1+2S)$. Sketch the Polar plot	L4	CO 3	1,2,3	4M

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Department of Electronics and Communication Engineering
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Vidya Jyothi Institute of Technology (Autonomous) **Subject Code:A15415**	R15

B.Tech. III Year I Semester Examination NOVEMBER-2019

SUBJECT NAME:Control System Engineering **BRANCH : ECE**

Time: 3 Hours **Max. Marks:75**

Note: This question paper contains two *Parts A and B.*

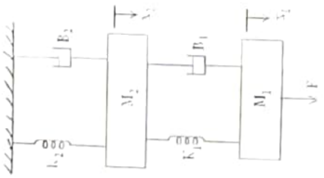
Part A is compulsory which carries 25 Marks. Answer all the questions

Part B consists of 5 questions. Answer all the questions

Bloom's Level:

Remember	L1	Analyze	L4
Understand	L2	Evaluate	L5
Apply	L3	Create	L6

	PART - A **ANSWER ALL THE QUESTIONS**	**Bloom's Level**	**25Marks**
1	Write any two properties of signal flow graph.	L4	2M
2	Write the analogous electrical elements in force voltage analogy for the elements of mechanical translational system	L2	3M
3	What are static error coefficients?	L1	2M
4	Define type and order of the system.	L1	3M
5	Find the phase angle of the transfer function G(s) = KS [illegible]	L2	2M
6	Determine the stability of the system which is represented by characteristic equation $S^4+8S^3+18S^2+16S+5=0$.	L3	3M
7	What are the advantages of Frequency Response Analysis?	L1	2M
8	Draw the Bode plot of a typical lag-lead compensator.	L5	3M
9	State the conditions of controllability in terms of the matrices A and B.	L1	2M
10	Define Phase and Gain cross-over frequency.	L6	3M

	PART - B **ANSWER ALL THE QUESTIONS**	**Bloom's Level**	**50Marks**
11.i	Derive the transfer function armature controlled DC servo motor and draw its block diagram.	L1	10M
	[OR]		
ii.a)	Compare open loop and closed loop systems.	L1	4M
b)	Write the Differential equation for the given mechanical system and obtain the Equivalent Force-Voltage Analogous system  	L4	6M
12.i.	Find the delay time, rise time, peak time, settling time and peak overshoot for unity feedback system with open loop transfer function $G(S) = \frac{16}{S(S-6)}$	L5	10M
	[OR]		
ii.	Find out the output of the undamped second order system when the input applied to the system is unit step input.	L3	10M

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13.i	Sketch the Root locus for the following T.F and find range of 'K' for system to be stable $G(S)H(S) = \dfrac{K}{S(S-4)(S-11)}$	L2	10M
	[OR]		
ii.	Explain the effects of adding poles and zeros to G(s)H(s) on the root loci by considering with an example.	L5	10M
14.i.	Sketch the bode plot for the given system whose T.F is given by $G(s)H(s)=\dfrac{K}{S(S+2)(S+10)}$	L3	10M
	a).Find gain margin b) Find the phase margin		
	[OR]		
ii.a)	Construct the complete Nyquist plot for a unity feedback control system whose open loop transfer function is $G(s)H(s)=\dfrac{100}{s(s+1)(s+2)}$	L4	10M
15.i.	Construct the state model for a system characterized the differential equation $\dfrac{d^3y}{dt^3}+6\dfrac{d^2y}{dt^2}+11\dfrac{dy}{dt}+6y=u$	L3	10M
	[OR]		
ii.	Determine the state controllability and observability of the system described by $\dot{x}(t)=\begin{vmatrix}-3 & 1 & 1\\ -1 & 0 & 1\\ 0 & 0 & 1\end{vmatrix}x(t)+\begin{vmatrix}0 & 1\\ 0 & 0\\ 2 & 1\end{vmatrix}u(t)$ $y(t)=\begin{bmatrix}0 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}x(t)$	L2	10M

VJIT(A)

Key

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Department of Electronics and Communication Engineering

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Content Beyond Syllabus MAPPING WITH Pos and PSOs

S. No	Name of the Topic	POs	PSO
1	NICOLS CHART	1,5,12	2
2	MATLAB SIMULATIONS	1,2,3,5,12	2

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Teaching Learning Methods

1. Black Board Teaching
2. Power point presentation
3. Mind map.
4. Think-Pair-Share
5. Brown bag

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Innovative/ Student Centric Teaching Method Form

Faculty Name: K.L.LOKESH

Course: B.Tech

Subject: Control Systems

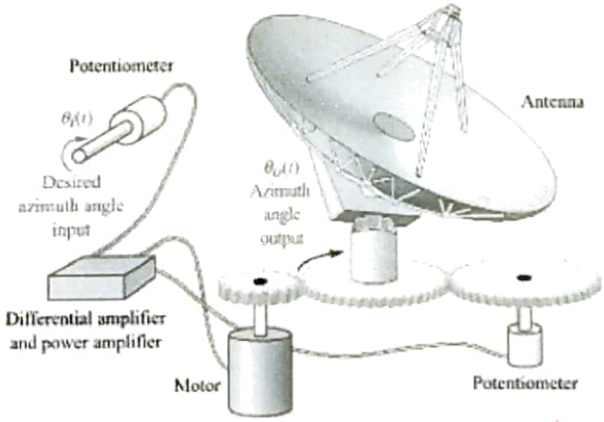
Class-Section: II-C

AY: 2020-21

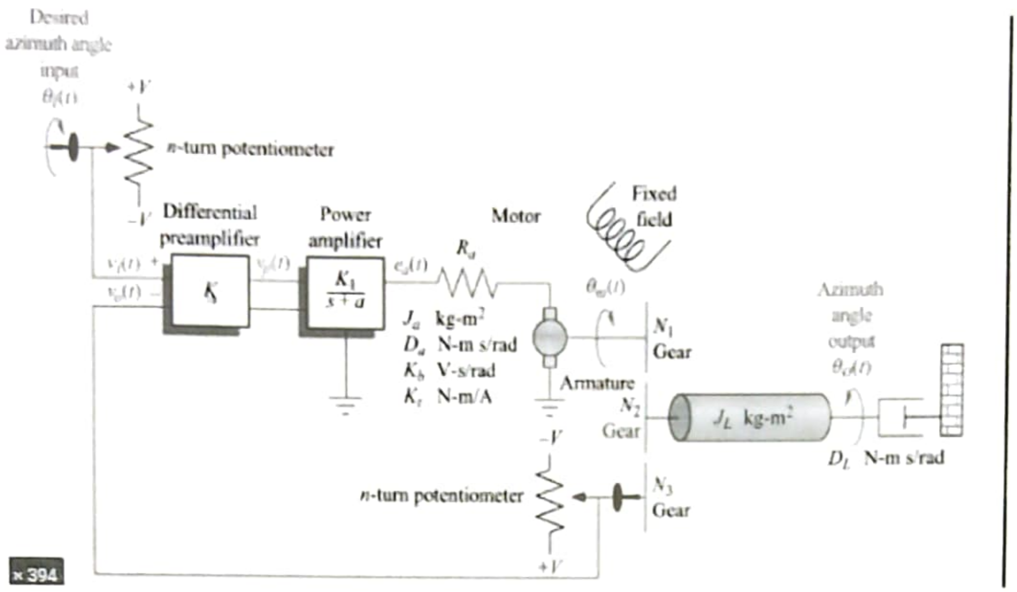
Mode of Innovative Teaching Mode: Mind Map

Description about the mode: Mind Maps can be used in class to brainstorm and generate discussions. This involves use of notes with keywords and images in classroom teaching.

Implementation: Following images is shown to students and they are asked to discuss it among themselves.



Topic Handled: Modelling Antenna Azimuth Control System



Outcome of the teaching mode:

This will encourage students not only to participate but also to fully understand a topic and its nuances by creating connections between ideas. This makes students remember the topic for a longer time.

Signature of Faculty

HOD-ECE

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Innovative/ Student Centric Teaching Method Form

Faculty Name: K.L.LOKESH

Course: B.Tech

Subject: Control Systems

Class-Section: II-D

AY: 2020-21

Mode of Innovative Teaching Mode: Think-pair-share

Description about the mode:

Think-pair-share (TPS) is a collaborative learning strategy where students work together to solve a problem or answer a question about an assigned reading.

Implementation: **Implementation**: In the classroom faculty has to give one question to the student and each student has to solve the question individually and after completion of solving the question student has to pair with one of his/her classmate and discuss each answer. Then one of the students in the class shares his/her thoughts with total class. This strategy requires students to think individually about a topic or answer to a question and share ideas with classmates.

Topic Handled: NYQUIST PLOT

Outcome of the teaching mode:

This strategy improves collaborative thinking enhances collective skills.

Signature of Faculty **HOD-ECE**

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Teaching Learning Methods

1. Black Board Teaching
2. Power point presentation
3. Flipped classroom
4. Think-Pair-Share
5. Brown bag

KH

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DIRECT ATTAINMENT OF THE COURSE

S.No	Reg.No	MID I Threshold 60%								MID II Threshold 60%								Threshold 60% (45M)
		ASM - I (5)	PART-A				PART-B			ASM - II (5)	PART-A				PART-B			
			Q1(2M)CO1	Q2(2M)CO2	Q3 A (1M)CO1	Q3 B (1M)CO2	Q4(5M)CO1	Q5(5M) CO2	Q6(4M)CO3		Q1(2M)CO4	Q2(2M)CO5	Q3 A (1M) CO4	Q3 B (1M) CO5	Q4(4M)CO3	Q5 (5M) CO4	Q6(5M)CO5	End Exam (75M)
1	16911A0484	5								5	1	1	1					7
2	16911A0485	5								5								A
3	16911A04A6	5								5								A
4	17911A0447	5	1	1	1	1	4	2	1	5								13
5	17911A0460	5								5								A
6	17911A0476	5								5								A
7	17911A0489	5								5								22
8	17911A0490	5								5								45
9	17911A0497	5								5								51
10	17911A04A1	5	1	1	1					5	1	1	1	1	1	1		20
11	17911A04A6	5	1	1						5	1	1	1	1	2	2	1	45
12	17911A04D5	5								5	1	1	1	1	1	1	1	A
13	17911A04F9	5								5	1	1	1	1	2	3	2	49
14	17911A04H2	5								5								A
15	17911A04H4	5								5								6
16	17911A04L1	5								5								16
17	17911A04P2	5								5	1	1	1	1	1	1	1	38
18	17911A04P3	5								5	1	1						24

19	18911A0401	5	2	2	1	1	2	2	2	5	1							34
20	18911A0402	5	2	1	1	1	5	5	4	5	1	1	1	1	2	5	5	49
21	18911A0403	5	2	2	1	1	5	3	4	5	1	1	1	1	4	2	5	45
22	18911A0404	5	2	2	1	1	4	3	2	5	1	1	1	1	4	2	4	37
23	18911A0405	5	2	1	1	1	5	5	4	5	2	1	1	1	4	5	5	46
24	18911A0406	5	1	1	1	1	1	1	1	5								15
25	18911A0407	5	2	2	1	1	4	3	2	5	1	1	1	1	1	1	1	11
26	18911A0408	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	47
27	18911A0409	5	2	2	1	1	4	3	3	5	1	1	1	1	4	2	5	44
28	18911A0410	5	2	2	1	1	5	3	4	5	1	1	1	1	2	4	2	39
29	18911A0411	5								5	1	1	1	1	2	1	1	37
30	18911A0412	5	1	1	1	1	4	2	1	5	1	1	1	1	4	5	5	63
31	18911A0413	5	1	1	1	1	2	2	2	5	1	1	1	1	2	4	2	40
32	18911A0414	5	2	2	1	1	4	2	1	5	1	1	1	1	2	3	2	48
33	18911A0415	5	2	2	1	1	4	3	2	5	1	1	1	1	4	2	3	43
34	18911A0416	5	2	2	1	1	4	4	3	5	1	1	1	1	4	2	5	39
35	18911A0417	5	2	2	1	1	4	2	1	5	1	1	1	1	4	2	3	41
36	18911A0418	5	2	2	1	1	5	5	4	5	1	1	1	1	2	1	1	61
37	18911A0419	5	2	2	1	1	5	3	4	5	1	1	1	1	2	4	2	41
38	18911A0420	5								5	1	1	1		1	1		51
39	18911A0421	5	2	2	1	1	4	3	2	5	1	1	1	1	2	3	2	34
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44	18911A0426	5	2	2	1	1	5	5	4	5	1	1	1	1	4	4	5	43

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45	18911A0427	5								5	1	1	1					46
46	18911A0428	5								5								48
47	18911A0429	5	2	1	1	1	5	5	4	5	2	1	1	1	4	5	5	42
48	18911A0430	5	2	2	1	1	5	5	4	5	1	1	1	1	4	5	5	68
49	18911A0431	5	2	2	1	1	5	5	4	5	1	1	1	1	4	5	5	62
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54	18911A0436	5	2	2	1	1	5	3	4	5	1	1	1	1	2	3	1	42
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75	18911A0457	5	2	2	1	1	5	3	4	5	1	1	1	1	4	2	5	51
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79	18911A0461	5	1	1	1	1				5	1	1	1	1	1	1	1	36
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111	18911A0498	5	1	1	1	1	1	1		5	1	1	1	1	4	2	3	45
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113	18911A04A0	5	2	2	1	1	4	2	2	5	1	1	1	1	2	3	1	40
114	18911A04A1	5	2	2	1	1	5	3	4	5	1	1	1	1	2	5	5	59
115	18911A04A2	5	2	2	1	1	4	4	3	5	1	1	1	1	2	4	2	52
116	18911A04A3	5	2	2	1	1	5	5	4	5	1	1	1	1	4	4	5	45
117	18911A04A4	5	2	2	1	1	4	4	3	5	1	1	1	1	4	2	4	56
118	18911A04A5	5	1	1	1	1				5	1	1						51
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120	18911A04A7	5	1	1	1	1	4	2	1	5	1	1	1	1	1	1		37
121	18911A04A8	5	2	2	1	1	4	4	3	5	1	1	1					39
122	18911A04A9	5	2	2	1	1	5	5	4	5	1	1	1	1	4	2	3	64

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123	18911A04B0	5	1	1	1	1	4	2	1	5	1	1	1	1	4	2	3	37
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126	18911A04B3	5	2	2	1	1	4	4	3	5	1	1	1	1	4	4	5	49
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129	18911A04B6	5	2	2	1	1	5	5	4	5	1	1	1	1	1	1	1	39
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133	18911A04C0	5								5								3
134	18911A04C1	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	45
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142	18911A04D0	5	1	1	1	1	1	1	1	5	1	1	1	1	1	1	1	6
143	18911A04D1	5	2	2	1	1	2	2	2	5	1	1	1	1	2	3	1	34
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151	18911A04D9	5	2	2	1	1	4	2	2	5	2	2	1	1	4	5	5	49
152	18911A04E0	5	2	2	1	1	5	3	4	5	2	1	1	1	4	5	5	46
153	18911A04E1	5								5	2	1	1	1	4	5	5	48
154	18911A04E2	5	2	2	1	1	4	2	2	5	1	1	1	1	2	2	1	3
155	18911A04E3	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	59
156	18911A04E4	5	2	2	1	1	4	3	3	5	1	1	1	1	4	2	3	48
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158	18911A04E6	5	1	1	1	1	2	2	2	5	1	1	1	1	2	3	1	49
159	18911A04E8	5	2	2	1	1	5	3	4	5	2	1	1	1	4	5	5	43
160	18911A04E9	5								5	1	1	1	1	4	4	5	42
161	18911A04F0	5	2	2	1	1	5	5	4	5	1	1	1	1	4	5	5	43
162	18911A04F1	5	2	2	1	1	4	2	2	5	1	1	1	1	4	2	4	47
163	18911A04F2	5	2	2	1	1	5	5	4	5	1	1	1	1	4	5	5	42
164	18911A04F3	5	2	2	1	1	5	3	4	5	1	1	1	1	4	2	3	47
165	18911A04F4	5	1	1	1	1	2	2	2	5	1	1	1	1	2	2	1	49
166	18911A04F5	5	2	2	1	1	2	2	2	5	1	1	1	1	4	2	5	38
167	18911A04F6	5	2	2	1	1	4	2	2	5	1	1	1	1	2	4	2	40
168	18911A04F7	5	2	1	1	1	5	5	4	5	2	1	1	1	4	5	5	61
169	18911A04F8	5	2	1	1	1	5	5	4	5	2	1	1	1	4	5	5	59
170	18911A04F9	5	1	1	1	1	2	1	1	5	1	1	1	1				2
171	18911A04G0	5	2	2	1	1	5	5	4	5	1	1	1	1	4	2	5	48
172	18911A04G1	5	2	2	1	1	5	5	4	5	2	1	1	1	4	5	5	49
173	18911A04G2	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	49
174	18911A04G3	5	2	2	1	1	4	3	3	5	1	1	1	1	4	4	5	34

175	18911A04G4	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	51
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177	18911A04G6	5	2	1	1	1	5	5	4	5	1	1	1	1	4	2	4	39
178	18911A04G7	5	2	1	1	1	5	5	4	5	1	1	1	1	4	5	5	49
179	18911A04G8	5	2	2	1	1	5	5	4	5	1	1	1	1	4	5	5	65
180	18911A04G9	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	52
181	18911A04H0	5	2	1	1	1	5	5	4	5	2	1	1	1	4	5	5	48
182	18911A04H1	5	2	2	1	1	4	3	3	5	1	1	1	1	2	1	1	34
183	18911A04H2	5	1	1	1	1	1	2	2	5	1	1	1	1	2	2	1	2
184	18911A04H3	5	2	1	1	1	5	5	4	5	1	1	1	1	4	5	5	43
185	18911A04H4	5	2	2	1	1	4	3	3	5	2	1	1	1	4	5	5	51
186	18911A04H5	5	2	2	1	1	5	3	4	5	1	1	1	1	4	5	5	51
187	18911A04H6	5	2	2	1	1	5	5	4	5	2	1	1	1	4	5	5	45
188	18911A04H7	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	43
189	18911A04H8	5								5								8
190	18911A04H9	5	2	2	1	1	5	5	4	5	1	1	1	1	4	4	5	45
191	18911A04J0	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	45
192	18911A04J1	5	2	2	1	1	5	5	4	5	2	1	1	1	4	5	5	44
193	18911A04J2	5								5	1	1	1	1	1	1	1	46
194	18911A04J3	5	2	2	1	1	4	3	3	5	1	1	1	1	1	1		35
195	18911A04J4	5	2	2	1	1	4	4	3	5	1	1	1	1	4	2	4	47
196	18911A04J5	5	2	2	1	1	5	5	4	5	1	1	1	1	4	5	5	41
197	18911A04J6	5	2	2	1	1	5	5	4	5	1	1	1	1	4	2	4	39
198	18911A04J7	5	2	2	1	1	4	2	2	5	1	1	1					48
199	18911A04J8	5	2	2	1	1	4	3	3	5	1	1	1					34
200	18911A04J9	5	2	1	1	1	5	5	4	5	1	1	1	1	2	4	2	34

201	18911A04K0	5	1	1	1	1	2	2	2	5	1	1	1	1	1	1		47
202	18911A04K1	5	2	2	1	1	5	5	4	5	1	1	1	1	3	3	2	52
203	18911A04K2	5	2	2	1	1	5	5	4	5	1	1	1	1	2	5	5	38
204	18911A04K3	5	2	2	1	1	5	3	4	5	1	1	1	1	4	2	4	45
205	18911A04K5	5	2	1	1	1	5	5	4	5	1	1	1	1	2	1	1	47
206	18911A04K6	5	2	1	1	1	5	5	4	5	1	1	1	1	3	5	4	40
207	18911A04K7	5	2	2	1	1	5	5	4	5	1	1	1	1	4	5	5	47
208	18911A04K8	5	2	2	1	1	5	3	4	5	1	1	1	1	4	5	5	51
209	18911A04K9	5	2	2	1	1	4	2	2	5	1	1	1	1	2	1	1	42
210	18911A04L0	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	71
211	18911A04L1	5	2	2	1	1	4	2	2	5	1	1	1	1	4	2	4	62
212	18911A04L2	5	2	1	1	1	5	5	4	5	1	1	1	1	4	2	4	61
213	18911A04L3	5	2	2	1	1	5	3	4	5	1	1	1	1	2	3	2	59
214	18911A04L4	5	2	1	1	1	5	5	4	5	1	1	1	1	4	2	4	41
215	18911A04L5	5	2	2	1	1	4	2	2	5	1	1	1	1	4	2	4	52
216	18911A04L6	5	2	2	1	1	4	4	3	5	1	1	1	1	4	5	5	48
217	18911A04L7	5	2	1	1	1	5	5	4	5	1	1	1	1	2	3	2	48
218	18911A04L8	5	2	2	1	1	4	4	3	5	1	1	1	1	4	5	5	55
219	18911A04M0	5	2	2	1	1	4	4	3	5	1	1	1	1	1	1	1	38
220	18911A04M1	5	2	2	1	1	5	5	4	5	2	1	1	1	4	5	5	53
221	18911A04M2	5	2	2	1	1	5	5	4	5	2	1	1	1	4	5	5	50
222	18911A04M3	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	56
223	18911A04M4	5	2	2	1	1	5	3	4	5	1	1	1	1	2	3	2	52
224	18911A04M5	5	2	2	1	1	4	2	2	5	1	1	1	1	2	4	2	59
225	18911A04M6	5	2	2	1	1	5	5	4	5	2	1	1	1	4	5	5	60
226	18911A04M7	5	2	2	1	1	4	3	3	5	1	1	1	1	4	4	5	54

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227	18911A04M8	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	57
228	18911A04M9	5	1	1	1	1	4	2	1	5	1	1	1	1	2	1	1	46
229	18911A04N0	5	1	1	1	1	4	2	1	5	1	1	1	1	4	4	5	46
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231	18911A04N2	5								5	1	1	1	1	4	4	5	42
232	18911A04N3	5	2	2	1	1	5	5	4	5	1	1	1	1	4	5	5	48
233	18911A04N4	5	2	2	1	1	5	3	4	5	1	1	1	1	3	4	4	9
234	18911A04N5	5	2	2	1	1	5	3	4	5	1	1	1	1	2	4	2	52
235	18911A04N6	5	2	2	1	1	5	5	4	5	2	1	1	1	4	5	5	57
236	18911A04N7	5	2	1	1	1	5	5	4	5	1	1	1	1	2	3	2	44
237	18911A04N8	5	2	1	1	1	5	5	4	5	1	1	1	1	2	1	1	40
238	18911A04N9	5	2	1	1	1	5	5	4	5	1	1	1					47
239	18911A04P0	5	2	2	1	1	4	4	3	5	1	1	1	1	3	3	2	50
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241	18911A04P2	5	2	1	1	1	5	5	4	5	1	1	1	1	2	2	1	56
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243	18911A04P4	5								5	1	1	1	1				A
244	18911A04P5	5	2	2	1	1	5	3	4	5	1	1	1	1	2	2	1	43
245	18911A04P6	5	2	2	1	1	5	5	4	5	2	2	1	1	4	5	5	53
246	18911A04P7	5	1	1	1	1	1	1		5	1	1	1	1	1	1		39
247	18911A04P8	5	2	1	1	1	5	5	4	5	2	1	1	1	4	5	5	63
248	18911A04P9	5	2	2	1	1	5	5	4	5	1	1	1	1	4	5	5	50
249	18911A04Q0	5	1	1	1	1	2	2	2	5								16
250	18915A0410	5	1	1	1	1	1	1	1	5								46
251	18915A0413	5	1	1	1					5								10
252	18915A0427	5								5	1	1	1					9

253	19915A0401	5	2	2	1	1	4	2	1	5	1	1	1	1	4	2	3	36
254	19915A0402	5	2	2	1	1	4	3	2	5	1	1	1	1	4	2	5	50
255	19915A0403	5	2	2	1	1	4	3	2	5	1	1	1	1	2	3	2	49
256	19915A0404	5	2	2	1	1	4	4	3	5	1	1	1	1	4	2	3	48
257	19915A0405	5	2	2	1	1	4	4	3	5	1	1	1	1	1	1		12
258	19915A0406	5	2	2	1	1	4	4	3	5	1	1	1	1	2	2	1	35
259	19915A0407	5	2	2	1	1	4	3	3	5	1	1	1	1	2	1	1	43
260	19915A0408	5	2	2	1	1	4	4	3	5	1	1	1	1	4	2	3	51
261	19915A0409	5	2	2	1	1	5	3	4	5	1	1						50
262	19915A0410	5	2	2	1	1	4	2	1	5	1	1	1	1	1	1	1	46
263	19915A0411	5	1	1	1	1	2	1	1	5	1	1	1	1	1	1	1	50
264	19915A0412	5	2	2	1	1	4	4	3	5	1	1	1	1	4	2	4	53
265	19915A0413	5	2	2	1	1	5	3	4	5	1	1	1	1	2	3	2	41
266	19915A0414	5	2	1	1	1	5	5	4	5	1	1	1	1	2	5	5	46
267	19915A0415	5	2	2	1	1	5	5	4	5	1	1	1	1	4	2	4	35
268	19915A0416	5								5								A
269	19915A0417	5	2	2	1	1	4	4	3	5	1	1	1	1	4	4	5	53
270	19915A0418	5	2	2	1	1	2	2	2	5	1	1	1	1	2	5	5	51
271	19915A0419	5	2	2	1	1	5	3	4	5	1	1	1		1	1		50
272	19915A0420	5	2	1	1	1	5	5	4	5	1	1	1	1	1	1		49
273	19915A0421	5	2	1	1	1	5	5	4	5	1	1	1	1	4	2	4	48
274	19915A0422	5	2	1	1	1	5	5	4	5	1	1	1	1	2	4	2	36
275	19915A0423	5	2	1	1	1	5	5	4	5	1	1	1	1	4	2	5	44
276	19915A0424	5	2	2	1	1	5	3	4	5	1	1	1	1	2	2	1	11

Average marks	**5.00**	**1.84**	**1.68**	**1.00**	**1.00**	**4.28**	**3.62**	**3.19**	**5.0**	**1.15**	**1.07**	**1.00**	**1.00**	**3.02**	**3.20**	**3.63**	**44.38**

KM

CO	Method	value	Avg	CO Attainment (Internal)a1	CO Attainment (End Exam)b1	Overall CO Direct Attainment =0.25(a1)+b1(0.75)
CO 1	ASM I	3	3.0	2.40	2.00	2.10
	MID I - PART A - Q1	3.0				
	MID I - PART A - Q3 A	3.0				
	MID I - PART B - Q4	3.0				
CO 2	ASM I	3	2.8			
	MID I - PART A - Q2	2.0				
	MID I - PART A - Q3 B	3.0				
	MID I - PART B - Q5	3.0				
CO 3	ASM I	3	2.5			
	ASM II	3.0				
	MID I - PART B - Q6	3.0				
	MID II - PART B - Q4	1.0				
CO 4	ASM II	3	1.8			
	MID II - PART A - Q1	0.0				
	MID II - PART A - Q3 A	3.0				
	MID II - PART B - Q5	1.0				
CO 5	ASM II	3	2.0			
	MID II- PART A - Q2	0.0				
	MID II - PART A - Q3 B	3.0				
	MID II - PART B - Q6	2.0				

K.l.lokesh

Course Coordinator

HOD-ECE

Average marks	5.00	1.84	1.68	1.00	1.00	4.28	3.62	3.19	5.00	1.15	1.07	1.00	1.00	3.02	3.20	3.63	44.38
No of students attemped	276	244	244	241	238	235	235	231	276	253	252	249	233	232	232	217	268
%of students scored 60% and above	100	83.61	67.62	100.00	100.00	88.51	75.74	72.73	100.00	14.62	6.75	100.00	100.00	58.19	56.03	68.66	61.19
CO ATTAINMENT LEVEL	3	3.0	2.0	3.0	3.0	3.0	3.0	3.0	3	0.0	0.0	3.0	3.0	1.0	1.0	2.0	2.0

VIDYA JYOTHI INSTITUTE OF TECHNOLOGY(Autonomous)

(Accredited by NAAC, Approved by AICTE New Delhi & Permanently Affiliated to JNTUH)
Aziz Nagar Gate, C.B. Post, Hyderabad-500 075

Department of Electronics and Communication Engineering
(Accredited by NBA)

INDIRECT ATTAINMENT

CSE	Slight (Low) 1	Moderate (Medium) 2	Substantial (High) 3	Total	Attainment
CO1	6	59	94	159	**2.55**
CO2	0	73	88	161	**2.55**
CO3	3	67	89	159	**2.54**
CO4	7	87	74	168	**2.40**
CO5	2	78	81	161	**2.49**
AVERAGE					**2.51**

HOD-ECE

19. Course closure report

Regulation	:	R18
Academic Year	:	2020-21
Program	:	B.Tech
Year/Sem	:	III Year I Sem
Course Name	:	CONTROL SYSTEMS ENGINEERING
Course Code	:	A15415
Contact Hours	:	Lectures / Tutorial/ Credits
No. of Students	:	251

OVERALL ATTAINMENT (80% DIRECT + 20% INDIRECT)	
DIRECT	2.1
INDIRECT	2.51
OVERALL ATTAINMENT	2.182(1.68 DIRECT+0.502 INDIRECT)

HOD-ECE

Unit - I

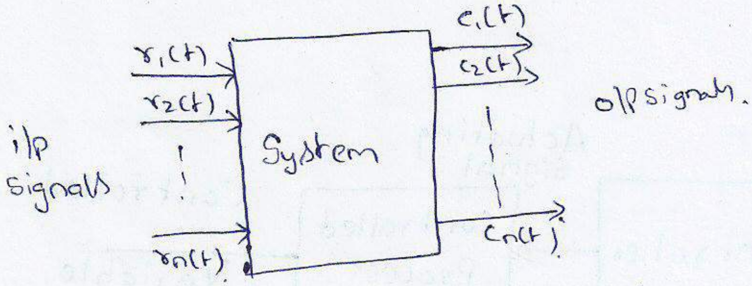
① Introduction and Transfer function ② Representation

Introduction :-

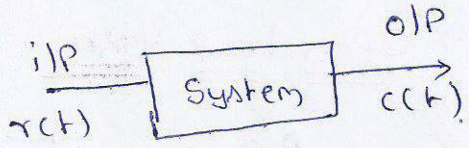
System:- A system is a combination or an arrangement of - different physical components which act together as an entire unit to achieve certain objective.

-> A system is one which operates on an input signal to produce an o/p signal.

-> A control system is an interconnection of physical components to provide a desired function, involving some kind of - controlling action in it.



Multi i/p Multi o/p System

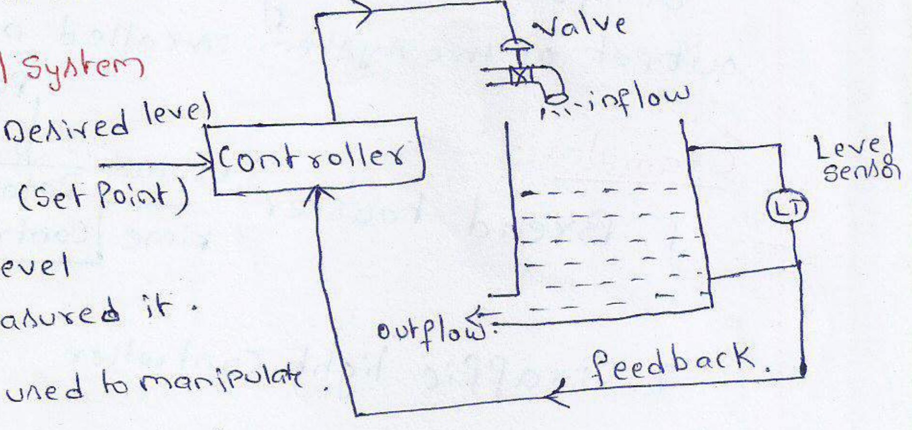


SISO

-> The prime objective of any control system is to maintain a particular variable of a process at a desired value.
↳ also known as controlled variable [c(t)]

-> The control system requires several elements like sensors and final control elements (actuators) to maintain controlled variable at reference level.

eg:- Liquid level Control System



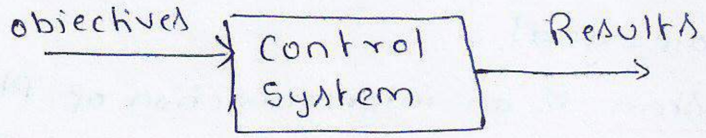
Controlledvariable
- Liquid level

-> Sensors are used to measured it.

-> The elements that are used to manipulate the controlled variable are called actuators or final control elements.

-> The i/p variables which can be manipulated to modify the controlled variable are known as Manipulated Variables.

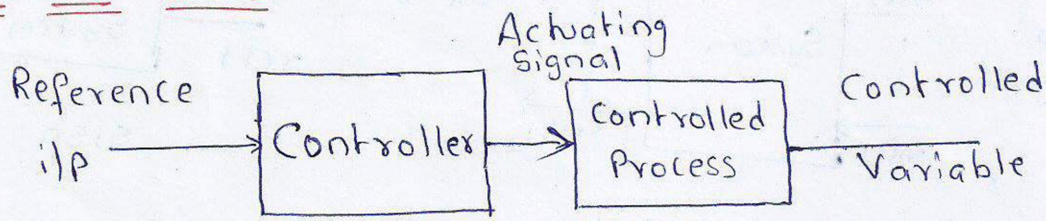
-> The control system compares the desired level (set point) with the present level (controlled variable) using level sensor. If there is an error, the controller takes the necessary action. (increase & decrease valve opening).

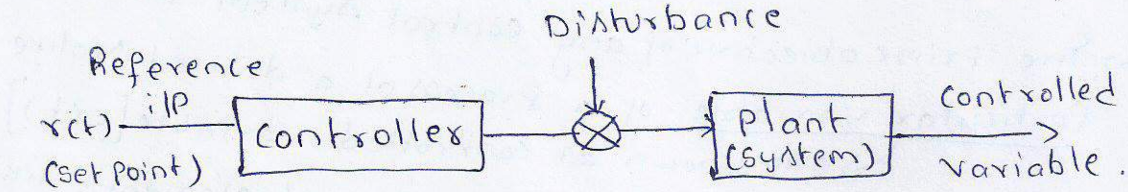


-> objectives can be identified with i/ps, or actuating signals.

->

Open Loop System :-

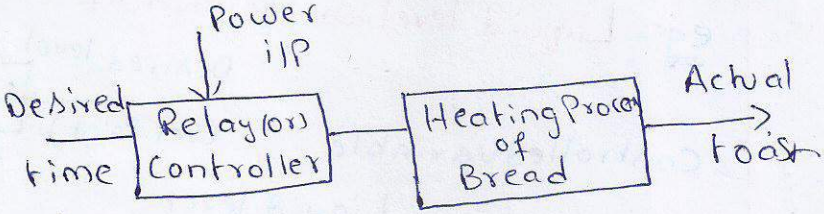




-> A system in which o/p is independent on i/p but controlling or input is totally independent of the o/p (or) changes in output of the system is called an open loop system.

Examples:-

1. Bread toaster.



2. Traffic light controller

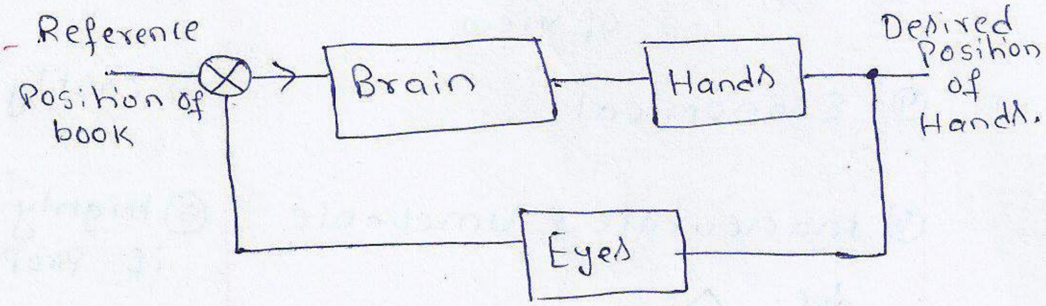
etc.

Closed Loop System:-

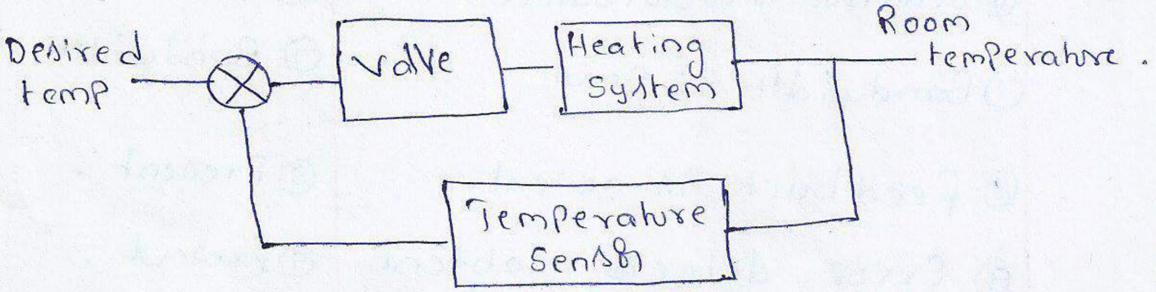
A System in which the controlling action or input is somehow dependent on o/p or changes in o/p is called closed loop system.

Examples:-

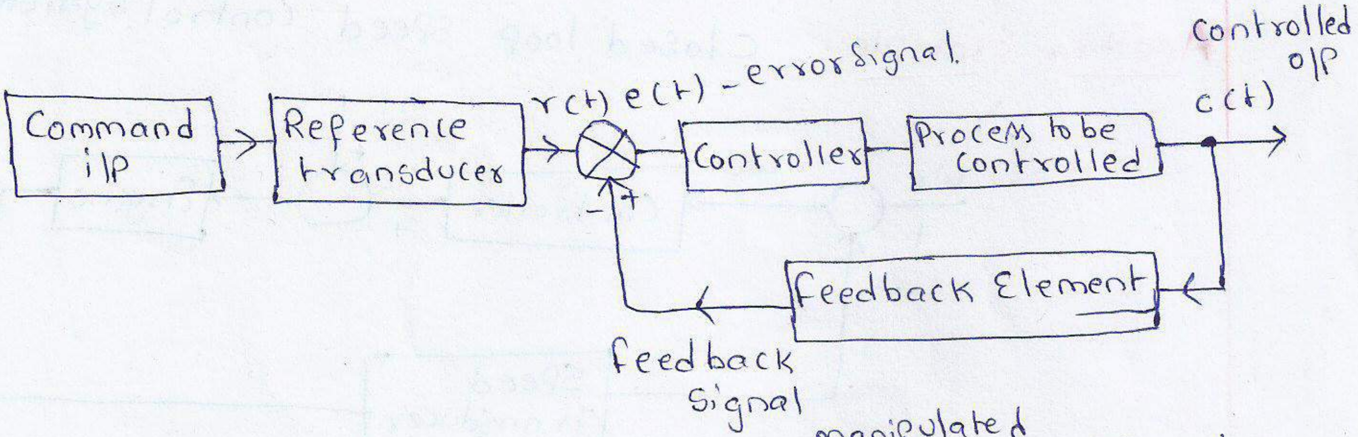
a) Human being:-



b) Home heating System:-



Representation:-



r(t) - reference i/p

e(t) - Error Signal

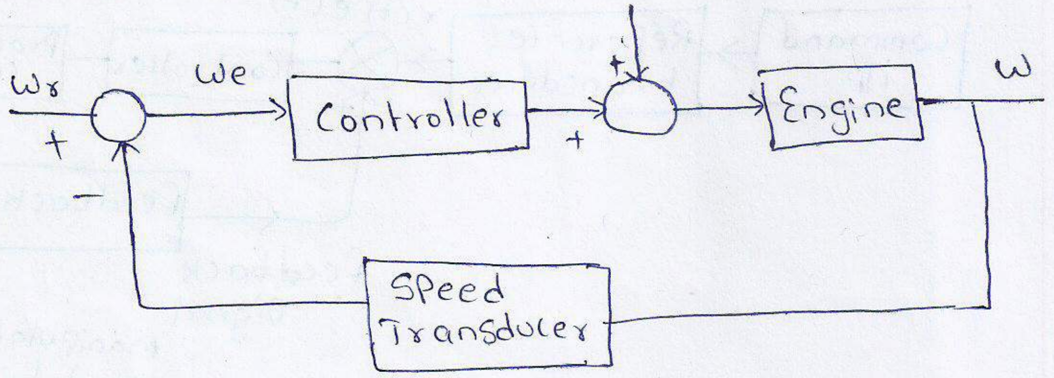
c(t) - Controlled o/p

m(t) - ~~modulated~~ manipulated Signal

b(t) - feedback Signal.

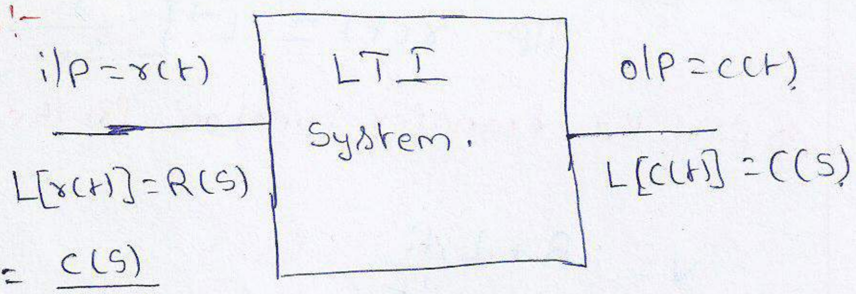
Open loop	closed loop
① Simple in construction	① complex.
② Generally not troubled with Problems of stability.	② Stability Problems are severe.
③ Easy from Maintenance Point of view	③ difficult.
④ Economical	④ costly.
⑤ inaccurate & Unreliable due. internal external disturbance.	⑤ Highly accurate & reliable if Properly designed.
⑥ Sensitive to disturbances	⑥ insensitive.
⑦ Bandwidth is small	⑦ Bandwidth is large.
⑧ feedback is absent	⑧ Present.
⑨ Error detects is absent	⑨ Present.
⑩ Highly affected by nonlinearites	⑩ Reduced effect of nonlinearites.

Another Example:- closed loop speed control system.



ω_r - reference value.

Transfer function:-



$$TF = \frac{L[\text{output}]}{L[\text{Input}]} = \frac{C(S)}{R(S)}$$

If $R(S) = 1$, i.e., impulse input $L[\delta(t)] = 1$

$$T.F = C(S)$$

→ Transfer function is also called impulse response of the system.

$$\boxed{\begin{matrix} IR = L^{-1}[TF] \\ TF = L[IR] \end{matrix}}$$

Problems:-

① Impulse response $= 10e^{-3t}u(t)$, the TF of the system is.

$$TF = L[IR] = L[10e^{-3t}u(t)]$$
$$= \frac{10}{S+3}$$

② $TF = \frac{1}{S^2+3S+2}$, IR = ?.

$$IR = L^{-1}[TF] = L^{-1}\left[\frac{1}{S^2+3S+2}\right] = L^{-1}\left[\frac{1}{(S+2)(S+1)}\right]$$
$$= L^{-1}\left[\frac{-1}{(S+2)} + \frac{1}{S+1}\right]$$
$$= \left[-1 \times e^{-2t} + e^{-t}\right] u(t)$$

③ Step response of system of a system $te^{-2t}u(t)$. The T.F is

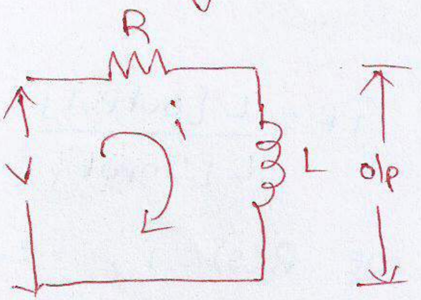
$$T.F = \frac{L[o/p]}{L[i/p]} = \frac{L[te^{-2t}]}{L[u(t)]} = \frac{\frac{1}{(S+2)^2}}{\frac{1}{S}} = \frac{S}{(S+2)^2}$$

④ The response of a system whose Impulse response is $e^{-t}u(t)$ is $te^{-t}u(t)$. find the input of the system.

$$T.F = L[e^{-t}u(t)] = \frac{1}{S+1}$$
$$C(S) = L[c(t)] = L[te^{-t}u(t)] = \frac{1}{(S+1)^2}.$$
$$iR(S) = \frac{C(S)}{TF} = \frac{\frac{1}{(S+1)^2}}{\frac{1}{S+1}} = \frac{1}{S+1}$$

i/p $r(t) = L^{-1}\left[\frac{1}{S+1}\right] = e^{-t}u(t)$.

⑤ Find the transfer function for the following ckt diagram.



$$V = iR + L\frac{di}{dt}.$$

$$V(S) = RI(S) + LS\,I(S)$$

$$V(S) = I(S) + R + LS\,I(S)$$

$$\boxed{\frac{I(S)}{V(S)} = \frac{1/L}{S+R/L}}.$$

Standard time domain test signals:-

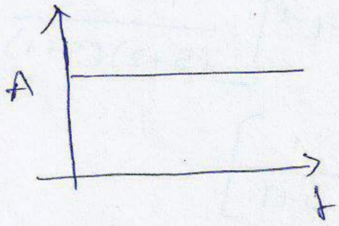
Step signal

$r(t) = Au(t)$

$u(t) = 1 \quad t>0$

$\quad = 0 \quad t<0$

$R(S) = \frac{A}{S}$.

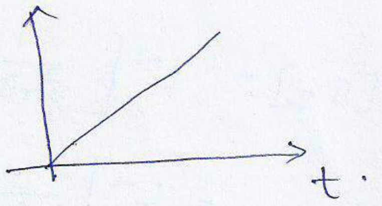


Ramp signal

$r(t) = At. \quad t>0$

$\quad = 0 \quad t<0$.

$LT[r(t)] = \frac{A}{s^2}$.

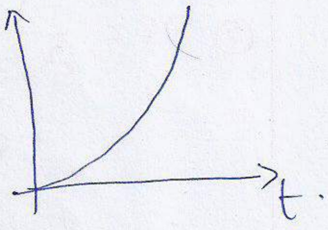


Parabolic signal.

$r(t) = \frac{At^2}{2} \quad t>0$

$\quad = 0 \quad t<0$.

$R(S) = A/s^3$

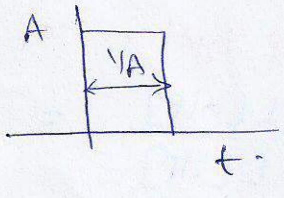


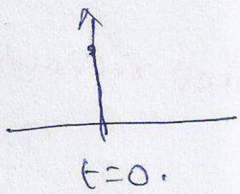
Impulse signal

$r(t) = 1 \quad t=0$

$\quad = 0 \quad t\neq 0$.

$L[r(t)] = 1$



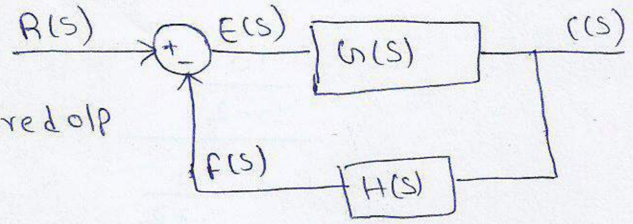


$\frac{d}{dt}$ (Parabolic response) = Ramp response.

$\frac{d}{dt}$ [Ramp response] = step response

$\frac{d}{dt}$ [step response] = Impulse response.

Transfer function of a closed loop System:-



$R(S)$ = i/p signal (or)
Reference i/p (or) Desired o/p

$C(S)$ = output

$E(S)$ = Actuating Error Signal

$F(S)$ = feedback signal

$G(S)$ = forward TF

$H(S)$ = Feedback T.F.

$G(S)H(S)$ = Openloop Transfer fn = OLTF.

$\frac{C(S)}{R(S)}$ = closed loop Transfer fn = CLTF.

from the above diagram,

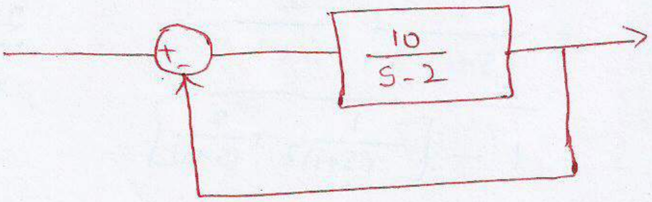
$$C(S) = E(S)G(S)$$
$$= [R(S) - F(S)]G(S)$$
$$= [R(S) - H(S)C(S)]G(S)$$
$$C(S) = R(S)G(S) - G(S)H(S)C(S)$$
$$C(S)[1 + G(S)H(S)] = R(S)G(S)$$
$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

$$\boxed{\begin{aligned} CLTF &= \frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)} \\ CLTF &= \frac{C(S)}{R(S)} = \frac{G(S)}{1-G(S)H(S)} \end{aligned}}$$

for -ve F.B.

for +ve F.B.

(Pb)



Find T.F.

$$G(S) = \frac{10}{S-2}, \quad H(S) = 1$$

$$CLTF = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\frac{10}{s-2}}{1+\frac{10}{s-2}} = \frac{10}{s+8}.$$

(Pb) The CLTF of an unity feedback system is $\frac{4}{s^2+7s+13}$ find the corresponding OLTF.

$$CLTF = \frac{4}{s^2+7s+13} \quad , \quad H(s)=1$$

$$CLTF = \frac{G(s)}{1+G(s)}$$

$$CLTF + CLTF\, G(s) = G(s)$$

$$CLTF = G(s)[1-CLTF]$$

$$G(s) = \frac{CLTF}{1-CLTF} = OLTF.$$ when $H(s)=1$

$$OLTFF = \frac{\frac{4}{s^2+7s+13}}{1-\frac{4}{s^2+7s+13}} \Rightarrow OLTF = \frac{4}{s^2+7s+9}.$$

(Pb) If a closed loop system I.R $-te^{-t}+2e^{-t} (t \geq 0)$. Find OLTF.

$$CLTF = L[IR]$$

$$= L[-te^{-t}+2e^{-t}]$$

$$= -\frac{1}{(s+1)^2} + \frac{2}{s+1}.$$

$$OLTF = \frac{-\frac{1}{(s+1)^2}+\frac{2}{s+1}}{1-\left[-\frac{1}{(s+1)^2}+\frac{2}{(s+1)}\right]} = \frac{2s+1}{s^2}.$$

Types of feedback Control Systems:-

According to method of analysis, design and control systems are classified as.

1. Linear or non linear.
2. Time varying or time invariant

According types of signals found in the system, reference is often made to

1. continuous data or discrete date
2. modulated or unmodulated systems.

According to main purpose of the system.

1. position - control system
2. velocity control system etc.

→ Strictly speaking, Linear systems do not exist in Practice, since all physical systems are nonlinear to some extent.

→ When the Magnitudes of signals are extended beyond the range of the linear operation, depending upon severity of nonlinearity, ithe system should no longer be considered linear.

eg:- amplifiers - saturation

→ for linear systems, good no. of analytical and graphical techniques for is available for analysis & design.

Non linear systems are difficult to analyse.

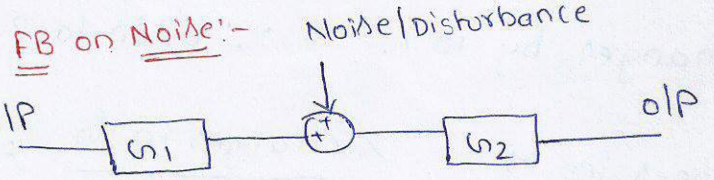
Feedback & it's Effects:-

(1) Effects of FB on gain:- For +ve feedback → gain is increased

-ve feedback → " " decreased.

(2) Effect of FB on Noise:-

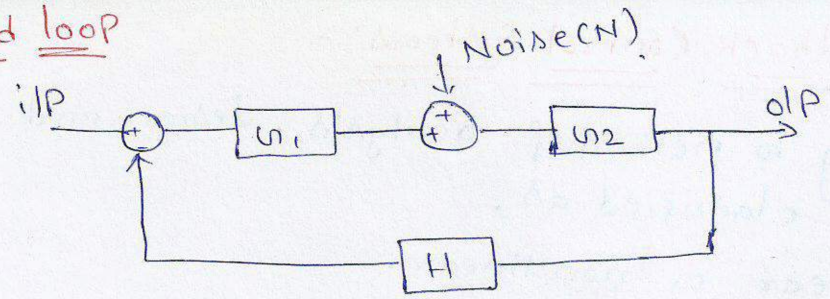
Open loop:-



o/p due to Noise $(G_1 + N) G_2 = \underset{\text{actual signal}}{G_1G_2} + G_2N$

$= G_2N$ → due to Noise.

closed loop



$$o/P = \frac{G_1 G_2}{1+G_1 G_2 H} \quad \& \quad o/P = \frac{(G_1 + G_2 N) G_2}{1 + G_1 G_2 H}$$

$$o/P \text{ due to noise} = \frac{G_2 N}{1+G_1 G_2 H}.$$

Effect of F.B on Sensitivity:-

$$M = \frac{C}{R} = \frac{G(S)}{1+G(S)H(S)}$$

Sensitivity of M w.r.t changes in 'G'.

$$S_G^M = \frac{\%\text{ changes in 'M'}}{\%\text{ changes in 'G'}} = \frac{\left(\frac{\partial M}{M}\right)\times 100\%}{\left(\frac{\partial G}{G}\right)\times 100)}$$

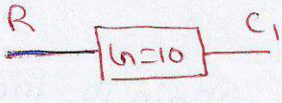
$$S_G^M = \frac{\partial M}{\partial G}\cdot\frac{G}{M}$$

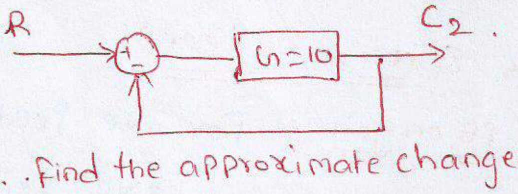
$$= \frac{G}{M}\frac{\partial}{\partial G}\frac{G}{1+GH} = \frac{G}{M}\frac{(1+GH)(1)-GH}{(1+GH)^2}.$$

$$S_G^M = \frac{1}{1+GH}$$

→ feedback reduces the sensitivity variation at the o/p.

(Pb)





If G changes by 10%. Find the approximate changes in C_1 & C_2.

C_1 changes by 10%. ∵ open loop.

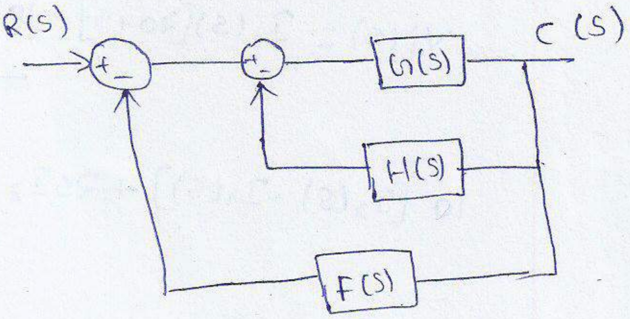
$$\%\text{ changes in } M = \frac{\%\text{ changes in } G}{1+GH} = \frac{10\%}{1+10(1)}$$

$$= \frac{10}{11}\% \cong 1\%.$$

Effect of feedback on stability:-

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

If $G(S)H(S) = -1$, the system becomes unstable, but the system can be stabilized by connecting a proper outer feedback loop as shown in fig below.



$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)+G(S)F(S)}$$

Problem:-

1. A control system whose step response is $0.5[1-e^{-2t}]$ is cascaded to another control block where IR is e^{-t}. what is the T.f of the Cascaded combination.

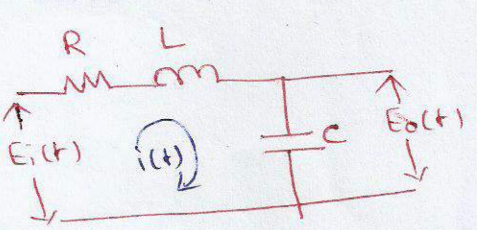
$$IR = \frac{d}{dt}(\text{step response})$$

$$= \frac{d}{dt}\left[0.5(1-e^{-2t})\right] = e^{-2t}.$$

$$TF_1 = \frac{1}{S+2}. \quad TF_2 = \frac{1}{S+1}$$

$$TF = TF_1 \times TF_2 = \frac{1}{(S+1)(S+2)}$$

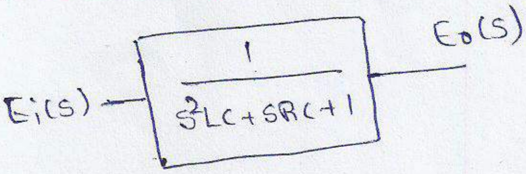
2. Find out the T.F of given network.



$$E_i(t) = iR + L\frac{di}{dt} + \frac{1}{c}\int i\,dt.$$

E Applying L.T

$$E_i(S) = I(S)\left[R+SL+\frac{1}{SC}\right]$$



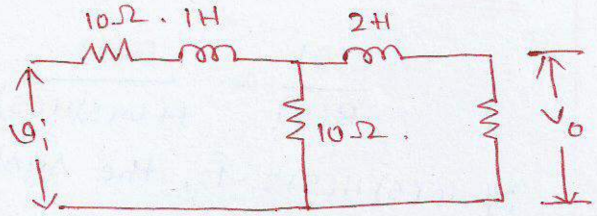
$$\frac{I(S)}{E_i(S)} = \frac{1}{R+SL+\frac{1}{SC.}}$$

$$E_o(t) = \frac{1}{c}\int i\,dt. \xrightarrow{} \text{ALT} \quad E_o(S) = \frac{1}{C}\frac{I_o(S)}{S.}$$

$$I(S) = sC\,E_o(S)$$

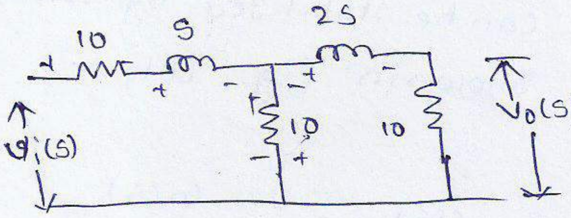
$$\frac{sC\,E_o(S)}{E_i(S)} = \frac{1}{sC\left[R+SL+\frac{1}{SC}\right]}$$

3. Derive the transfer function for the electrical nlw given below



The s-domain of Loop is

$$-V_i(s) + 10I_1(s) + sI_1(s)$$
$$+ 10(I_1(s) - I_2(s)) = 0.$$



$$V_i(s) = I_1(s)[20+s] - 10I_2(s) \quad —(1)$$

$$10[I_2(s) - I_1(s)] + 2sI_2(s) + \cancel{V_0(s)} = 0.$$
$$+ I_2(s)10 = 0.$$

$$20I_2(s) + 2sI_2(s) = 10I_1(s)$$

$$I_1(s) = \left[\frac{20+2s}{10}\right]I_2(s) \quad —(2)$$

$$V_i(s) = I_2(s)\left[\frac{20+2s}{10}\right][20+s] - 10I_2(s)$$

$$I_2(s) = \frac{V_i(s)}{0.2s^2+6s+30}$$

$$V_0(s) = 10I_2(s) = \frac{10V_i(s)}{0.2s^2+6s+30}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{50}{s^2+30s+150.}$$

Mathematical Models of Control Systems:-

→ The set of' mathematical equations, describing the dynamic characteristics of a system is called mathematical model of the system.

→ To analyse systems, it is necessary to convert such systems into mathematical models based on transfer function approach.

→ Mechanical systems < Translational / Rotational

Translation motion:-

→ The elements dominantly involved in the analysis of translational motion systems.

i) Mass ii) spring iii) friction.

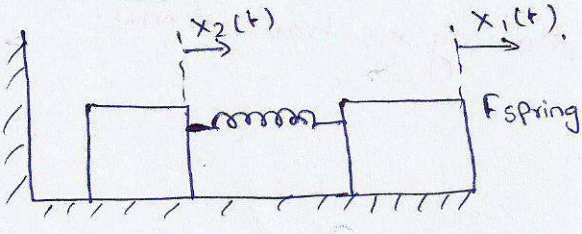
→ from Newton's law of motion, sum of forces applied on rigid body (or) system must be equal to sum of forces consumed to produce displacement, velocity and acceleration in various elements of the system.

i) Mass (m):- The displacement of mass always takes place in the direction of the applied force.

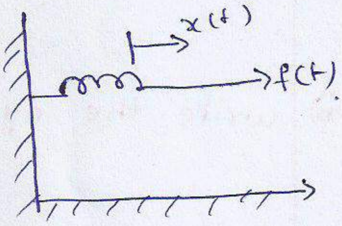
→ $F_1 = m \times acceleration = m\frac{d^2x(t)}{dt2}. = Ms^2X(s)$

Linear Spring:- All springs are basically nonlinear in nature but of small deformations their deformations can be approximated as linear one

Force required to cause displacement x(t) $\boxed{f(t) = Kx(t)}$

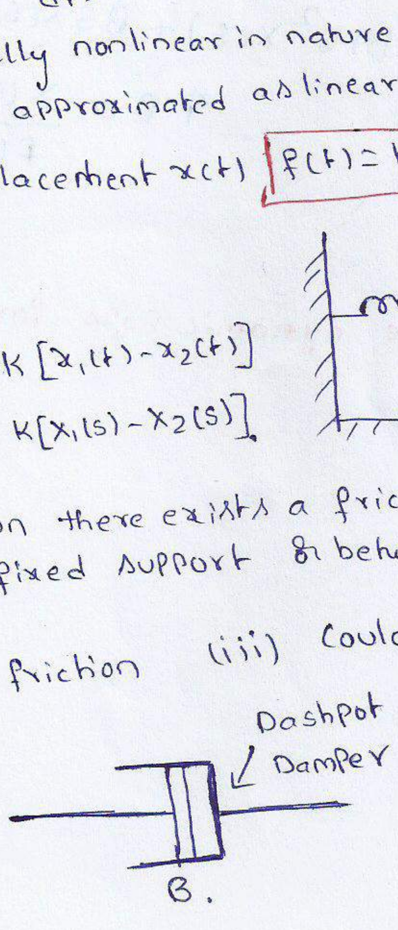


$F_{spring} = K[x_1(t) - x_2(t)]$
$= K[X_1(s) - X_2(s)]$.



Friction:- Whenever there is a motion there exists a friction. Friction may be between moving element and fixed support & between two ~~fixed two~~ moving surfaces

(i) Viscous friction (ii) Static friction (iii) Coulomb friction

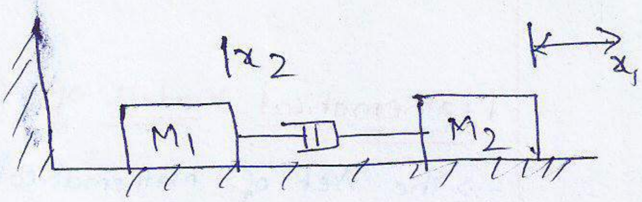


→ $F_{frictional} = \frac{B\,dx(t)}{dt}$.

$F_{friction}(s) = BSX(s)$.

$f_{frictinal} = B\left[\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt}\right]$

$f_{frictional} = Bs\left[X_1(S) - X_2(S)\right].$



D'Alembert's Principle:-

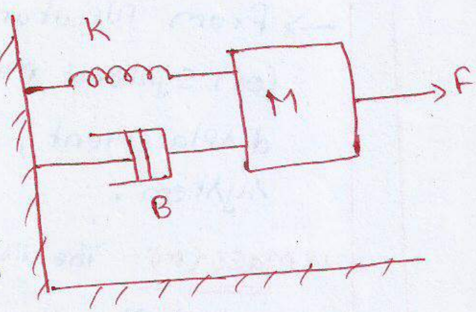
Algebraic sum of the forces at a point is equal to zero i.e., the applied force is equal to the sum of the forces resisting the motion of the body.

$$F = F_I + F_D + F_K.$$

F = Applied force F_I - Inertial force.

F_D = Damping force F_K - Spring force.

(Pb) For the following mass-spring damping system
① write the dynamic Equation
② Transfer f^n.

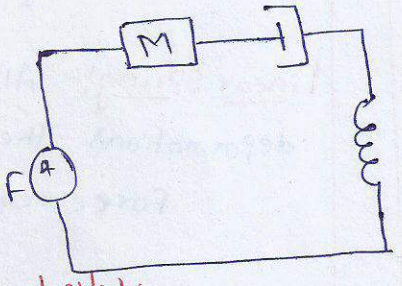


① $F = F_I + F_D + F_K.$

$= M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx.$ - dynamic Eq^n

② $Ms^2X(S) + BSX(S) + KX(S) = F(S)$

$T.F = \frac{X(S)}{F(S)} = \frac{1}{Ms^2 + BS + K}.$



(Pb) write the dynamic Eq^{ns} for following mechanical N/w.

Rotational Motion:-

-> Motion about a fixed axis

-> In this systems, the force gets replaced by a moment by a about fixed axis i.e., Torque. (force x distance from axis)

-> Newton's law states that sum of the torques consumed by the different elements of the system in order to produce angular displacement (θ), angular velocity (ω) and angular acceleration (α) in them.

->. Spring & friction behaves in same manner in rotational systems.

$$T_I = J\frac{d^2\theta}{dt}.$$

'J' -> moment of inertia.

Analogous Elements

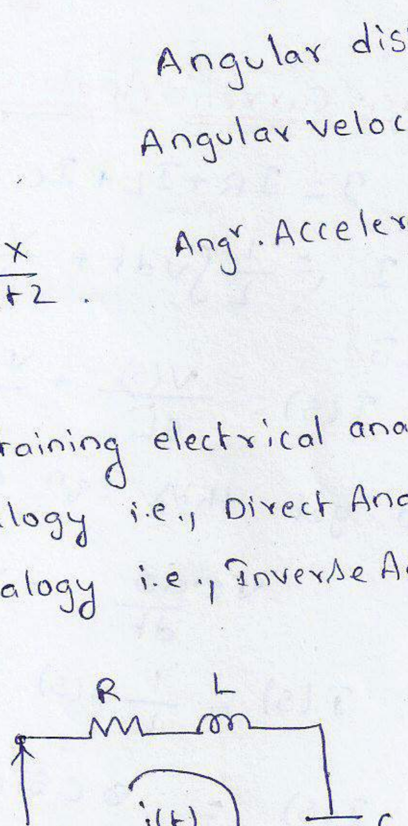
S.NO	Translation Motion	Rotational motion.
1.	Mass (M)	moment of Inertia (J)
2.	friction (B)	Friction (B)
3.	Spring (K)	Spring (K)
4.	Force (F)	Torque (T)
5.	Displacement (x)	Angular displacement (θ)
6.	Velocity $v = \frac{dx}{dt}$	Angular velocity ($\omega = \frac{d\theta}{dt}$)
7.	Acceleration = $\frac{d^2x}{dt2}$.	Angr. Acceleration ($\alpha = \frac{d^2\theta}{dt2}$)

Analogous Systems:-

There are two methods of obtaining electrical analogous n/ws

1. Force - Voltage Analogy i.e., Direct Analogy
2. Force - Current Analogy i.e., Inverse Analogy

Force voltage Analogy:-



$$V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt.$$

Applying Laplace transform,

$$V(S) = RI(S) + LSI(S) + \frac{I(S)}{CS}.$$

->To compare F(S) & V(S), change V(S) to the same form of F(S)

$i(t) = \frac{dq}{dt}$ $\quad I(S) = S\,Q(S)$ (or) $Q(S) = \frac{I(S)}{S.}$

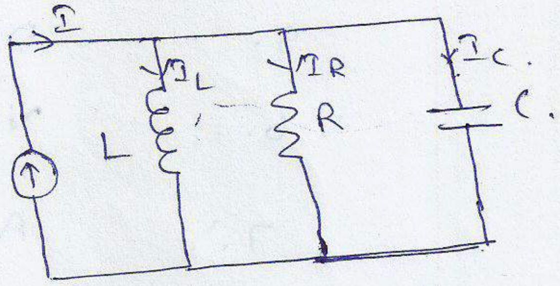
$$V(S) = LS^2 Q(S) + RS\,Q(S) + \frac{1}{C} Q(S)$$

$$F(S) = MS^2 X(S) + BS X(S) + K X(S)$$

Translational	Rotational	Electrical.
force.	Torque.	Voltage.
Mass M	moment of Intertia J	Inductance.
Friction Constant (B)	B	Resistance.
Spring constant (k) N/m.	K.	Reciprocal of C (1/c).
Displacement x	θ.	charge.
velocity $\frac{dx}{dt}$.	$\omega = \frac{d\theta}{dt}$.	current. $\frac{dq}{dt}$.

Force - Voltage Analogy.

→ Force Current Analogy :-



$$I = I_R + I_L + I_C.$$

$$I = \frac{1}{L}\int v\,dt + \frac{V}{R} + C\frac{dv}{dt}.$$

ALT

$$I(S) = \frac{V(S)}{SL} + \frac{V(S)}{R} + SCV(S)$$

To get this eqn similar inform to F(S).

$$V = \frac{d\phi}{dt}. \Rightarrow V(S) = S\,\phi(S)$$

$$I(S) = \frac{1}{L}\phi(S) + \frac{S\phi(S)}{R.} + CS^2\phi(S).$$

$$I(S) = CS^2\phi(S) + \frac{1}{R}S\phi(S) + \frac{1}{L}\phi(S).$$

$$F(S) = MS^2X(S) + BS\,X(S) + KX(S)$$

Force current Analogy.

Translation	Rotational	Electrical.
1. Force.	T	current.
2. M	J	Capacitor
3. B	B	Reciprocal of R (1/R).
4. K.	K	1/L.
5. x	θ	Flux.
6. $v = \frac{dx}{dt}$.	ω	Voltage.

→ Inertia can be defined as a property or tendency of an object that resists any change to the state of motion. Thus a body stays at rest or continues it's motion, unless ~~acted on by an~~ external force is applied.

→ In order to change body's state of rest or uniform motion in a straight line, some external force may act on the body for a given duration.

→. For a body rotating about an axis, it's Moment of Inertia with respect axis, is the resistance to change of it's state of motion about the axis unless acted upon by an external force.

It depends on the body's mass distribution and the axis chosen, with larger moments requiring more torque to change the body's rotation.

→ Torque: is defined as the tendency of a force, to rotate an object about ~~an ax~~ the axis

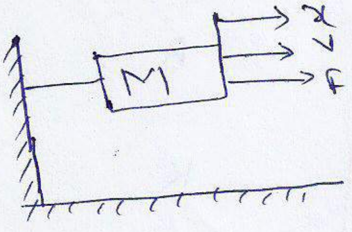
eg:- steering wheel.

Summary on Translational & Rotational System:-

Translational System:-

Input = force (F) output = linear displacement (x) (or) Linear velocity.

1. mass element

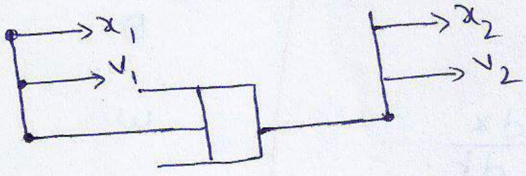


a) $f = M\frac{dv}{dt}$

$f = M\frac{d^2x}{dt^2}$.

2. Damper Element.

friction

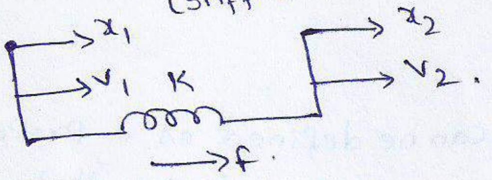


a) $f = B(v_1 - v_2)$

b) $f = B\frac{d}{dt}(x_1 - x_2)$

3) spring Element.

(stiffness)



a) $f = K(x_1 - x_2) = Kx$.

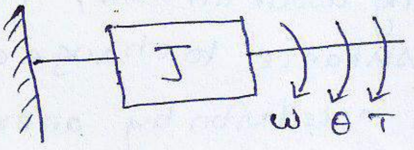
$x = x_1 - x_2$.

$= K\int(v_1 - v_2)dt$.

Rotational Systems:-

Input = Torque (T), output = angular displacement (or) angular velocity.

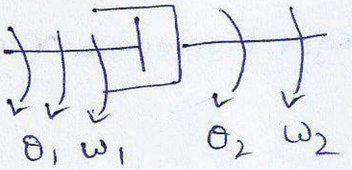
① Inertial Element



a) $T = J\frac{d\omega}{dt}$

b) $T = J\frac{d^2\theta}{dt^2}$

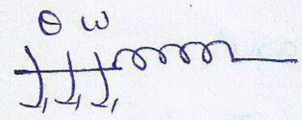
② Torsional Damper Element.



a) $T = B(\omega_1 - \omega_2)$

b) $T = B\frac{d}{dt}(\theta_1 - \theta_2)$

③ Torsional Spring Element.



a) $T = \omega K\int\omega\, dt$

b) $T = K\theta$.

Steps to Solve Problems on Analogous Systems. "Nodal Method"

① No. of nodes = No. of displacements.

② Take an additional node which is a reference node. Connect the mass (or) inertial mass element between the Principle node and reference node only
$\geq$1 Elements under Same displacement.

→ Elements causing change in displacement.

③ Connect the spring (or) damping element either b/n the Principle nodes (or) b/n the principle node and reference node — depending on their Position. (

④ Obtain the Nodal diagram and write the describing differential equation at each node.

⑤ In F-V analogy, use following replacements and rewrite eqns.

$F \rightarrow V$	$M \rightarrow L$	$B \rightarrow R$	$K \rightarrow 1/C$	$X \rightarrow q$	$\dot{x} \rightarrow i$

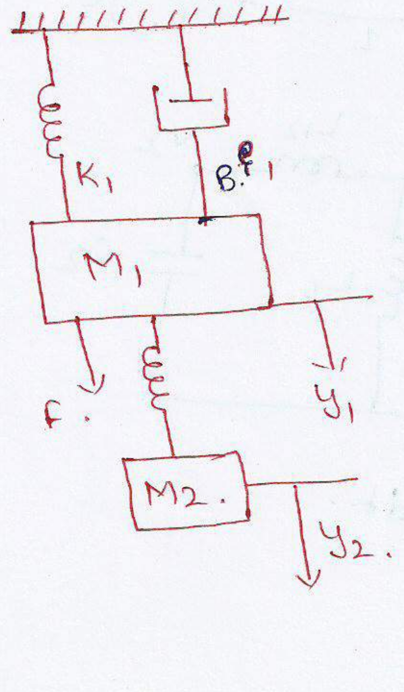
⑥ ~~Simulate~~. Draw the Electric n/w depending on eqns
No. of displacements = no. of loop currents.

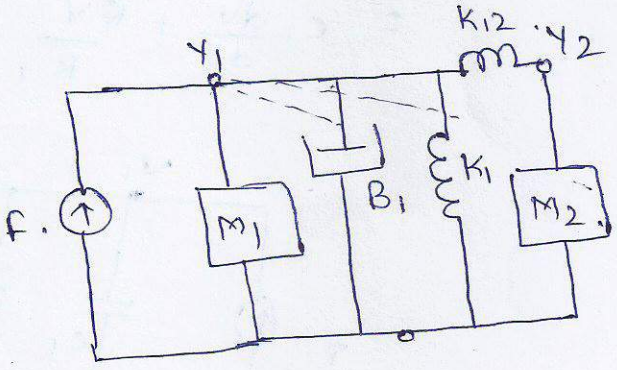
⑦ In F-I analogy, use following replacements and rewrite Eqns.

$F \rightarrow I$	$M \rightarrow C$	$B \rightarrow 1/R$	$K \rightarrow 1/L$	$X \rightarrow \phi$	$\dot{x} \rightarrow V$

⑦ Draw Electric n/w from Eqns.
No. of displacements = Node voltages.

Pb ①





At node Y_1

$$F = M_1\frac{d^2y_1}{dt^2} + B_1\frac{dy_1}{dt} + K_1y_1 + K_{12}(Y_1-Y_2)$$

At node Y_2.

$$0 = M_2\frac{d^2Y_2}{dt^2} + K_{212}(Y_2-Y_1)$$

Applying L.T to Eqn ①

$$F(S) = (M_1 S^2 + B_1 S + K_1 + K_{12})\, Y_1(S) - K_{12} Y_2(S) \quad — (3)$$

Laplace to ②

$$0 = (M_2 S^2 + K_2)\, Y_2(S) - K_{12} Y_1(S)$$

$$Y_2(S) = \left(\frac{K_{12}}{M_2 S^2 + K_{12}}\right) Y_{1S} \quad — (4).$$

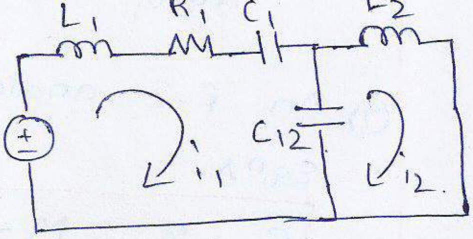
Sub $Y_2(S)$ in ③.

$$F(S) = \left[(M_1 S^2 + B_1 S + K_1 + K_{12}) - \frac{K_{12}^2}{M_2 S^2 + K_{12}}\right] Y_1(S)$$

$$\frac{Y_1(S)}{F(S)} = \frac{M_2 S^2 + K_{12}}{(M_1 S^2 + B_1 S + K_1 + K_2)(M_2 S^2 + K_{12}) - K_{12}^2}.$$

Force voltage analogy:-

$$V = L_1 \frac{di_1}{dt} + \cancel{\phi} R I_1 + \frac{1}{C_1}\int i_1\, dt + \frac{1}{C_{12}}\int (i_1 - i_2)\, dt$$

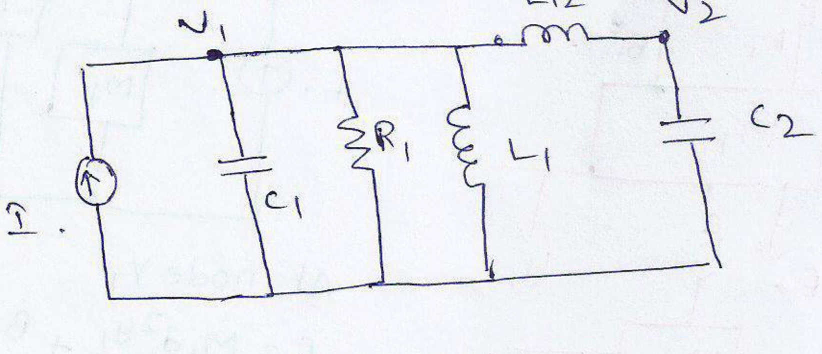


$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_{12}}\int (i_2 - i_1)\, dt.$$

Force-current Analogy:-

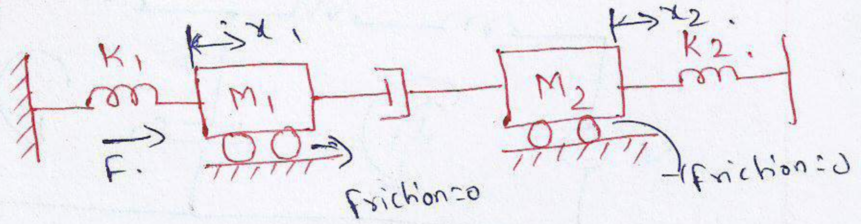
$$I_1 = C_1 \frac{d^2\phi}{dt^2} + \frac{\cancel{B}1}{R}\frac{d\phi_1}{dt} + K_1\phi_1 + K_{12}(\phi_1 - \phi_2)$$

$$= C_1 \frac{dv}{dt} + \frac{\cancel{B}V}{R_1} + \frac{\int v_1\, dt}{L} + \frac{1}{L_{12}}\int (v_1 - v_2)\, dt.$$

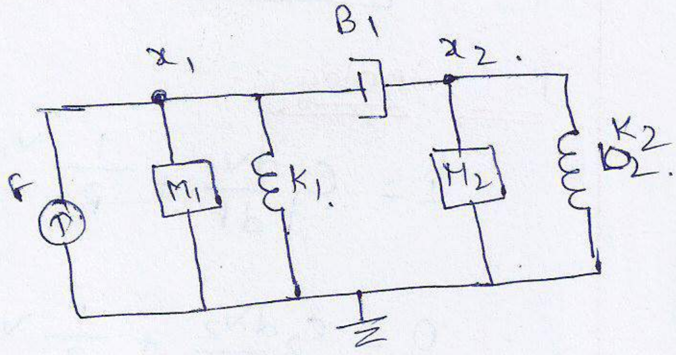


$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_{12}}\int (V_2 - V_1)\, dt.$$

② find the f-v analogy and f-I analogy for the following translational System:-



Nodal diagram:-



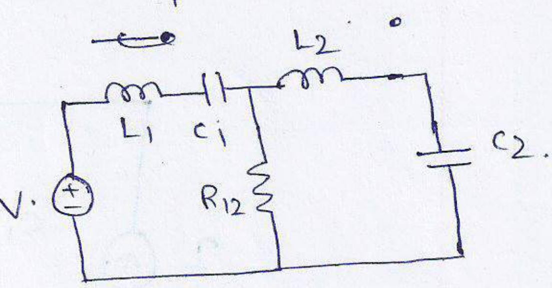
$$f = M_1\frac{d^2x_1}{dt^2} + K_1x_1 + B_{12}\left(\frac{d(x_1-x_2)}{dt}\right).$$

$$0 = M_2\frac{d^2x_2}{dt^2} + K_2x_2 + B_{12}\frac{d(x_2-x_1)}{dt}f$$

Force voltage Analogy:-

$$V = L_1\frac{di_1}{dt} + \frac{1}{C_1}\int i_1 dt + R_{12}(i_1-i_2)$$

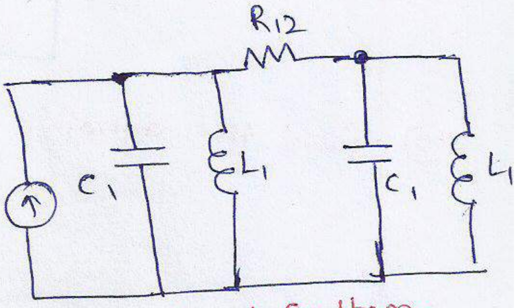
$$0 = L_2\frac{di_2}{dt} + \frac{1}{C_2}\int i_2 dt + R_{12}(i_2-i_1).$$



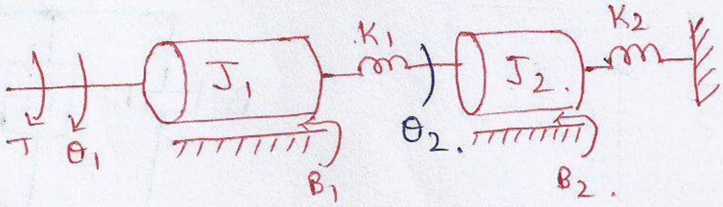
Force current Analogy:-

$$I = C_1\frac{dV}{dt} + \frac{1}{L_1}\int V_1 dt + \frac{1}{R_1}(V_1-V_2)$$

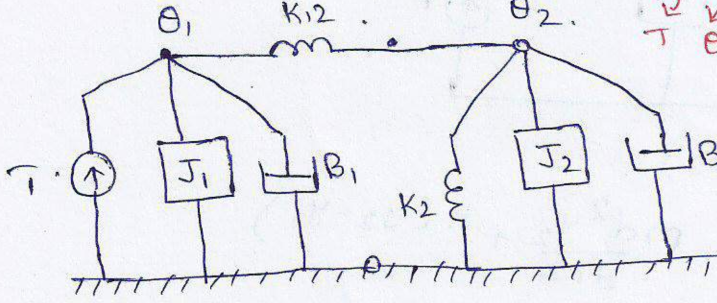
$$0 = C_2\frac{dV}{dt} + \frac{1}{L_2}\int V_2 dt + \frac{1}{R_2}(V_2-V_1).$$



③ find the R-N I@ T-V & T-I analogy. for given rotational system.



Nodal diagram



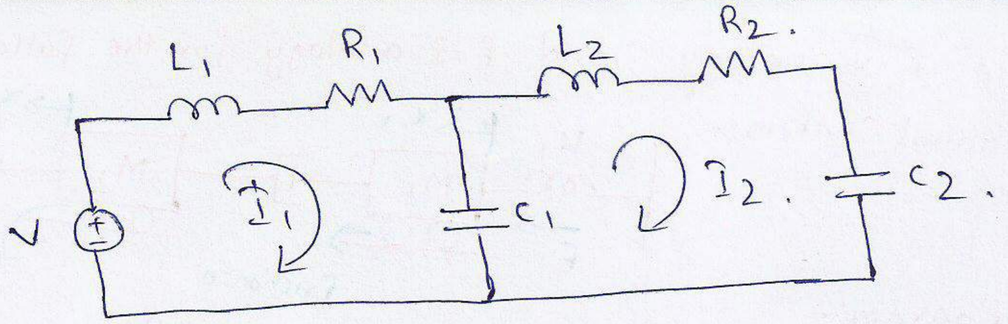
$$T = J_1\frac{d^2\theta_1}{dt} + B_1\frac{d\theta_1}{dt} + K_{12}(\theta_1-\theta_2)$$

$$0 = J_2\frac{d^2\theta_2}{dt} + B_2\frac{d\theta_2}{dt} + K_{12}(\theta_2-\theta_1) + K_2\theta_2.$$

T-V - analogy

$$V = \cancel{M} K_1 d^2\theta_1\ L_1\frac{dI_1}{dt} + R_1I_1 + \frac{1}{C_1}\int(I_1-I_2)dt.$$

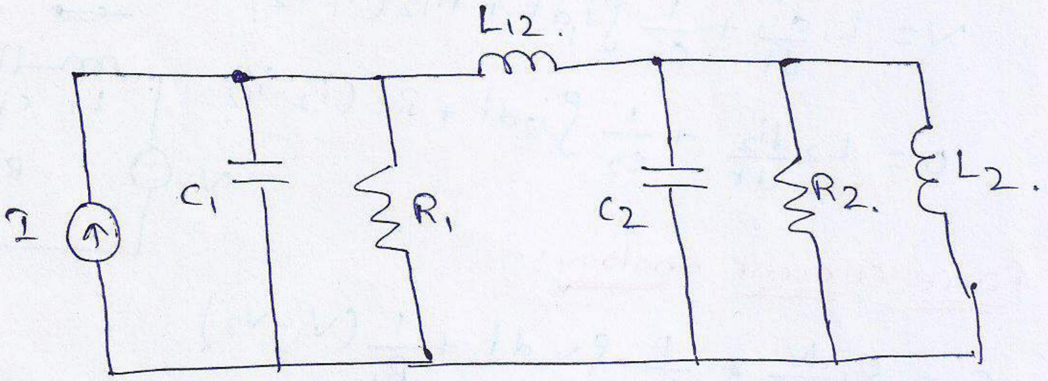
$$0 = L_2\frac{d I_2}{dt} + R_2I_2 + \frac{1}{C_1}\int(I_2-I_1)dt + \frac{1}{C_2}\int I_2 dt.$$



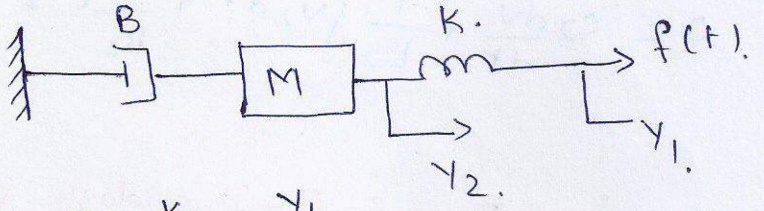
I-I Analogy:-

$$I = C_1\frac{dv_1}{dt} + \frac{1}{R_1}v_1 + \frac{1}{L_{12}}\int(v_1-v_2)\,dt.$$

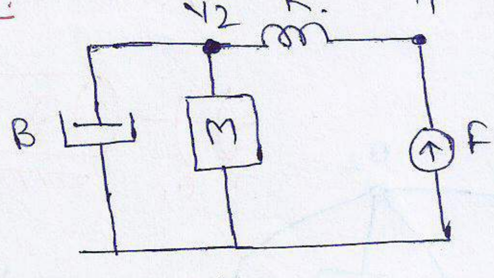
$$0 = C_2\frac{dV_2}{dt} + \frac{1}{R_2}V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{L_{12}}\int(V_2-V_1)\,dt.$$



④ Find the dynamic Equations for mechanical system.



Nodal diagram:-

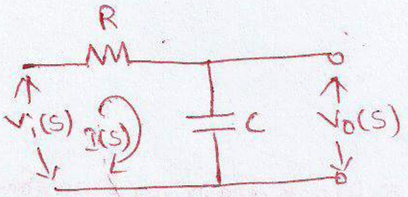


$$f(t) = K(y_1-y_2)$$

$$0 = B\frac{dy_2}{dt} + m\frac{d^2y_2}{dt^2} + K(y_2-y_1)$$

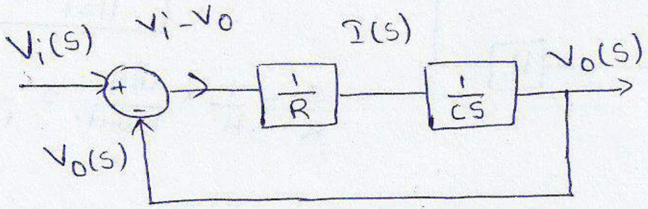
Block diagram Reduction

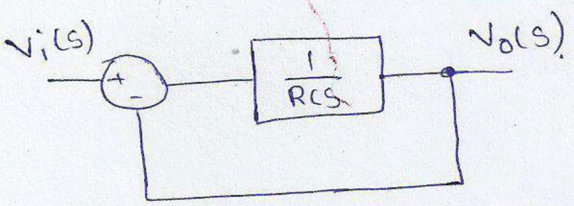
Ⓠ obtain the Block diagram of the following Electrical n/w.



$$I(S) = \frac{V_i(S) - V_o(S)}{R}$$

$$V_o(S) = I(S) \cdot \frac{1}{Cs}$$



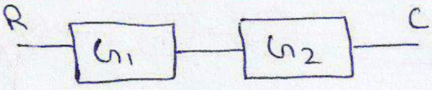


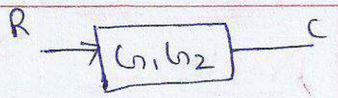
Rules

original B.D

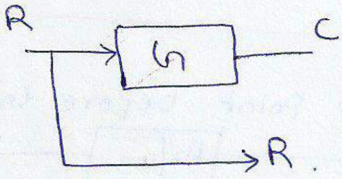
Equivalent. B.D.

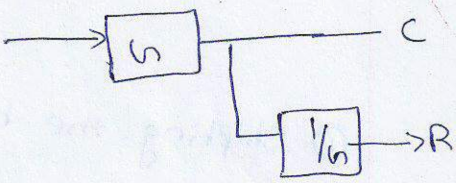
1. Cascading of Blocks



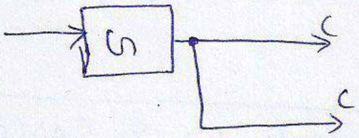


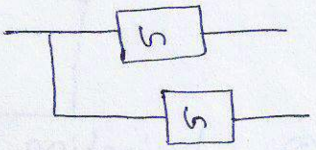
2. Shifting takeoff Point Pickoff Point after the block.



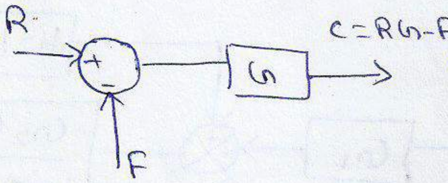


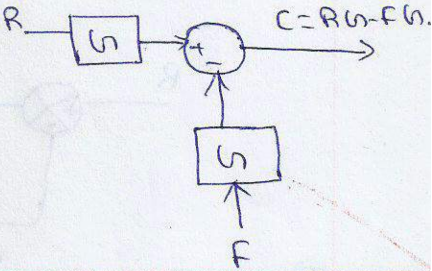
3. Shifting take off /Pickoff before the block.



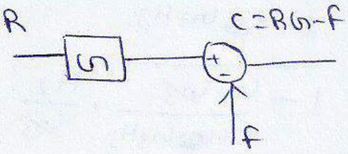


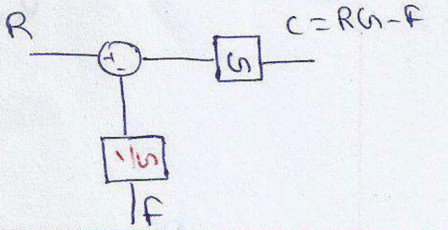
4. Shifting Summing Point after the block.



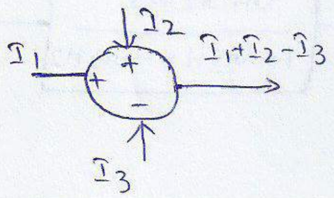


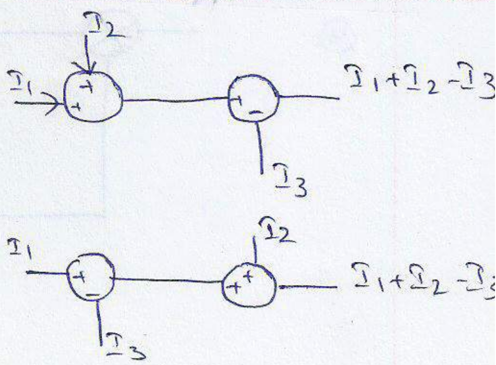
5. shifting Summing point before the block.



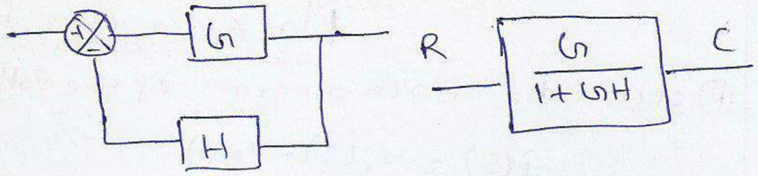


6. Equivalence of Summing Points.

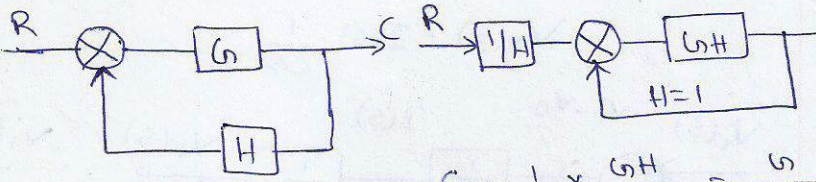




⑦ Eliminating feedbackloop



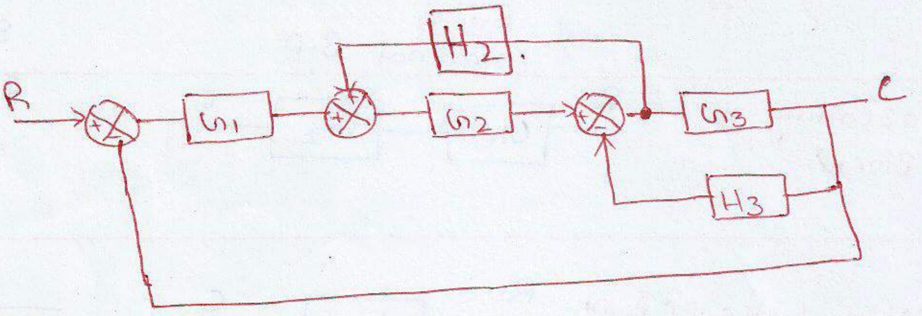
⑧ Block diagram transform



$$\frac{C}{R} = \frac{1}{H} \times \frac{GH}{1 \pm GH} = \frac{G}{1 \pm GH}.$$

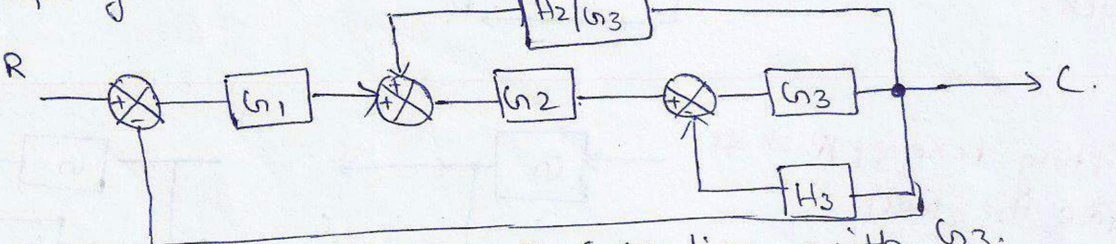
~~Critical roles~~:-

Problem ①

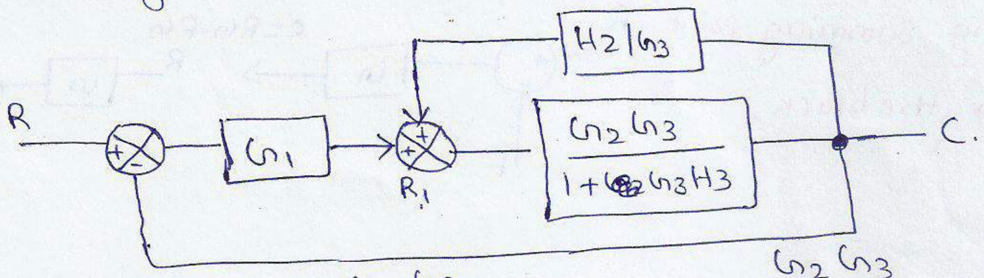


$$\frac{C}{R} = ?$$

① Shifting the take off point before G_3 to After G_3.

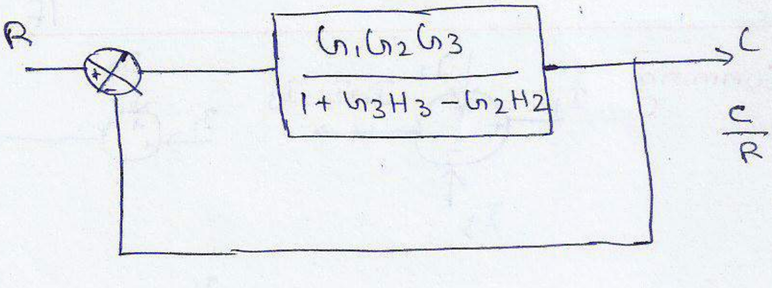


② Eliminating FB loop H_3 & Cascading with G_3.



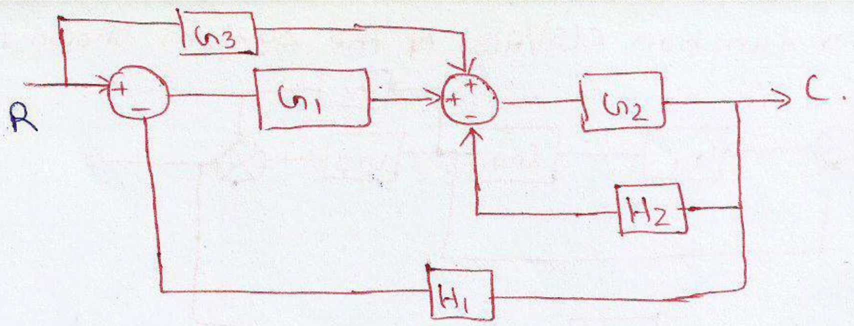
③ $$\frac{C}{R_1} = \frac{\frac{G_2G_3}{1+G_3H_3.}}{1-\frac{G_2\cancel{G_3}}{1+G_3H_3}\cdot\frac{H_2}{\cancel{G_3}}} = \frac{G_2G_3}{1+G_3H_3-G_2H_2.}$$

④

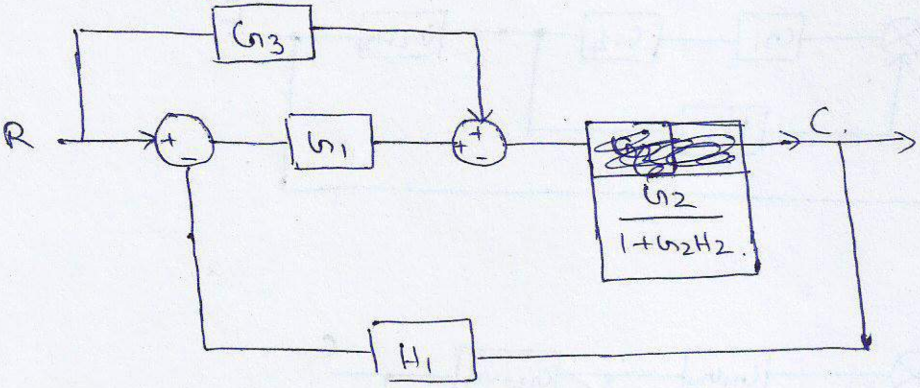


$$\frac{C}{R} = \frac{G_1G_2G_3}{1+G_3H_3-G_2H_2+G_1G_2G_3.}$$

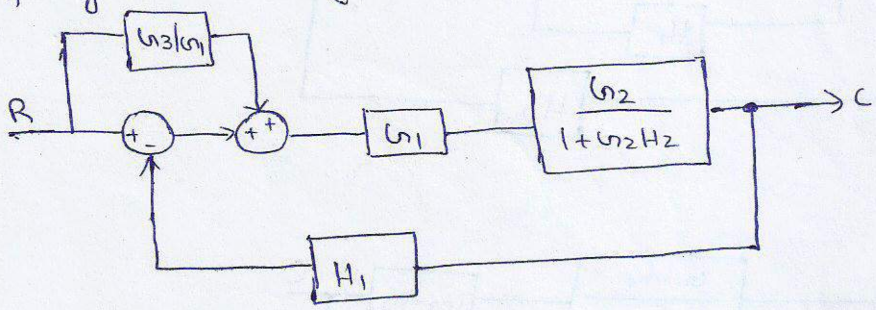
②



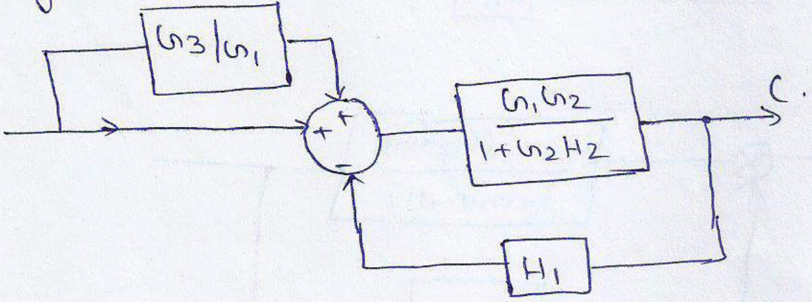
① Eliminating feedback G2 H2.



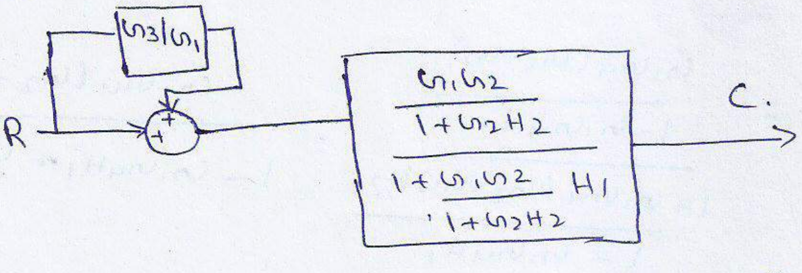
②· Shifting the summing point after G1 to before G1



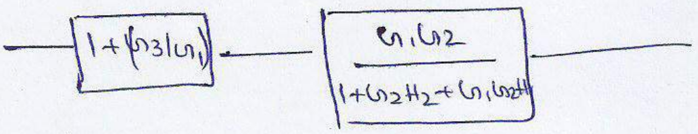
③ Combining the summing points.



④ Eliminate the feedback loop.

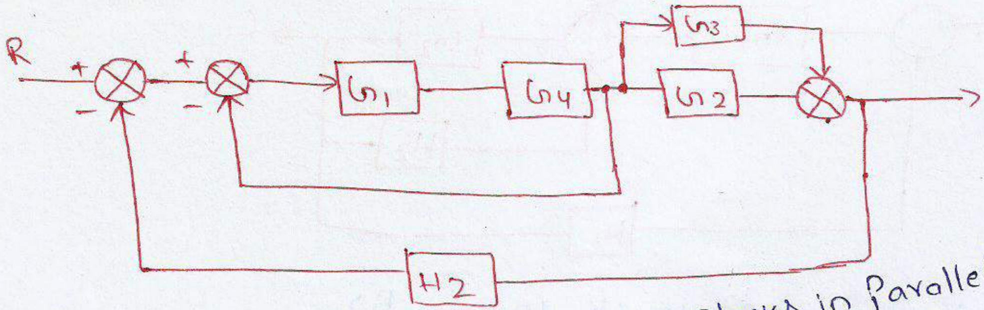


⑤

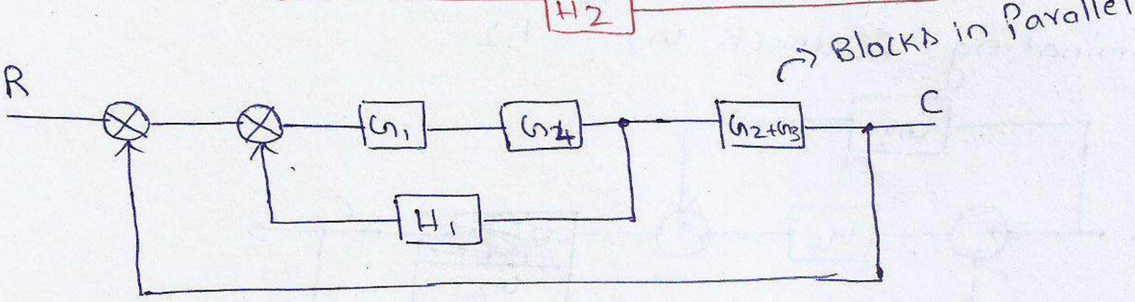


$$\frac{C}{R} = \frac{G_1G_2 + G_3G_2}{1+G_1G_2H_1+G_2H_2}.$$

③ Determine the transfer function C(S)/R(S) of the system shown in the fig.



Sol':-



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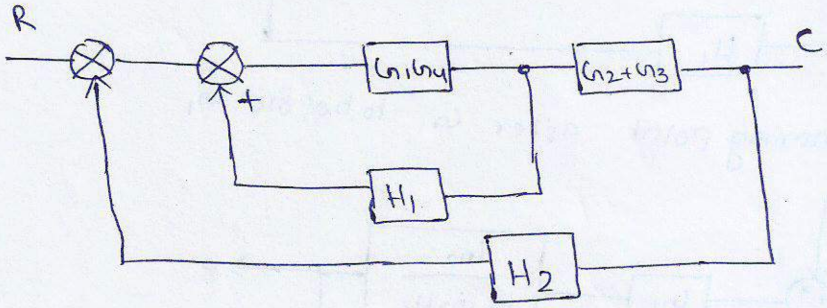

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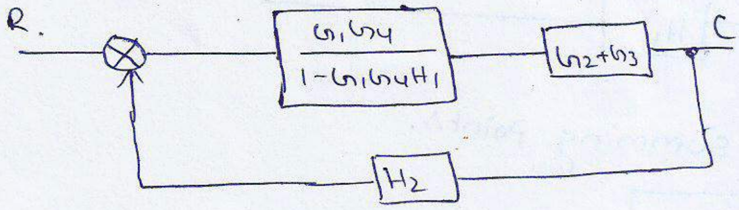

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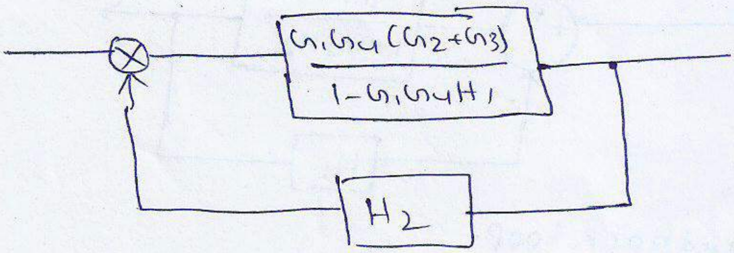
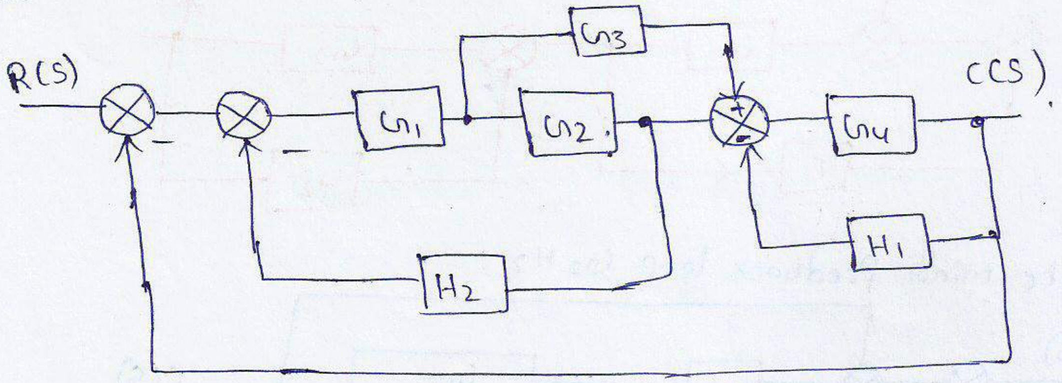


->.

$$\frac{C(S)}{R(S)} = \frac{\dfrac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1}}{\dfrac{1+G_1G_4(G_2+G_3)H_2}{1-G_1G_4H_1}} = \frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1+G_1G_4(G_2+G_3)H_2}$$

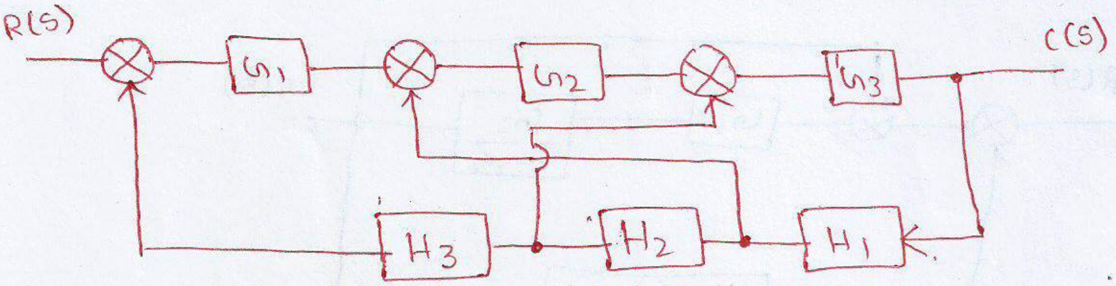
Problems

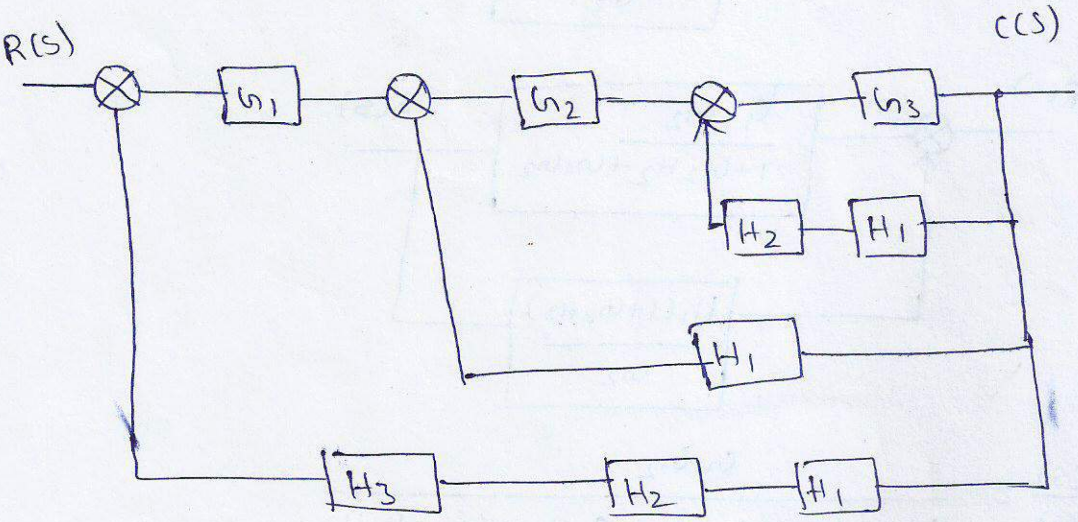
① Reduce BD to its simple form and hence obtain C(S)/R(S).



Ans $\frac{C(S)}{R(S)} = \frac{G_1G_4(G_2+G_3)}{1+G_1G_2H_2+G_4H_1+G_1G_2G_4H_1H_2+G_1G_4(G_2+G_3)}$

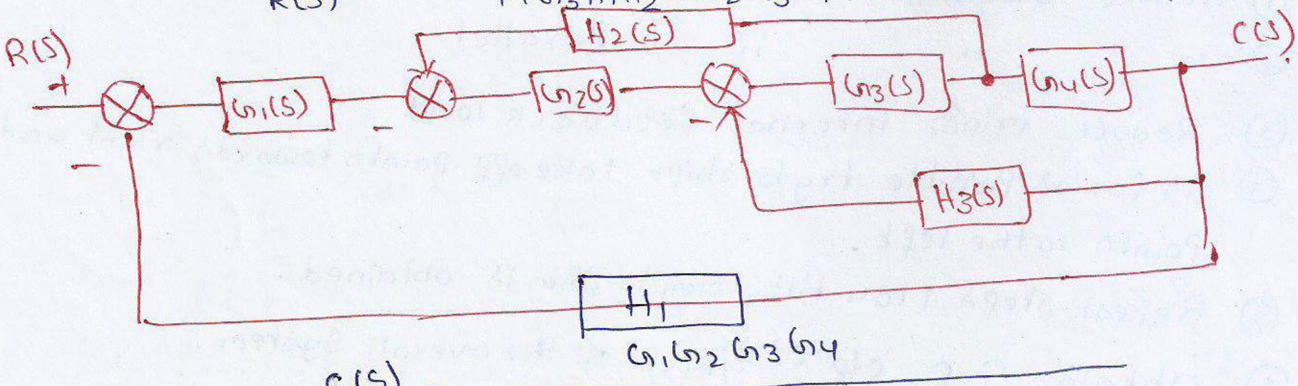
②





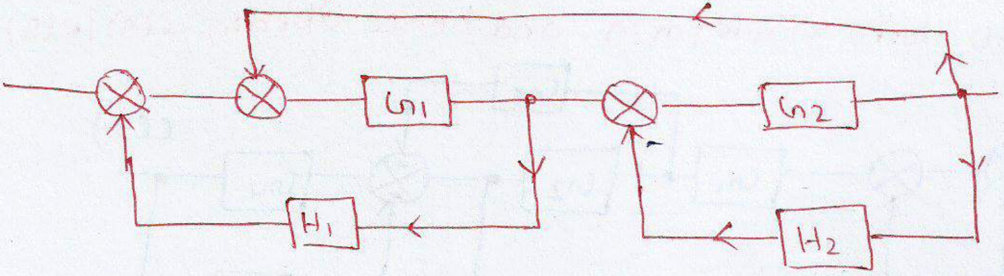
Ans = $\frac{C(S)}{R(S)} = \frac{G_1G_2G_3}{1+G_3H_1H_2+G_2G_3H_1+G_1G_2G_3H_1H_2H_3}$

③

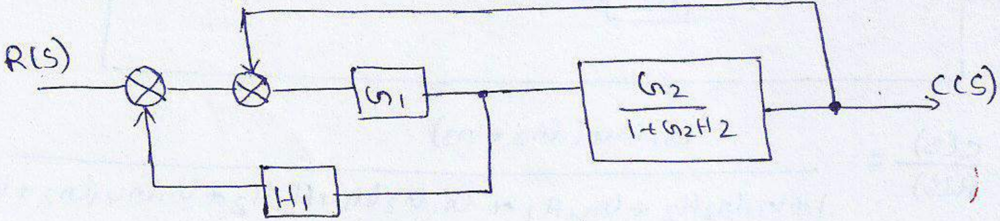


$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_4}{1+G_3G_4H_3+G_2G_3H_2+G_1G_2G_3G_4H_1}$

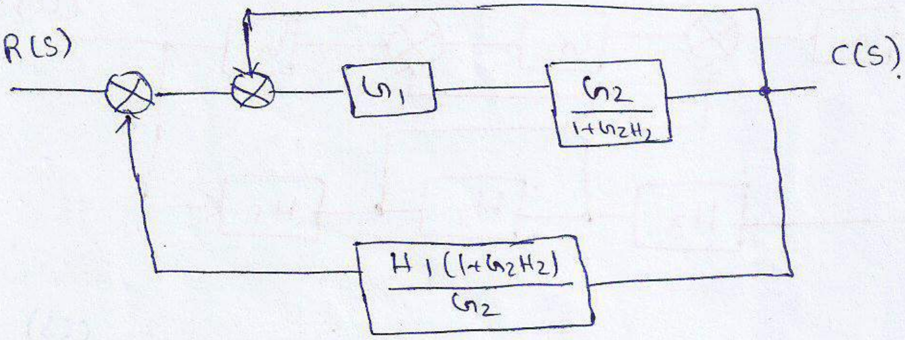
④ Reduce the block diagram and obtain its closed loop T.F C(S)/R(S).



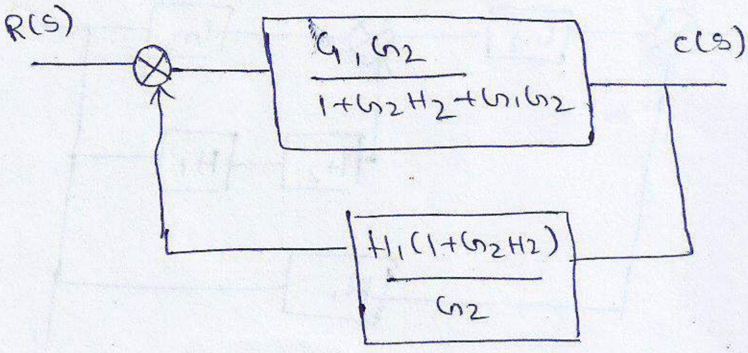
① Eliminate minor feedback loop G_2H_2.



② Shifting the takeoff to the Right.



③



c/s $$\frac{C(S)}{R(S)} = \frac{G_1G_2}{1+G_1G_2+G_2H_2+G_1H_1+G_1G_2H_1H_2}.$$

Procedure to solve Block Diagram Reduction Rules:-

① Reduce blocks connected in Series

② " " " " Parallel

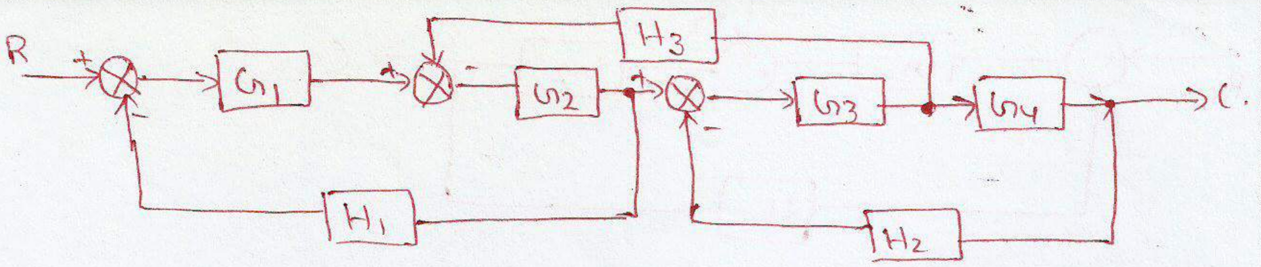
③ Reduce minor internal feedback loops

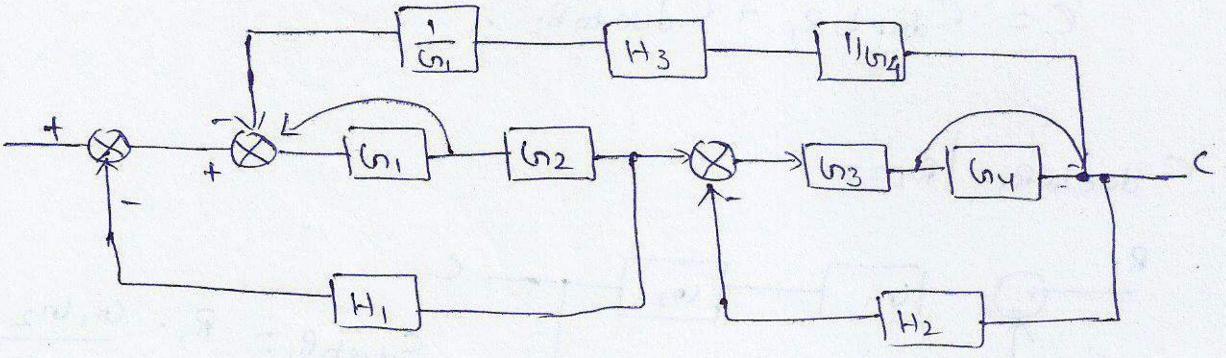
④ As far as Possible try to shift take off Points towards right and Summing Points to the left.

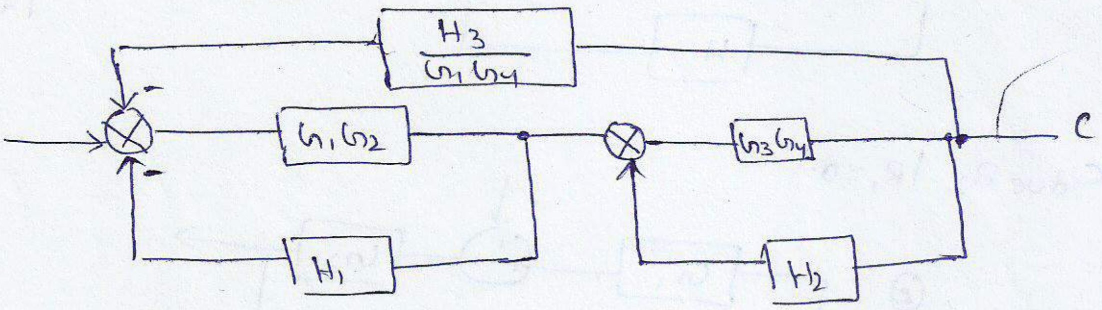
⑤ Repeat steps 1 to 4 till simple form is obtained.

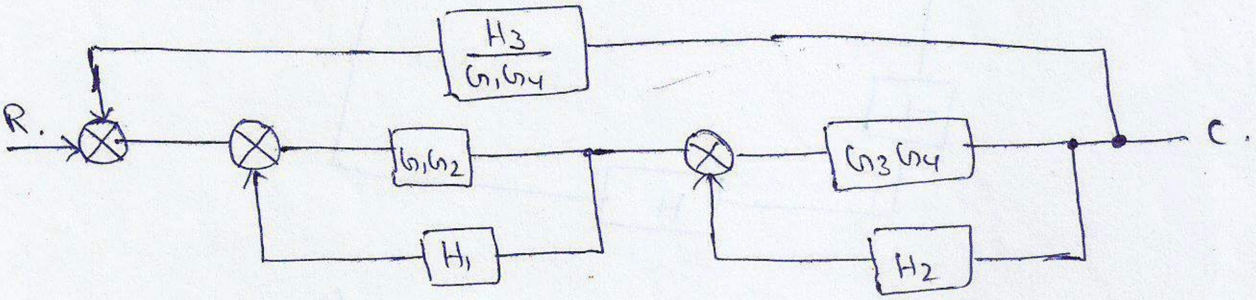
⑥ obtain T.F ~~o/p~~ C(S)/R(S) of the overall System.

Pb



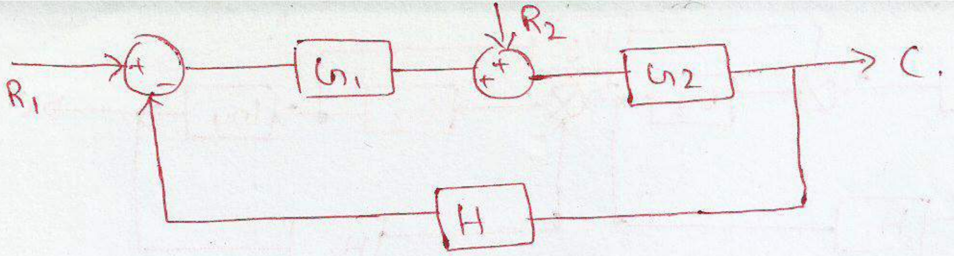






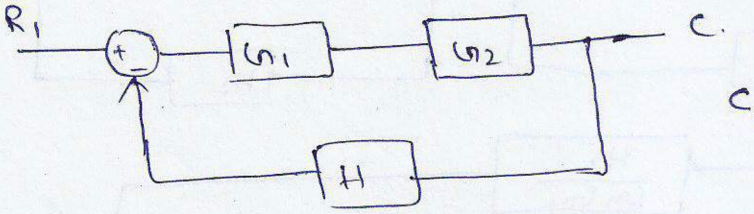
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_2 + G_1 G_2 G_3 G_4 H_1 H_2 + G_2 G_3 H_3}.$$

⑤



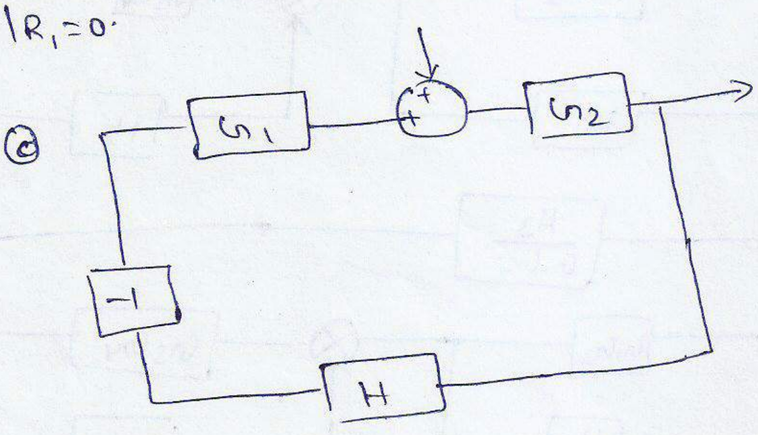
$C = C_{\text{due to } R_1} + C_{\text{due to } R_2}$.

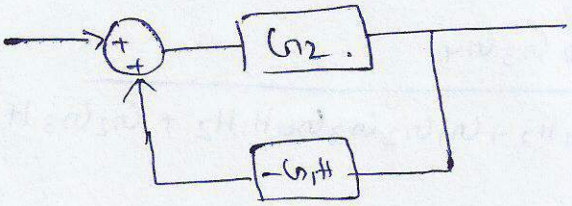
$C_{\text{due to } R_1} \big| R_2 = 0.$



$$C_{\text{due to } R_1} = R_1 \cdot \frac{G_1 G_2}{1 + G_1 G_2 H}.$$

$C_{\text{due } R_2} \, | R_1 = 0.$



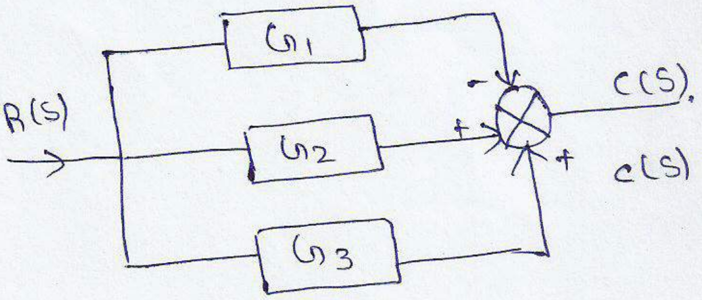


$$\frac{C}{R_2} = \frac{G_2}{1 - [G_2(G_1 H)]}$$

$$= \frac{G_2}{1 + G_1 G_2 H}.$$

$$C = \frac{R_1 G_1 G_2 + R_2 G_2}{1 + G_1 G_2 H}.$$

Note:-



$c(S) = R(S)\left[G_2 + G_3 - G_1\right].$

Signal flow Graph

SFG is a graphical representation of the control system inwhich nodes represent the system variable which are connected by - direct branches.

(Pb) obtain SFG of the system described by the following Equations

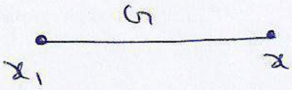
$$x_2 = a_{12}x_1 + a_{32}x_3$$

$$x_3 = a_{23}x_2 + a_{43}x_4$$

$$x_4 = a_{34}x_3 + a_{24}x_2 + a_{44}x_4.$$

Sol:- x_1, x_2, x_3, x_4 are called state variables. (eg:- voltages, currents.)

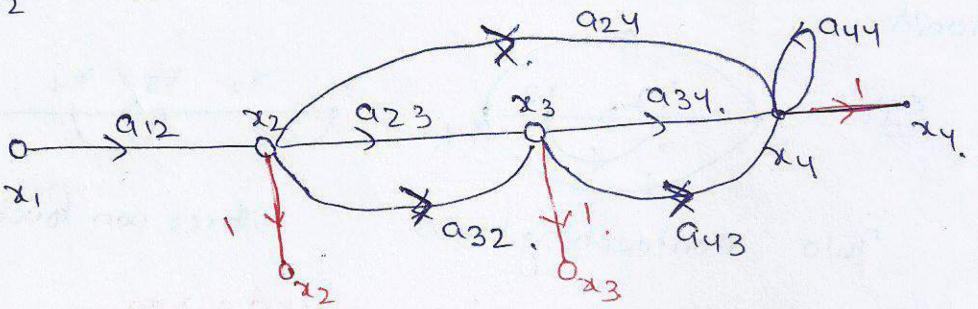
a_{12}, a_{32}, a_{23} - - - . are called branch gains (or) transmittances



G: Branch gain.

x_1 to x_2 gain is a_{12}

x_3 to x_2 " a_{32} - - - .



Terminology:-

① I/P (or) Source node:- A node with only outgoing branches.

eg:- x_1

② o/P (or) Sink node:- A node with only incoming branches except. Except i/P node any node can be converted to an o/P node.

eg:- x_2, x_3, x_4.

③ forward Path:- A Path which starts at the i/P & terminates at the o/P along which no node is repeated more than once.

eg:- Let x_4 be o/P node.

forward Path		forward Path gain.
$x_1 - x_2 - x_3 - x_4$	–	$a_{12}\,a_{23}\,a_{34}$
$x_1 - x_2 - x_4$	–	$a_{12}\,a_{24}$.

④ **Loop (or) FB loop (or) Individual loop:-**

A Path which starts and terminates at the same point along which no node is repeated more than once.

eg:-

$x_2 - x_3 - x_2$. — $a_{23} \cdot a_{32}$

$x_3 - x_4 - x_3$ — $a_{34} \cdot a_{43}$

$x_4 - x_4$ — a_{44}

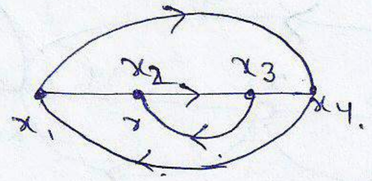
$x_2 - x_4 - x_3 - x_2$ — $a_{24} \cdot a_{43} \cdot a_{32}$.

⑤ **Self Loop:-** A feedback loop consisting of only one node is called self loop.

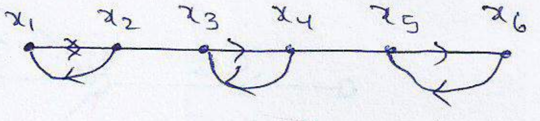
eg:- a_{44} at x_4.

⑥ **Non-touching loops:-** If there is no node common in between the two or more ~~norx~~ loops, such loop are called non touching loops.

eg:-



Two nontouching loops



Three non touching loops.

Procedure to obtain SFG from Blockdiagram

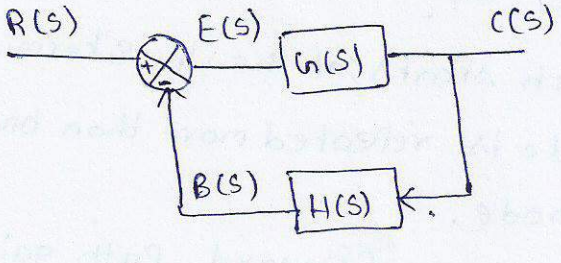
① Name all the summing points and take off point in the BD.

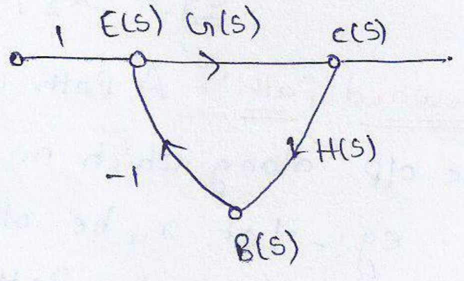
② Represent each SPs & take of points. by a separate node in SFG.

③ connect them by branches instead of blocks, indicating block transfer function as the gains of corresponding branches.

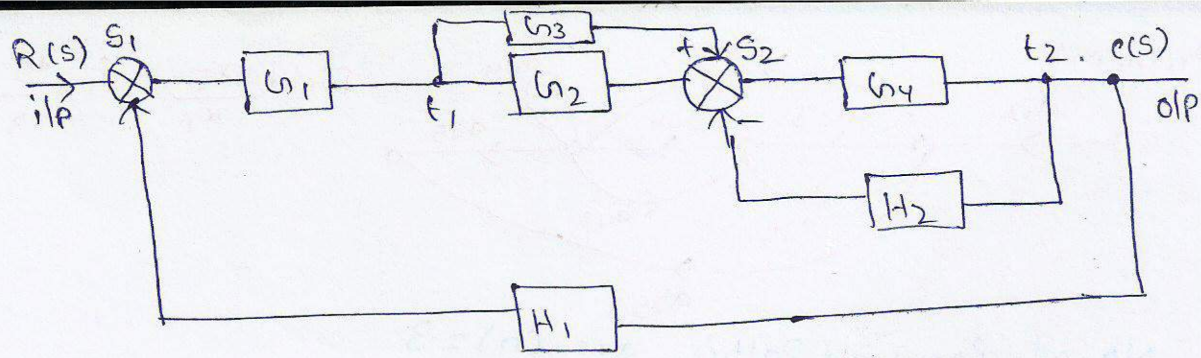
④ Show the i/p & o/p nodes separately if required, to complete "SFG"..

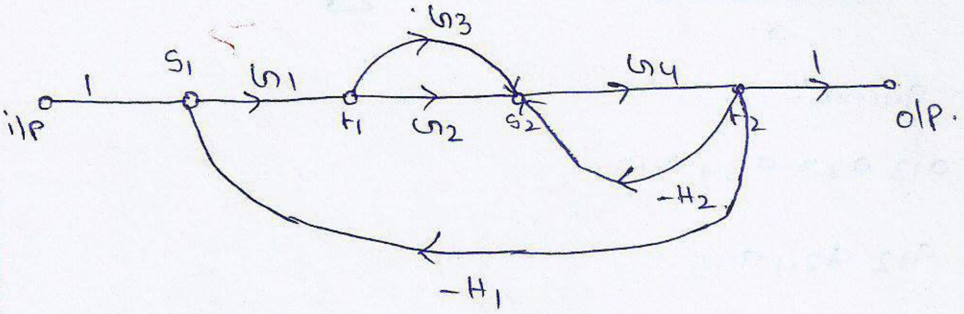
pb





②





Mason's Gain Formula:-

Overall gain (or) Transfer function = $\frac{o/p}{i/p} = \frac{\sum_{n=1}^{n} M_K \Delta_K}{\Delta}$

$n \rightarrow$ no. of forward paths.

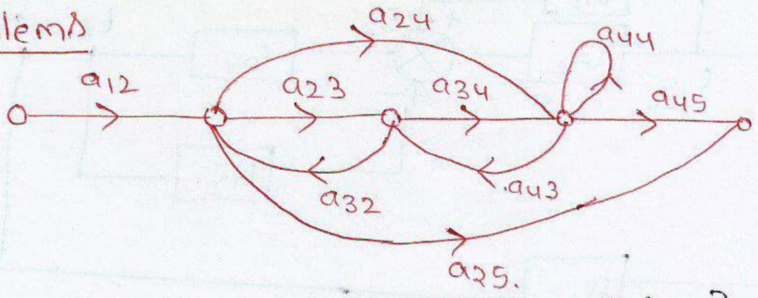
$M_K \rightarrow K^{th}$ forward path gain

$\Delta_K \rightarrow$ The value of Δ, which is not touching the k^{th} forward path.

$\Delta = 1 -$ (sum of loop gains) + (sum of gain product of two non touching loops) ~~+~~ − (sum of gain product of 3 non touching loops) + - - - .

Δ is called the determinant.

Problems

① 

find $\frac{x_5}{x_1}$ & $\frac{x_5}{x_3}$

Sol:- No. of forward Paths are (n) = 3

$$\therefore \frac{x_5}{x_1} = \frac{\sum_{k=1}^{3} M_K \Delta_K}{\Delta} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3}{\Delta}$$

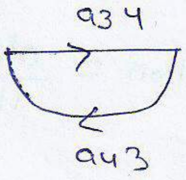
Forward Path gains:

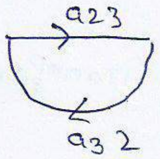
$M_1 = a_{12}\, a_{23}\, a_{34}\, a_{45}.$

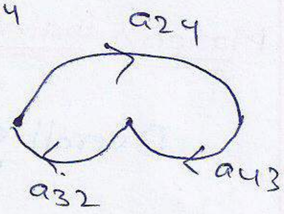
$M_2 = a_{12}\, a_{24}\, a_{45}$

$M_3 = a_{12}\, a_{25}.$

Individual loops.



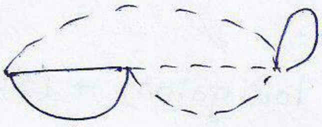




$L_1 = a_{34} a_{43}$ $L_2 = a_{23} a_{32}$ $L_3 = a_{44}$ $L_4 =$

$L_4 = a_{24} a_{43} a_{32}.$

Gain Product of two non touching loops.

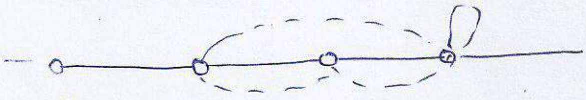


$L_2 L_3 = a_{23} a_{32} a_{44}.$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_2 L_3)$

$= 1 - (a_{34}a_{43} + a_{23}a_{32} + a_{44} + a_{24}a_{43}a_{32}) + a_{23}a_{32}a_{44}.$

Δ_1 = The value of Δ not touching 1st forward Path.



$\Delta_1 = 1 - (0+0+0+0) + 0 = 1$

$\Delta_2 = 1$

$\Delta_3 = 1 - (a_{34}a_{43} + a_{44})$

$$\therefore \frac{x_5}{x_1} = a_{12}a_{23}a_{34}a_{45}(1) + a_{12}a_{24}a_{45}(1) + a_{12}a_{23}\left[1-(a_{34}a_{43}+a_{44})\right] \quad -(1)$$

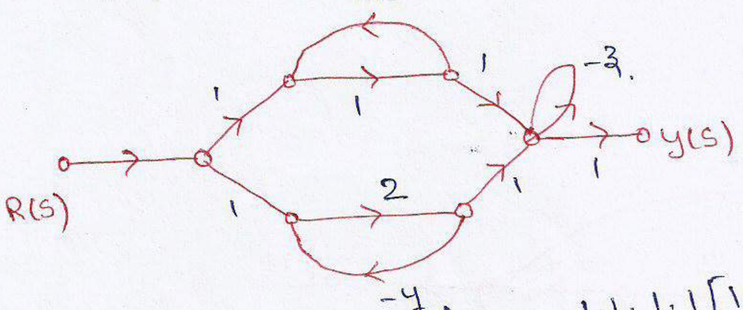
② Mason's gain formula can't be applied between x_5 & x_3 as both are o/p nodes.

$$\therefore \frac{x_5}{x_3} = \frac{x_5}{x_1} \div \frac{x_3}{x_1}$$

$$\frac{x_3}{x_1} = \frac{a_{12}a_{23}(1-a_{44}) + a_{12}a_{24}a_{43}}{\Delta} \quad -(2)$$

$$\frac{x_5}{x_3} = \frac{x_5/x_1}{x_3/x_1} = \frac{(1)}{(2)}$$

②

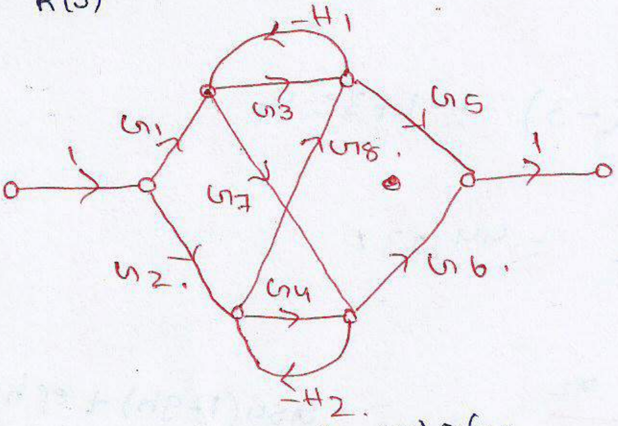


$$\frac{Y(s)}{R(s)} = \frac{M_1\Delta_1 + M_2\Delta_2}{\Delta} = \frac{1.1.1.1\left[1-(-2y)\right] + (1.2.1.1)\left[1-(-x)\right]}{\Delta}$$

$$\Delta = 1 - [-x - 2y - 3) + (x3 - 2y3 + 2xy) - (-2xy3)$$

$$\frac{Y(s)}{R(s)} = \frac{(1)\left[1+2y\right] + 2(1+x)}{\Delta}.$$

③



No. of forward paths (n) = 6.

$$\frac{C}{R} = \frac{\sum_{n=1}^{6} M_K\Delta K}{\Delta} = \frac{M_1\Delta_1 + M_2\Delta_2 + M_3\Delta_3 - - - + M_6\Delta_6}{\Delta}$$

Forward Path gains

$M_1 = G_1G_3G_5$ $\quad M_2 = G_1G_7G_6$ $\quad M3 = G_1G_7(-H2)G_8G_5.$

$M3 = G_2G_4G_6$ $\quad M_4 = G_2G_8G_5$ $\quad m6 = G_2G_8(-H_1)G_7G_6$

$L_1 = -G_3H_1$ ~~G~~ $L_2 = G_4(-H_2$ $L_3 = G_7(-H_2)G_8(-H_1)$

$= -G_4H_2.$ $= G_7G_8H_1H_2.$

2 non touching Loops $L_1L_2 = G_3G_4H_1H_2.$

$\Delta = 1-(-G_3H_1 - G_4H_2 + G_7G_8H_1H_2) + G_3G_4H_1H_2,$

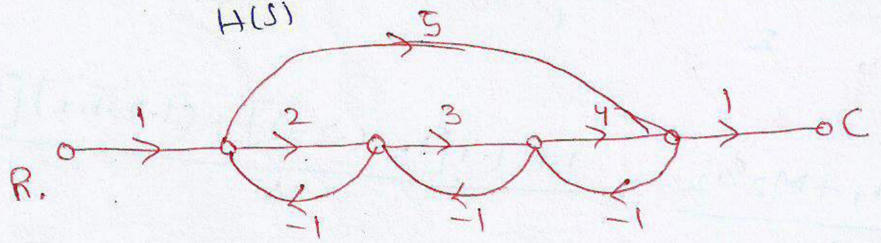
MY1/F

$\Delta = 1 - (-G_3H_1 - G_4H_2 + G_7G_8H_1H_2) + G_3G_4H_1H_2.$

$\Delta_1 = 1 + G_4H_2.$ $\Delta_3 = 1 + G_3H_4.$ $\Delta_2 = \Delta_5 = \Delta_4 = \Delta_6 = 1$

$\frac{C(S)}{H(S)} =$

④



$\Delta \frac{C}{R} =$ ~~7M1/~~ $1-(-2-3-4.-5)+(8)$

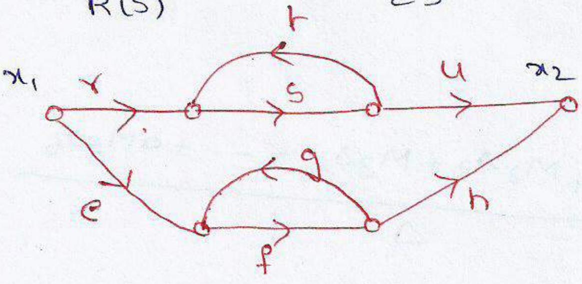
$= 1+14+18 = 23$

$M_1 = 24$ $M_2 = 5.$

$\Delta_1 = 1$ $\Delta_2 = 1-(-3) = 1+3 = 4.$

$\frac{C(S)}{R(S)} = \frac{24+20}{23} = 44/23.$

⑤



$\frac{x_2}{x_1} = \frac{rsu(1-gh) + efh(1-st)}{1-(st+gf)+stgf}.$

$= \frac{rsu(1-gh)}{(1-st)(1-fg)} + \frac{efh(1-st)}{(1-st)(1-fg)}$

$= \frac{rsu}{1-st} + \frac{efh}{1-fg}.$

Unit-II
TIME RESPONSE ANALYSIS

Syllabus:- standard test signals - Time response of order systems - characteristic Eqn of FB control systems - Transient Response of second order system - Time domain specifications - Steady state Response - Steady state errors and error constants - - Effects of PD, PI systems.

→ The o/p variation w.r.t time is analyzed by applying standard test signals.

Test Signals	$r(t)$	$L[r(t)] = R(S)$
Impulse - shock	$\delta(t)$.	1
step - Position	$u(t)$	$1/s$.
ramp - velocity	$t u(t)$	$1/s^2$.
Parabolic - acceleral	$\frac{t^2}{2} u(t)$	$1/s^3$

$\int_0^t$ (Impulse response) dt = Step Response. $\quad \frac{d}{dt}(P.R) = R.R.$

$\int_0^t$ (Step Response) dt = Ramp Response. $\quad \frac{d}{dt}(R.R) = S.R$

$\int_0^t$ (Ramp Response) dt = Parabolic response. $\quad \frac{d}{dt}(S.R) = I.R.$

Total Response = Transient + Steady State Response.

Time Response:- The response given by the system which is a function of the time, to the applied excitation is called time response of control system.

Transient Response:- The o/p variation during the time, it takes to achieve it's final value is called transient period.

→ from this Response speed of the system is Analyzed.

Steady state Response:- It is that part of the time response which remain after complete transient response vanishes from system o/p.

→ Steady state Accuracy of system is Analyzed.

Total time ~~AP~~ response,

$$c(t) = c_{ss} + c_t(t).$$

Initial value of the response c(t):-

$$\text{Initial value } c(0) = \underset{t \to 0}{Lt}\ c(t)$$

$$\boxed{c(0) = \underset{s \to \infty}{Lt}\ s\,C(s)}$$ Initial value theorem

* To apply initial value theorem, the no. of poles of C(s) should be more than no. of zeroes i.e., C(s) must be strictly proper.

Final value of the response c(t):-

All poles of sC(s) should be lie in Left side of s-plane.

$$c(\infty) = \underset{t \to \infty}{Lt}\ c(t)$$

$$\boxed{c(\infty) = \underset{s \to 0}{Lt}\ s\,C(s)}$$ final value theorem.

Problems

① Find the Initial and final values of the following systems to a step i/p ?

a) $T.F = \dfrac{10(s+2)}{(s+4)(s^2+4s+5)}$. b). $T.F = \dfrac{4}{s^2+1}$

a) $$\frac{C(s)}{R(s)} = \frac{10(s+2)}{(s+4)(s^2+4s+5)} \Rightarrow C(s) = \frac{10(s+2)}{s(s+4)(s^2+4s+5)}$$

$$c(0) = \underset{s \to \infty}{Lt}\ s\,C(s) = \underset{s \to \infty}{Lt}\ \frac{10(s+2)}{(s+4)(s^2+4s+5)} = \frac{\cancel{10 \times 2}}{\cancel{4 \times 5}}$$

$$= \underset{s \to \infty}{Lt}\ \frac{10 \cdot \cancel{s}}{s \cdot s^2} = \underset{s \to \infty}{Lt}\ \frac{10}{s^2} = 0 \,//$$

$$c(\infty) = \underset{s \to 0}{Lt}\ s\,C(s) = \frac{10 \times 2}{4 \times 5} = 1.$$

b) $$\frac{C(s)}{R(s)} = \frac{4}{s^2+1} \Rightarrow C(s) = \frac{4}{s(s^2+1)}.$$

$$c(0) = \underset{s \to \infty}{Lt}\ \frac{4}{s^2+1} = 0.$$

$$c(\infty) = \underset{s \to 0}{Lt}\ \frac{4}{s^2+1}$$ Poles on Imaginary axis c(∞) can't be determined.

Time Response of 1st order Systems.:-

Type & order of the System:-

1. Every transfer function representing controlsystem has - certain type & order
2. Steady state Analysis depends on type of Control system
3. The type of the system is obtained from OLTF $G(S)H(S)$.
4. The no.of OL Poles occuring at origin gives the type of the control system.

$$\text{Let} \quad G(S)H(S) = \frac{K(1+T_a S)}{S^P(1+T_1 S)}$$

P=0 → Type-0 System

P=1 → Type-1 System

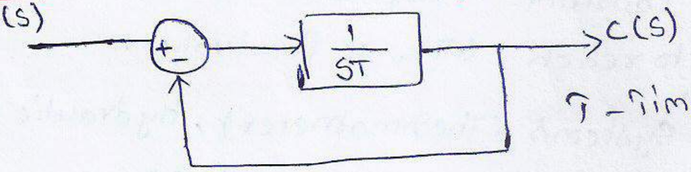
5. The transient state Analysis depends on order of system
6. The ~~transfer of~~ order of control System is obtained from CLTF

$$CLTF = \frac{G(S)}{1+G(S)H(S)}$$

7. Highest power of characteristic eqn $1+G(S)H(S)=0$, determin order of the control system.

First order Systems

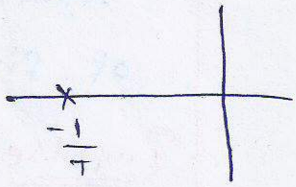
eg:- $\tau = RC$, RC circuit.



T - Time constant of the syste

$$T.F = \frac{C(S)}{R(S)} = \frac{1}{1+ST}$$

$$C(S) = \frac{1}{1+ST} R(S)$$



Step Response:-

$r(t) = u(t)$; $R(S) = 1/S$

$$C(S) = \frac{1}{1+ST} \times \frac{1}{S} = \frac{A}{S} + \frac{B}{1+ST}$$

$$A = \frac{1}{1+ST}\Big|_{S=0} = 1 \qquad B = \frac{1}{S}\Big|_{S=-1/T} = -T$$

$$C(S) = \frac{1}{S} - \frac{T}{1+ST} = \frac{1}{S} - \frac{1}{S+1/T}$$

$c(t) = (1 - e^{-t/T})\,u(t)$

↳ Transient response

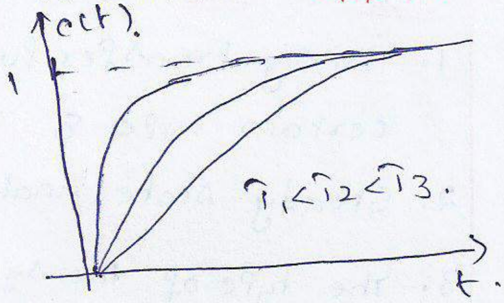
↳ steady state response.

1 − 0.367

1 − 0.0497 (e3)

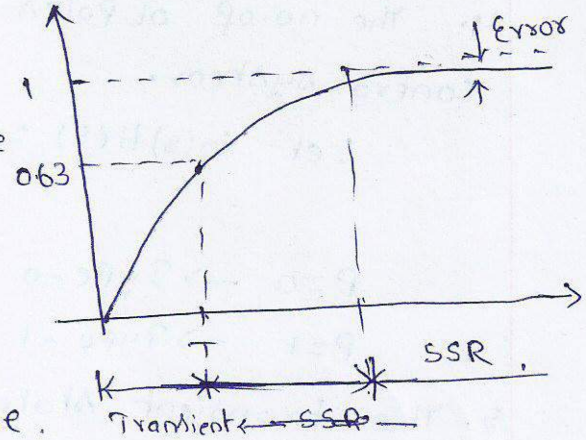
1 −

99.3%.



→ $c(0) = 0$, $c(\infty) = 1$.

$$\text{Error} = \lim_{t \to \infty} [\underset{\text{i/p}}{r(t)} - \underset{\text{o/p}}{c(t)}] = 0$$



$c(t)|_{t=T} = [0.63][c(\infty)] = 0.63$

63% of steady state value

$c(t)|_{t=3T} = 95\%$ of $c(\infty)$.

$c(t)|_{t=5T} = 99.3\%$ of $c(\infty)$

→ For the system to settle 4 to 5 time constants are required.

Transient time ≃ settling time.

→ For stable system TR / Transient response approaches to zero when $t \to \infty$.

→ The time constant is defined as time taken by response of the system to reach 63% of final value.

→ Thermal systems, (thermometer), hydraulic system (liquid level system) and pneumatic systems (pressure vessels) are examples of first order systems. $\boxed{X_o(t) = K_o + (K_i - K_o)e^{-t/T}}$

K_i – initial value.

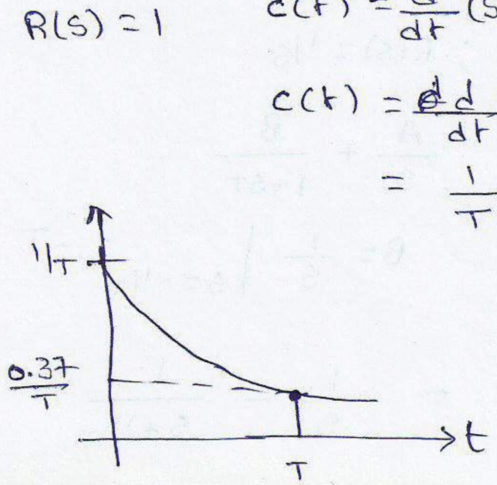
K_o – final value.

Impulse Response:-

$R(S) = 1$ $\quad c(t) = \frac{d}{dt}$ (step response)

$$c(t) = \frac{d}{dt}(1 - e^{-t/T})\,u(t)$$

$$= \frac{1}{T}e^{-t/T}u(t)$$



Ramp Response:-

$R(S) = \frac{1}{S^2}$ $\quad c(t) = \int_0^t (\text{stepresponse})\, dt$

$= \int_0^t (1-e^{-t/T})dt$

$= t - \left.\frac{e^{-t/T}}{-1/T}\right]_0^t = t + Te^{-t/T} - T$

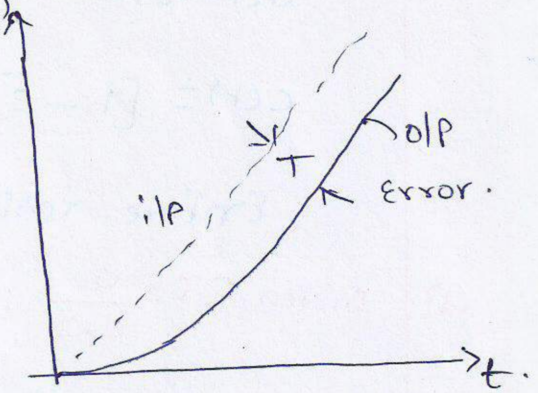
$= t - T[1-e^{-t/T}]$

$c(t) = t - T[1-e^{-t/T}]$

$c(\infty) = \lim_{t\to\infty} c(t) = (t-T)$; $\text{Error} = \lim_{t\to\infty} [r(t)-c(t)]$

$= t-(t-T) = T$

-> By decreasing time constant T, the Error can be reduced.



(Pb) $T.F = \frac{10}{S+2}$; obtain & sketch response for $u(t)$ & $2u(t)$.

(1). $c(0) = \lim_{S\to\infty} S.C(S)$.

$= \lim_{S\to\infty} S.T.F.R(S) = \lim_{S\to\infty} (S).\frac{10}{S+2}.\frac{1}{S}. = 0.$

$\frac{10}{2(\frac{1}{2}S+1)}$

$c(\infty) = \lim_{S\to 0} \frac{10}{S+2} = 10/2 = 5.$

$c(t) = 5 - 5e^{-t/T}$ $\qquad T = 1/2.$

$c(t) = 5 - 5e^{-2t}.$

(ii) $c(0) = 0$

$c(\infty) = 10/2 \times 2 = 5\times 2 = 10$

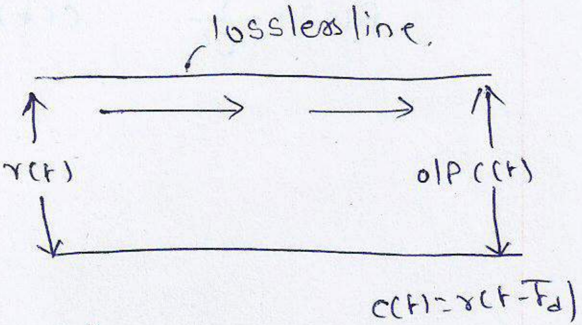
$\therefore c(t) = 10\,(1-e^{-2t}).$

Transfer fn of a System with Pure delay (or) Transportation lag (or) dead time (Td) :-

$c(t) = r(t - T_d)$

$C(S) = e^{-ST_d} R(S)$

$T.f = \frac{C(S)}{R(S)} = e^{-ST_d}$

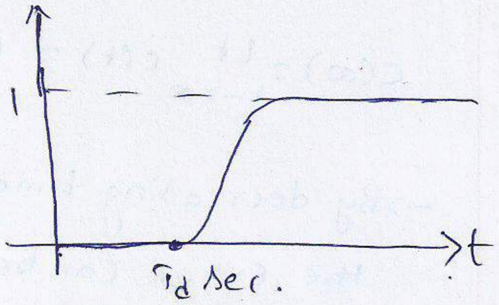


Pb ① Given $T.f = \frac{e^{-ST_d}}{1+ST}$ obtain step respons.

$u(t) \longleftrightarrow \frac{1}{S}$

$u(t - T_d) \longleftrightarrow \frac{1}{S} e^{-ST_d}$

$c(t) = \left[1 - e^{-\left(\frac{t - T_d}{t}\right)}\right] u(t - T_d)$



Entire response is delayed by Td sec.

② Given $T.f = \frac{4}{S+2}$; obtain the response for u(t-3).

$r(t) = u(t-3)$

$R(S) = L[u(t-3)] = \frac{e^{-3S}}{S}.$

$\frac{C(S)}{R(S)} = \frac{4}{S+2}. \Rightarrow C(S) = \frac{4}{S+2} . R(S) = \frac{4}{S+2} \frac{e^{-3S}}{S}.$

$\frac{C(S)}{e^{-3S}} = \frac{4}{S(S+2)}. = \frac{2}{S} - \frac{2}{S+2}.$

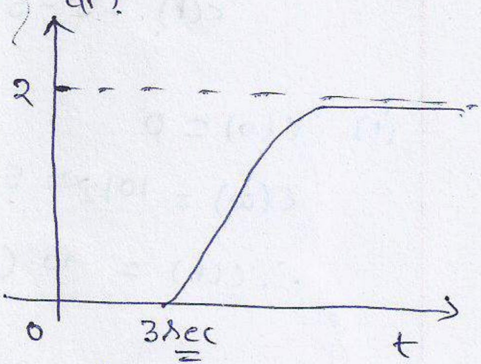
$C(S) = \frac{2e^{-3S}}{S} + \frac{2e^{-3S}}{S+2}.$

$c(t) = L^{-1}[C(S)]$

$= L^{-1}\left[\frac{2e^{-3S}}{S} - \frac{2e^{-3S}}{S+2}\right]$

$= 2u(t-3) - 2e^{-2(t-3)} u(t-3).$

$= 2\left[1 - e^{-2(t-3)}\right] u(t-3)$



Second Order Systems

→ The response of second order and higherorder systems exhibit continuous and sustained oscillations above the steady state value of i/p with frequency known as undamped natural frequenly. (ω_n) rad/s.

→ These oscillations in the response are damped in the nature.

→ Examples ① PMMC.

i/p – deflecting Torque (T_d)

o/p – Angular deflection of Pointer (θ).

Torques acting in opposition to rotation are

Inertial torque, frictional Torque, Spring Torque.

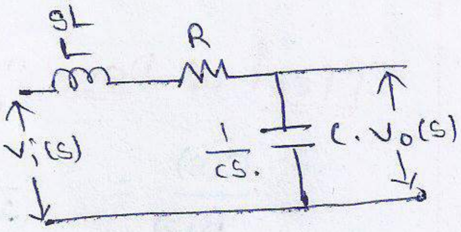
$$T_d = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta .$$

$$\boxed{\begin{aligned}\omega_n &= \sqrt{\frac{K}{J}}\\ \delta &= \frac{B}{2\sqrt{KJ}}\end{aligned}}$$

$$T_d(S) = [JS^2 + BS + K]\,\theta(S)$$

$$\frac{\theta(S)}{T_d(S)} = \frac{1}{JS^2+BS+K} = \frac{1/J}{S^2+\frac{B}{J}.S+\frac{K}{J}} .$$

→ ② Series RLC circuit.



$$V_i(S) = I(S)\left[R + LS + \frac{1}{CS}\right]$$

$$= I(S)\left[\frac{LCS^2 + RCS + 1}{CS}\right]$$

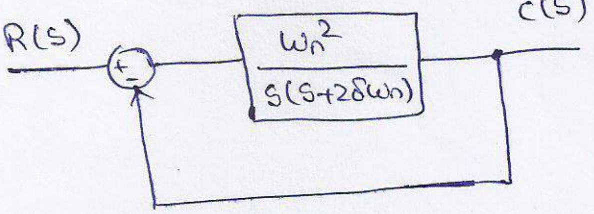
$$\boxed{\begin{aligned}\omega_n &= \frac{1}{\sqrt{LC}}\ \text{r/s.}\\ \delta &= \frac{R}{2}\sqrt{\frac{C}{L}}.\end{aligned}}$$

$$V_o(S) = \frac{1}{CS}\,I(S)$$

$$\frac{V_o(S)}{V_i(S)} = \frac{1}{LCS^2+RCS+1} = \frac{1/LC}{S^2+\frac{R}{L}S+\frac{1}{LC}.}$$

Standard (or) Prototype Second order System:-

$$T.F = \frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\delta\omega_n S+\omega_n^2}$$



δ → damping ratio

ω_n → undamped natural frequency

$S^2+2\delta\omega_n S+\omega_n^2 = 0$ charteristic eqn.

$$s_{1,2} = -\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$$

$$\frac{-2\delta\omega_n \pm \sqrt{4\delta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$-\delta\omega_n \pm \omega_n\sqrt{\delta^2-1}.$$

$$-\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}.$$

$$s_{1,2} = -\alpha \pm j\omega_d.$$

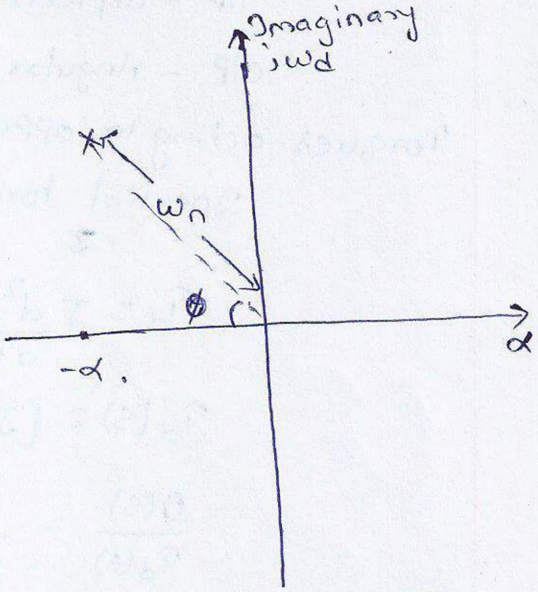
$\alpha = \delta\omega_n \rightarrow$ Damping factor

$\omega_d = \omega_n\sqrt{1-\delta^2} \rightarrow$ Damped frequency

Time constant $= \dfrac{1}{\alpha}$

$$|s_{1,2}| = \sqrt{(-\alpha)^2 + (\pm\omega_d)^2} = \delta^2\omega_n^2 + \omega_n^2(1-\delta^2)$$

$$= \omega_n \quad \text{for} \quad 0 \le \delta \le 1.$$



$$\cos\phi = \frac{\alpha}{\omega_n} = \frac{\delta\omega_n}{\omega_n} = \delta.$$

$$\phi = \cos^{-1}\delta \quad \text{(or)}$$

$$\cos^2\phi = \delta^2.$$

$$1 - \sin^2\phi = \delta^2 \Rightarrow \phi = \sin^{-1}\sqrt{1-\delta^2}.$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{\sqrt{1-\delta^2}}{\delta}\right)$$

Effect of Damping on Nature of Response :- (i/p = unit step).

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\delta\omega_n s + \omega_n^2)}$$

$$s_{1,2} = -\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$$

$$s^2 + 2\delta\omega_n s + \omega_n^2 = 0$$

Det $b^2 - 4ac = 0$

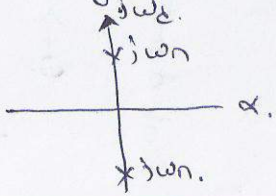
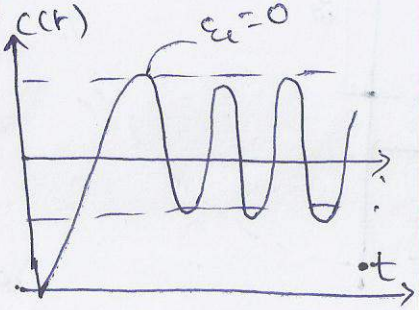
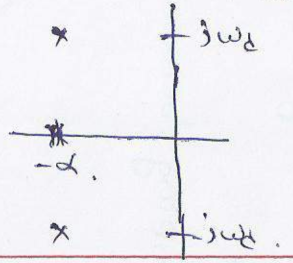
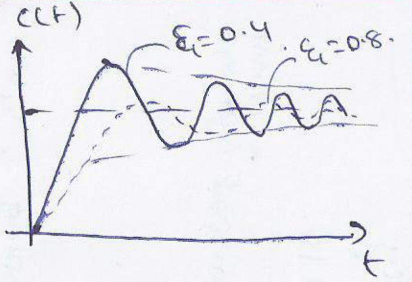
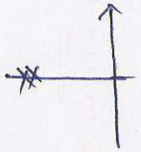
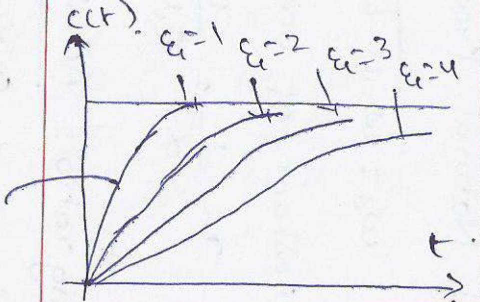
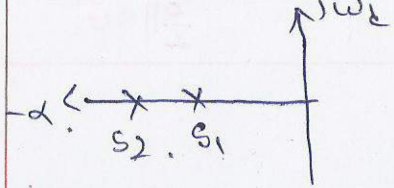
$$4\delta^2\omega_n^2 - 4.\omega_n^2 = 0.$$

$$D = \delta^2 - 1 = 0 \Rightarrow \xi = 1$$

$$D = \delta^2 - 1 < 0 \Rightarrow \delta < 1$$

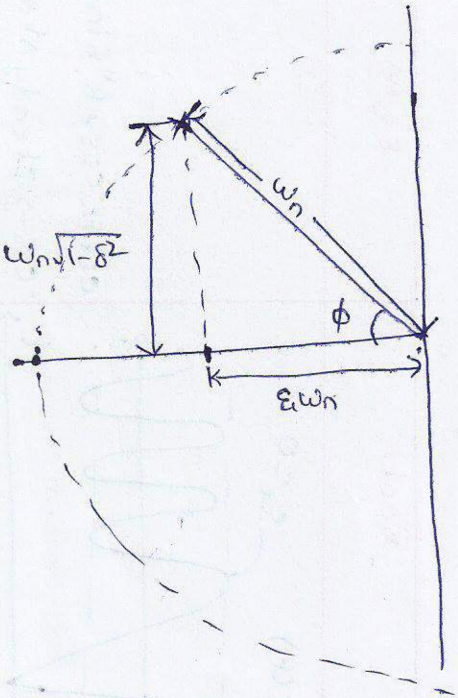
$$D = \delta^2 - 1 > 0 \Rightarrow \delta > 1$$

$s_{1,2} = -\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$

S.No.	Damping Ratio	Type of closed loop Poles.	Nature of Response.	Roots	Eq^ns.
1	$\xi\xi=0$	Purely Imaginary $s_{1,2} = \pm j\omega_n$	Oscillations with constant frequency & Amplitude 		$c(t)=c_{ss}+K''\sin(\omega_n t+\theta)$ $c_{ss}\to$ steady state value.
2	$0<\xi<1$	complex conjugates with -ve real Parts $s_{1,2}=-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$	Damped oscillations 		$c(t)=c_{ss}+Ke^{-\xi\omega_n t}\cos(\omega_d t+\theta)$
3	$\xi=1$	Roots are real, Equal & Negative $s_1=-\omega_n$ $s_2=-\omega_n$	Critical & Pure Exponential  Critically damped.		compared to overdamped, settling time required is less. $c(t)=c_{ss}+Bte^{-\omega_n t}+Ce^{-\omega_n t}$
4.	$1<\xi<\infty$ $\xi>1$	Roots are real & unequal $s_1=-\delta\omega_n+\omega_n\sqrt{\delta^2-1}$ $s_2=-\delta\omega_n-\omega_n\sqrt{\delta^2-1}$	Pure Exponential Slow and Sluggish 	"	$c(t)=c_{ss}+Be^{-K_1t}+Ce^{-K_2t}$

Characteristics of Underdamped System:-

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$



① $\cos\phi = \frac{\xi\omega_n}{\omega_n} = \xi \Rightarrow \phi = \cos^{-1}\xi$

② Damping Coefficient (or) Damping factor (or) Actual damping $\alpha = \xi\omega_n$.

③ Damped Natural frequency

$\omega_d = \omega_n\sqrt{1-\delta^2}$ r/s.

④ Time constant of underdamped ~~system~~ response

$T = \frac{1}{\alpha} = \frac{1}{\xi\omega_n}$

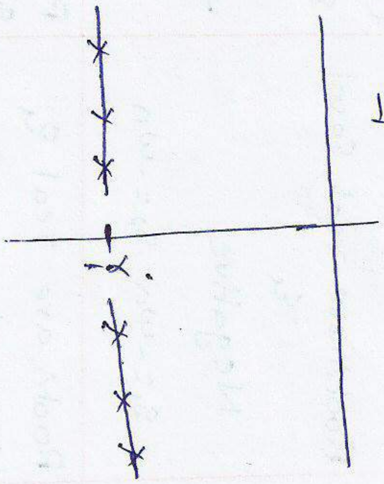
⑤ Damping ratio = $\frac{\text{Actual damping}}{\text{Critical damping}} = \frac{\xi\omega_n}{\omega_n} = \xi$

Actual damping $= \xi\omega_n$

At $\xi = 1$, Actual damping becomes critical damping (ω_n).

Note:-

Constant α locus:-

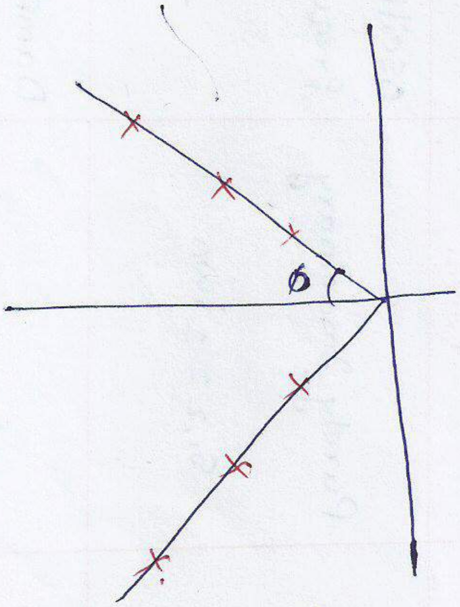


$\alpha = \delta\omega_n$

$\downarrow \delta\omega_n = k$.

$\delta\downarrow$

$\phi \to$ constant.

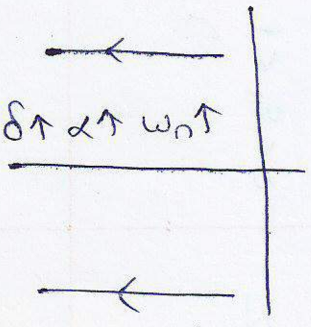


$\xi\downarrow$, $\omega_n\uparrow$ $\omega_d\uparrow$

$\alpha = \xi\omega_n\uparrow$

If $\delta\downarrow$, $\omega_d\uparrow$

If $\delta\downarrow$, $\omega_n\uparrow$ $\omega_d\uparrow$.



$\omega_d \to$ constant.

$\omega_n\uparrow$ $\alpha\uparrow$ $\omega_d\uparrow$.

$\uparrow\alpha = \delta\omega_n\uparrow$

$\to \omega_d = \uparrow\omega_n\sqrt{1-\delta^2}\uparrow$

Step Response of the Standard 2nd Order System:-

$$R(s) = \frac{1}{s} \qquad C(s) = \frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2} R(s) \Big|_{R(s)=1/s}$$

$$C(s) = \frac{\omega_n^2}{(s+\delta\omega_n)^2+\omega_d^2} \cdot \frac{1}{s}$$

$$\boxed{\begin{aligned} &= s^2+2\delta\omega_n s+\omega_n^2 . \\ &(s+\delta\omega_n+j\omega_d)\,(s+\delta\omega_n-j\omega_d) \\ &= (s+\delta\omega_n)^2+\omega_d^2 . \end{aligned}}$$

$$C(s) = \frac{c_1}{s} + \frac{c_2 s+c_3}{(s+\delta\omega_n)^2+\omega_d^2} .$$

$$c_1 = \frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}\Big|_{s=0} = 1$$

$$\boxed{\begin{aligned} L\{e^{at}\sin\omega t\} &= \frac{\omega}{(s-a)^2+\omega^2} \\ L\{e^{at}\cos\omega t\} &= \frac{s-a}{(s-a)^2+\omega^2} . \end{aligned}}$$

$$\frac{1}{s} + \frac{c_2 s+c_3}{(s+\delta\omega_n)^2+\omega_d^2} = \frac{\omega_n^2}{s(s^2+2\delta\omega_n s+\omega_n^2)}$$

$$s^2+2\delta\omega_n s+\cancel{\omega_n^2}+c_2 s+c_3 = \cancel{\omega_n^2} .$$

Put $s=1$, $\quad c_2+c_3 = -1-2\delta\omega_n$

Put $s=-1$, $\quad c_2-c_3 = -1+2\delta\omega_n$

$$C(s) = \frac{1}{s} + \frac{-s-2\delta\omega_n}{(s+\delta\omega_n)^2+\omega_d^2} .$$

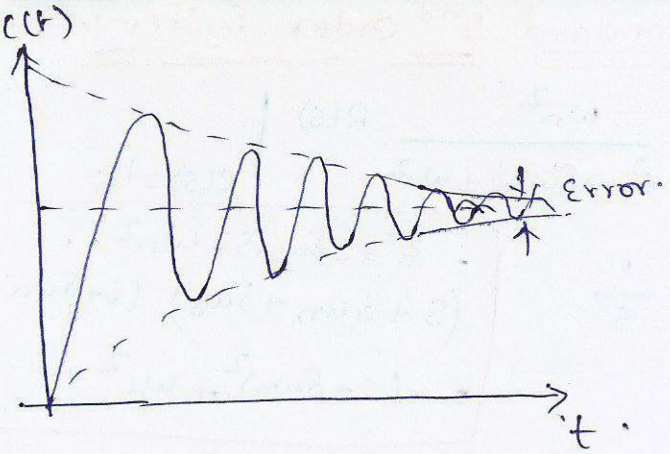
$$= \frac{1}{s} - \frac{s+\delta\omega_n}{(s+\delta\omega_n)^2+\omega_d^2} . - \frac{\delta\omega_n}{(s+\delta\omega_n)^2+\omega_d^2} .$$

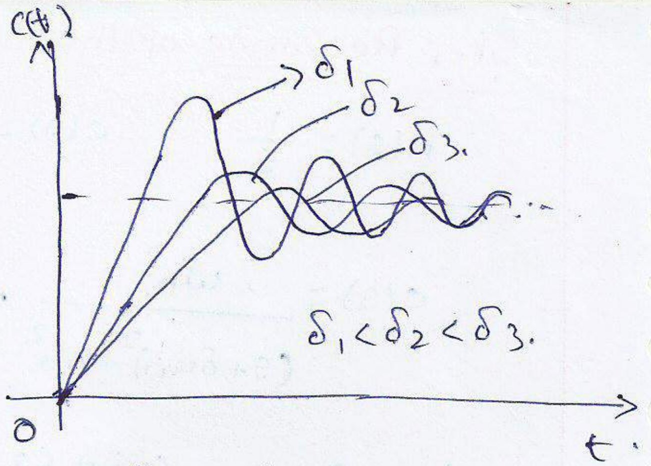
$$c(t) = L^{-1}[C(s)] = 1 - e^{-\delta\omega_n t}\cos\omega_d t - \frac{\delta\omega_n}{\omega_d} \cdot e^{-\delta\omega_n t} . \sin\omega_d t$$

$$= 1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\left[\sqrt{1-\delta^2}\cos\omega_d t + \delta\sin\omega_d t\right]$$

$$\boxed{c(t) = \underbrace{1}_{\text{Steady state response}} - \underbrace{\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}} \sin(\omega_d t+\phi)}_{\text{Transient response}}}$$

$$A\sin\omega t + B\sin\omega t = \sqrt{A^2+B^2}\,\sin(\omega t+\tan^{-1}(B/A))$$



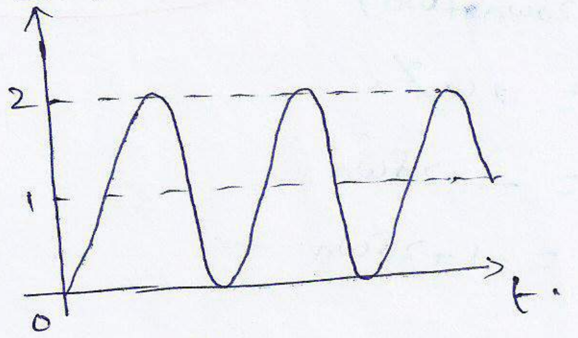


$0<\delta<1.$ underdamped Response

If $\delta=0$ $\quad c(t) = 1 - \frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}} \sin(\omega_d t+\phi)$ $\quad \phi = \tan^{-1}\frac{\sqrt{1-\delta^2}}{\delta}.$

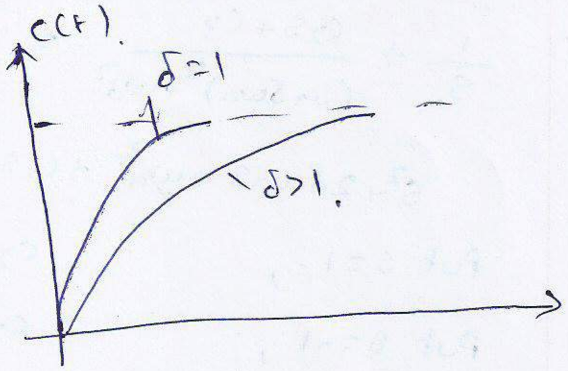
$$c(t) = 1 - \sin(\omega_n t + 90^\circ)$$

$$= 1 - \cos(\omega_n t)$$



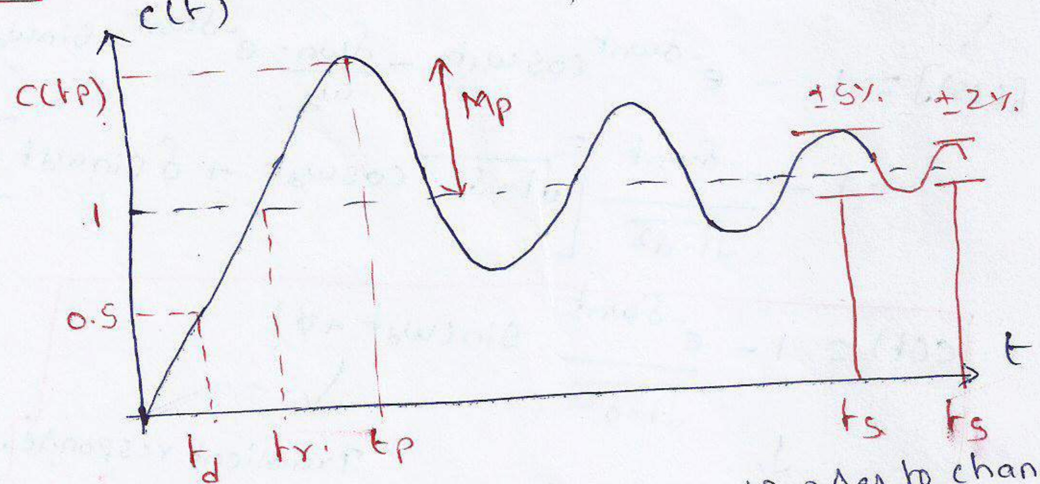
$\delta=0.$

undamped.



critically & overdamped.

Time domain Specifications (or) Parameters:-



Delay time:- It is time taken by the responses to change from the 0 to 50% of the final steady state value.

$$\boxed{t_d \simeq \frac{1+0.7\delta}{\omega_n}}$$

Risetime:- It is time taken by the response to change from 0 to 100% of the final value.

for $\delta = 1$, Critically damped System. 5% to 95%

$1 < \delta < \infty$ over " " 10% to 90%.

$$C(t)\Big|_{t=t_r} = 1 = 1 - \frac{e^{-\delta\omega_n t_r}}{\sqrt{1-\delta^2}} \sin(\omega_d t_r + \phi) \qquad \boxed{\phi = \cos^{-1}\delta}$$

$$\sin(\omega_d t_r + \phi) = 0 \cdot \Rightarrow \boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$

Peak time:- It is time taken by the response to reach maximum value.

$$\frac{dc(t)}{dt}\Big|_{t=tp} = 0.$$

$$0 - \left[-\delta\omega_n e^{-\delta\omega_n tp} . \sin(\omega_d tp + \phi) + e^{-\delta\omega_n tp}(\omega_d)\cos(\omega_d tp + \phi)\right] = 0$$

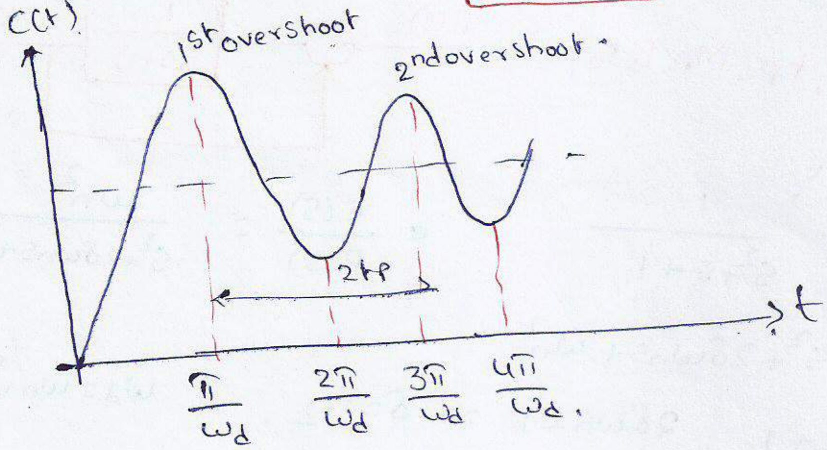
$$\omega_n e^{-\delta\omega_n tp}\left[\delta \sin(\omega_d tp + \phi) \mp \sqrt{1-\delta^2}\cos(\omega_d tp + \phi)\right] = 0.$$

$$\sin(\omega_d tp + \phi - \phi)$$

$$\sin \omega_d tp = n\pi$$

$$tp = \frac{n\pi}{\omega_d}, \quad n = 1, 2, 3 - - - - -$$

$n = 1. \rightarrow$ first peak. $\boxed{tp = \frac{\pi}{\omega_d}}$ $\delta\uparrow \quad \omega_d\downarrow \quad tp\uparrow.$



Maximum (cos) Peak Over Shoot:- It is the maximum error at the o/p.

$$Mp = C(tp) - C(\infty)$$

$$= C(tp) - 1$$

$$= \cancel{1} - \frac{e^{-\delta\omega_n tp}}{\sqrt{1-\delta^2}} \sin(\omega_d tp + \phi) - \cancel{1}$$

$$= -\left[\frac{e^{-\delta \cancel{\omega_n} \cdot \frac{\pi}{\cancel{\omega_n}\sqrt{1-\delta^2}}}}{\sqrt{1-\delta^2}} \sin\left(\cancel{\omega_d} \cdot \frac{\pi}{\cancel{\omega_d}} + \phi\right)\right]$$

$$\boxed{M_p = e^{-\delta\pi/\sqrt{1-\delta^2}}}$$

Mp depends only on δ. Mp ↓es when δ ↑es.

$$\boxed{\%M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%}$$

→ If i/p magnitude is doubled, steady state value double. Mp doubles.

→ If " " , % Mp remains same.

Settling time:- It is the time taken by the response to 5% (or) 2% tolerance bands.

$$\tau = T = \frac{1}{\delta\omega_n}$$

3 time constants required for 5%. $t_s = \frac{3}{\delta\omega_n}$

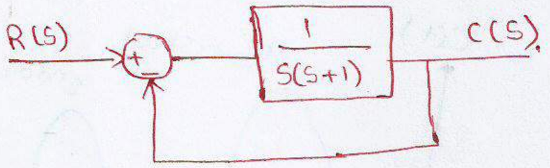
5 " " " 2%. $t_s \simeq \frac{4}{\delta\omega_n}$

$$\overset{\omega_d\downarrow}{\boxed{\delta\uparrow \quad t_r\uparrow \quad BW\downarrow, \; t_p\downarrow, \quad M_p\downarrow, \quad \omega_d\downarrow \quad \alpha\uparrow \quad \text{stability}\uparrow}}$$

$$\uparrow.\; t_r = \frac{\pi-\phi}{\omega_d} = \frac{\pi-\phi}{\omega_n\sqrt{1-\delta^2}\uparrow}$$

Problems:-

① Find t_r, t_p, M_p & t_s.



$$\frac{C(S)}{R(S)} = \frac{1}{S^2+S+1} \qquad \frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\delta\omega_n S+\omega_n^2}$$

$$C.E = S^2 + 2\delta\omega_n S + \omega_n^2 \qquad \omega_d = \omega_n\sqrt{1-\delta^2} = \sqrt{3}/2$$

$$\omega_n = 1 \qquad 2\delta\omega_n = 1 \Rightarrow \delta = 1/2$$

$$\phi = \cos^{-1}\delta = \pi/3$$

$$t_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - \pi/3}{\sqrt{3}/2} = \frac{4\pi}{3\sqrt{2}}$$

$$= 2.42 \text{ sec.}$$

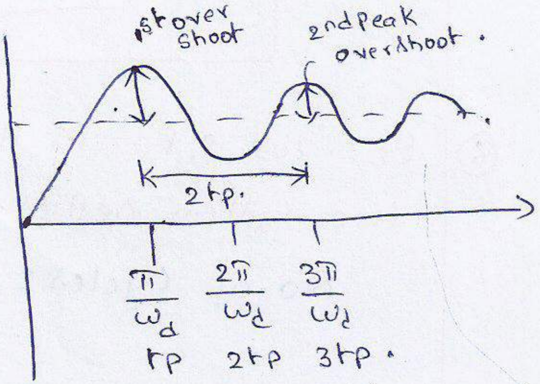
$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\sqrt{3}/2} = 3.62 \text{ sec.}$$

$$M_p = e^{-\delta\pi/\sqrt{1-\delta^2}}\Big|_{\delta=0.5} = 0.163.$$

$$t_s = \frac{3}{\delta\omega_n} = 6 \text{ sec.} \qquad t_s = \frac{5}{\delta\omega_n} = 10 \text{ sec.}$$

② diff For a 2nd order system (underdamped) subjected to a unit step i/p, The time response shows first peak to be 4 times of the 2nd overshoot. Determine the damping ratio of the system ? Expected max. overshoot.

$$\cancel{M_p = \frac{e^{-\delta \overset{\pi}{\omega_n} t_p}}{\sqrt{1-\delta^2}}}$$



$M_p = c(t_p) - 1.$

1st overshoot : $M_p = e^{-\delta\pi/\sqrt{1-\delta^2}} = e^{-x}$

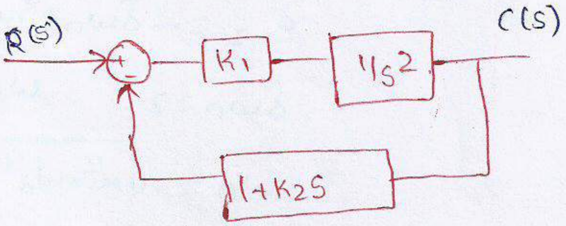
2nd overshoot $= e^{-3\delta\pi/\sqrt{1-\delta^2}} = e^{-3x}.$

Let $x = \frac{\delta\pi}{\sqrt{1-\delta^2}}.$

$M_p = c(t_p) - 1$

$e^{-x} = 4e^{-3x} - 1 \Rightarrow M_p = 4M_p^3 - 1.$

③ For a step i/p, the peak time and overshoot of the response respectively are 2sec & 25%. Find. ω_n, K_1 & K_2.



$$\frac{C(s)}{R(s)} = \frac{K_1/s^2}{1 + \frac{(1+K_2 s)K_1}{s^2}.}$$

Characteristic Eqn $CE = s^2 + K_1K_2 s + K_1.$

$\omega_n^2 = K_1$ $\quad 2\delta\omega_n = K_1K_2.$

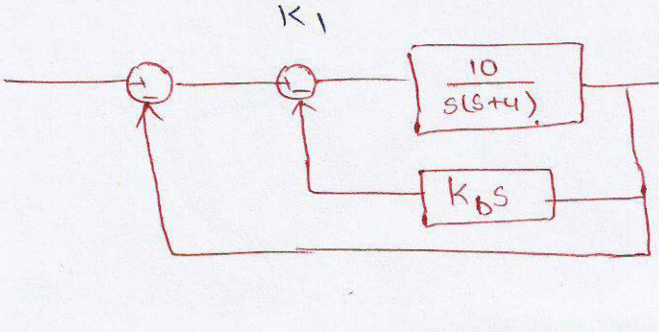
Given $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\delta^2}}. = 2sec.$ $\quad \therefore \omega_n = 1.7$ rad/se.

$M_p = e^{-\delta\pi/\sqrt{1-\delta^2}.} = 0.25. \Rightarrow \delta = 0.4.$

$K_1 = \omega_n^2.$

$K_2 = \frac{2\delta\omega_n}{K_1}$

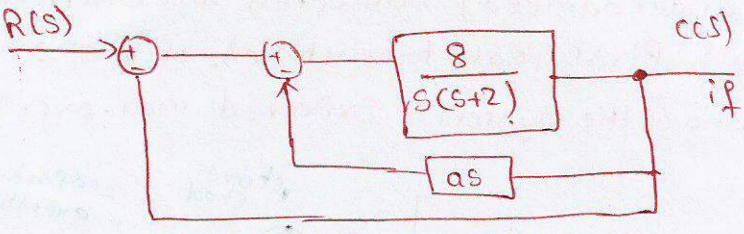
④



If $\delta = 0.8$, find K_b, t_p, M_p, ω_d, t_s for 2%.

$C.E = s^2 + (10K_b + 4) + 10 = 0.$

⑤



if $\delta = 0.707$, find value of 'a'.

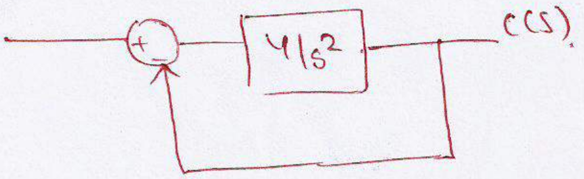
$a = 0.25$.

⑥ $t_s = 20s$ & $t_p = 1s$ the approximate no. of damped cycles, before the system settles.

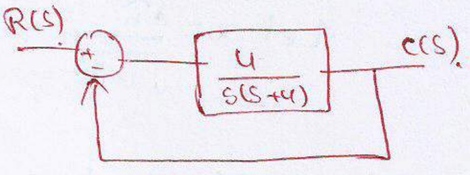
$$\text{No. of cycles} = \frac{\text{settling time}}{\text{one damped cycle period}} = \frac{t_s}{2t_p}.$$

$$= \frac{20}{2s} = 10 \,//$$

⑦



⑧



Find Mp. Ans (1)

Mp=0

⑨ Poles are at $-2 \pm j3$ find ω_d, δ.

$S_{1,2} = -\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$.

$\omega_d = 3$

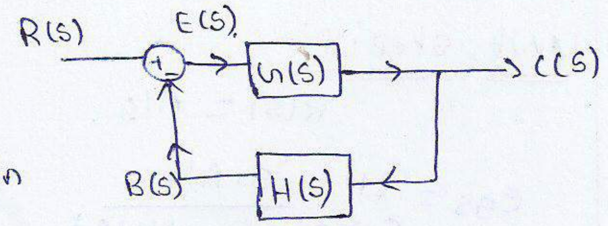
$\delta =$

$\delta\omega_n = 2$ $\omega_n\sqrt{1-\delta^2} = 3$.

$\omega_n = \sqrt{\alpha^2 + \omega_d^2} = \sqrt{13}$.

$\alpha = \delta\omega_n \Rightarrow \delta = \frac{2}{\sqrt{13}}$

Steady state Error Analysis:-



Steady state error:- (ess)

It is deviation from the desired o/p. It is the difference b/n actual o/p & desired o/p.

E(s) = Error signal   B(s) = feedback signal

$$E(S) = R(S) - B(S)$$

$$B(S) = C(S)H(S)$$

$$E(S) = R(S) - C(S)H(S) \quad ; \quad C(S) = E(S)G(S)$$

$$E(S) = R(S) - E(S)G(S)H(S)$$

$$\boxed{E(S) = \frac{R(S)}{1+G(S)H(S)} \qquad E(S) = \frac{R(S)}{1+G(S)} \text{ for UFB}}$$

$$e_{ss} = \underset{t\to\infty}{Lt}\; e(t) = \underset{S\to 0}{Lt}\; SE(S) \qquad \boxed{e_{ss} = \underset{S\to 0}{Lt} \frac{S \cdot R(S)}{1+G(S)H(S)}}$$

$$e(t) = L^{-1}\{E(S)\}$$

ess depends on 1) R(S) i/p 2) OLTF G(S)H(S).

classification of Steady state Error:-

1. Static Error constants
2. Dynamic or Generalized error constants.

Static Error constants:-

→ Position Error constant is measure of steady state error b/

i/p & o/p when i/p is unit step. " "

→ Velocity Error constant. "

" " " " ramp i/p. " "

"

→ Acceleration " " Parabolic i/p.

" "

Steady state Error for different types of i/p

(i) **Unit step i/p:-**

$$R(S) = A/s.$$

$$e_{ss} = \mathop{Lt}_{S\to 0} \frac{S\cdot A/s}{1+G(S)H(S)} = \frac{A}{1+\mathop{Lt}_{S\to 0} G(S)H(S)}$$

$$e_{ss} = \frac{A}{1+K_P}$$; K_P -> Position error constant.

$$K_P = \mathop{Lt}_{S\to 0} G(S)H(S).$$

(ii) **Ramp i/p:-** $R(S) = A/s^2.$

$$e_{ss} = \mathop{Lt}_{S\to 0} \frac{S\cdot A/s^2}{1+G(S)H(S)} = \frac{A}{\mathop{Lt}_{S\to 0} S + \mathop{Lt}_{S\to 0} S\,G(S)H(S)}$$

$$e_{ss} = \frac{A}{K_v}$$; K_v = velocity error constant

$$K_v = \mathop{Lt}_{S\to 0} S\,G(S)H(S).$$

(iii) **Parabolic i/p:-** $R(S) = A/s^3$

$$e_{ss} = \mathop{Lt}_{S\to 0} \frac{S\cdot A/s^3}{1+G(S)H(S)} = \frac{A}{\mathop{Lt}_{S\to 0} S^2 + \mathop{Lt}_{S\to 0} S^2 G(S)H(S)}$$

$$e_{ss} = \frac{A}{K_A}$$; K_A = Acceleration Error Constant.

$$\cancel{K_0}\, K_A = \mathop{Lt}_{S\to 0} S^2 G(S)H(S).$$

Steady State Error for different types of Systems:-

1) **step i/p :-**

Type-0-System. $K_P = \mathop{Lt}_{S\to 0} G(S)H(S) = \mathop{Lt}_{S\to 0} \frac{K(1+T_1 S)}{S^0(1+T_a S)} = K.$

$$e_{ss} = \frac{A}{1+K}. \qquad K_P = K.$$

Type-1- & higher order systems

$$K_P = \mathop{Lt}_{S\to 0} \frac{K(1+T_1 S)}{S(1+T_a S)} = \infty. \quad \therefore e_{ss} = 0.$$

-> Hence type 0 system can respond to step i/p with finite Error $\frac{A}{1+K}$ whereas type ① & higher order systems can respond without SSE.

2) Ramp i/p:-

Type-0-System:- $K_v = \underset{s\to 0}{Lt}\ s\,G(s)H(s) = \underset{s\to 0}{Lt}\ \frac{s \cdot K(1+T_1 s)}{s^0(1+T_1 s)} = 0.$

$$e_{ss} = \frac{1}{K_v} = \infty \Rightarrow e_{ss} = \infty.$$

Type-1-System:- $K_v = \underset{s\to 0}{Lt}\ \cancel{s} \cdot \frac{K(1+T_1 s)}{\cancel{s}(1+T_1 s)} = K.$

$$e_{ss} = 1/K.$$

Type-2 & higher order systems:- $K_v = \underset{s\to 0}{Lt}\ \cancel{s} \cdot \frac{K(1+T_1 s)}{s^2(1+T_1 s)} = \infty.$

$$e_{ss} = 0.$$

- → for type 'o' system e_{ss} is ∞. i.e., type 0 system is incapable to respond to unit ramp
- → Type 1 responds with finit error
- → Type 2 & higher system responds w/o SSE.

3) Parabolic i/p:-

Type 'o' & '1' : b $K_A = \underset{s\to 0}{Lt}\ s^2 G(s)H(s)$

$$= \underset{s\to 0}{Lt}\ \frac{s^2 K(1+T_1 s)}{s(1+T_a s)} = 0. \ ; \ e_{ss} = \infty.$$

Type '2' $K_A = \underset{s\to 0}{Lt}\ \frac{s^2 \cdot K(1+T_1 s)}{s^2(1+T_a s)} = K.$

$$e_{ss} = \frac{A}{K}.$$

Type '3 & higher system $K_A = \underset{s\to 0}{Lt}\ s^2 \cdot \frac{K(1+T_1 s)}{s^3(1+T_a s)} = \infty.$

$$e_{ss} = 0.$$

- → Type 'o' & '1' system cannot respond to parabolic i/p.
- → Type '2' system can respond with an Error of 1/k.
- → Type '3 & higher " " without " ".

i/p Type.	unit step		u Ramp		Parabolic.	
	Kp	$ess = \frac{A}{1+Kp}$	K_v	ess	KA	ess.
Type 0	K	$\frac{A}{1+K.}$	0	∞	0	∞
Type 1	∞	0	K	A/K.	0	∞.
Type 2	∞	0	∞	0	K	A/K.
Type 3	∞	0.	∞	0	∞	0.

Problems

① A unity FB has $G(S) = \frac{K}{S^2(S+2)(S+5)}$ find (i) Type of System
(ii) Error constants
(iii) ess for step, ramp, Para

And $Kp = \infty$ $Kv = \infty$ $Ka = K/10.$

$ess = 0$ $ess = 0$ $ess = 10/K.$

② A unity FB system has $G(S) = \frac{10(S+2)}{S(S+3)(S+4)}$

③ $G(S) = \frac{S+10}{S(S^3+7S^2+12S)}$

1) Type 2) Error constant.
3) $r(t) = 2t\,u(t)$
4) $r(t) = 4t^2 u(t)$.

④ find Kp, Kv, KA for the following systems. find ess. for $r(t) = 1+t$.

a) $G(S) = \frac{10}{S(S+1)}$ b) $G(S) = \frac{100}{S^2(S+2)(S+5)}$

Sol:- a)

$Kp = \infty.$ $r(t) = 1+t.$

$Kv = 10$ $R(S) = \frac{1}{S} + \frac{1}{S^2}.$

$KA = 0$

$$ess = \frac{1}{1+Kp} + \frac{1}{Kv.} = \frac{1}{1+\infty} + \frac{1}{10} = 0.1$$

b) $Kp = \infty$

$Kv = \infty.$ $ess = \frac{1}{1+Kp} + \frac{1}{Kv} = 0.$

$KA = 10.$

Note:-

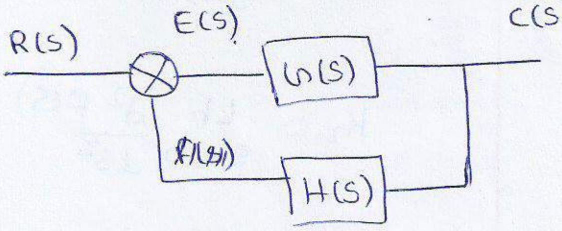
1. $ess \propto \frac{1}{K.}$ i.e., K↑ ess↓,
2. Max. Type no. for Linear control system is '2' beyond type '2' system exhibits nonlinear c/ls more dominantly. The common physical nonlinearities are saturation, friction, deadtime etc.

Dynamic Error Coefficients (or) Generalised Error Coefficients:-

The static error constants Kp, Kv, Ka are used only for Particular i/p. Generalised error coefficients can be applied to any arbitrary i/p.

→ Kp, Kv, KA gives definite values of Errors, either zero, a finite value (or) ∞ They donot give. infn regarding the variation of error with time.

→ The dynamic error coefficient gives complete Picture of error, as a fn of time.



$$E(S) = \frac{R(S)}{1+G(S)H(S)}$$

$$E(S) = R(S) \cdot F(S)$$

Let $F(S) = \frac{1}{1+G(S)H(S)}$; $E(S) = R(S)F(S)$

$$e(t) = \int_0^\infty f(T)\, r(t-T)\, d\tau.$$

Expanding $r(t-T)$ Using Taylor's Expansion.

$$r(t-T) = r(t) - T\dot{r}(t) + \frac{T^2}{2!}\ddot{r}(t) - \frac{T^3}{3!}\dddot{r}(t) + ----$$

$$e(t) = r(t)\int_0^\infty f(T)\,dT - \dot{r}(t)\int_0^\infty T f(T)\,dT + \frac{\ddot{r}(t)}{2!}\int T^2 f(T)\,dT ----$$

$$\boxed{\begin{gathered} e(t) = K_0 r(t) + K_1 \dot{r}(t) + \frac{K_2}{2!}\ddot{r}(t) + ----- \\ K_0 = \cancel{k}\int_0^\infty f(T)\,dT \;;\; K_1 = \int_0^\infty T f(T)\,dT \;;\; K_2 = \int_0^\infty T^2 f(T)\,dT \\ e_{ss} = \lim_{t\to\infty} e(t) \end{gathered}}$$

Determination of Error constants:-

$$L\{f(T)\} = F(S) = \int_0^\infty f(T)\, e^{-ST}\, dT$$

$$\lim_{S\to 0} F(S) = \lim_{S\to 0}\int_0^\infty f(T)\, e^{-ST}\, dT = \int_0^\infty f(T)\, dT = K_0.$$

$$\rightarrow \frac{d}{dS} F(S) = \frac{d}{dS} \int_0^{\infty} e f(T) e^{-ST} dT = \int_0^{\infty} -T f(T) dT$$

$$\rightarrow \underset{S\to 0}{Lt} \frac{d}{dS} F(S) = \underset{S\to 0}{Lt} - \int_0^{\infty} T f(T) de^{-ST} dT = - \int_0^{\infty} T f(T) dT$$

$$= K_1$$

$$K_0 = \underset{S\to 0}{Lt} F(S) \qquad K_1 = \underset{S\to 0}{Lt} \frac{d}{dS} F(S)$$

$$K_2 = \underset{S\to 0}{Lt} \frac{d^2 F(S)}{dS^2} \qquad \text{where } F(S) = \frac{1}{1+G(S)H(S)}$$

Relationship between Static & Dynamic Error constants:-

$$G(S)H(S) = \frac{100}{S(S+2)}$$

Static Error constants:-

$$K_P = \underset{S\to 0}{Lt} \frac{100}{S(S+2)} = \infty$$

$$K_V = \underset{S\to 0}{Lt} \frac{S.100}{S(S+2)} = 50$$

$$K_A = \underset{S\to 0}{Lt} \frac{S^2.100}{S(S+2)} = 0$$

II Dynamic Error constants:-

$$F(S) = \frac{1}{1+G(S)H(S)} = \frac{1}{1+\frac{100}{S(S+2)}}$$

$$K_0 = \underset{S\to 0}{Lt} F(S) = \underset{S\to 0}{Lt} \frac{1}{1+\frac{100}{S(S+2)}} = \frac{1}{1+\underset{S\to 0}{Lt}\frac{100}{S(S+2)}} = 0.$$

$$\boxed{K_0 = e_{ss}\Big|_{\text{for step i/p}} = \frac{1}{1+K_P.}}$$

$$K_{1_1} = \underset{s\to 0}{Lt} \frac{d}{ds} F(s)$$

$$\frac{d\,F(s)}{ds} = \frac{d}{ds} \frac{1}{1+\frac{100}{s(s+2)}} = \frac{d}{ds}\left[\frac{s(s+2)}{s^2+2s+100}\right]$$

$$K_1 = \underset{s\to 0}{Lt}\frac{d}{ds}F(s) = \underset{s\to 0}{Lt}\frac{(s^2+2s+100)(2s+2) - s(s+2)(2s+2)}{(s^2+2s+100)^2}.$$

$$= \frac{100\times 2 - 0}{100^2} = \frac{100\times 2}{100^2} = \frac{1}{50}.$$

$$\boxed{K_0 = \frac{1}{1+K_P} \qquad K_1 = \frac{1}{K_V} \qquad K_2 = \frac{1}{K_A}.}$$

① Given $G(s)H(s) = \frac{100}{s(s+2)}$ find ess for $r(t) = 5+2(t)$

I Error Ration.

$$e_{ss.} = \underset{s\to 0}{Lt} \frac{s\,R(s)}{1+G(s)H(s)}. \qquad R(s) = \frac{5}{s} + \frac{2}{s^2}.$$

$$e_{ss} = \underset{s\to 0}{Lt} \frac{s.\left(\frac{5s+2}{s^2}\right)}{1+\frac{100}{s(s+2)}} = \frac{2}{50} \text{ unit.}$$

II Error Series

$$e_{ss} = \underset{t\to\infty}{Lt}\left[K_0 r(t) + K_1 \mathring{r}(t) + \frac{K_2}{2!}\ddot{r}(t) + ---\right]$$

$$r(t) = 5+2t \Rightarrow K_0 = \underset{s\to 0}{Lt} \frac{1}{1+\frac{100}{s(s+2)}} = 0.$$

$$\dot{\mathring{r}}(t) = 5. \Rightarrow K_1 = \frac{1}{K_V} = \frac{1}{50}.$$

III $R(s) = \frac{5}{s} + \frac{2}{s^2}.$

$$e_{ss} = \underset{t\to\infty}{Lt}\left[0\times(5+2t) + 2\times {}^1/_{50}\right]$$

$$e_{ss} = \frac{A}{1+\underset{\infty}{K_P}} + \frac{A}{K} =$$

$$= 0 + \frac{A}{K} = \frac{2}{50} \text{ units.}$$

① find Dynamic Error coefficients of $G(s) = \frac{600(1+s)}{s(1+50s)}$

$$F(s) = \frac{1}{1+G(s)H(s)} = \frac{1}{1 + \frac{600(1+s)}{s(1+50s)}} = \frac{s(1+50s)}{s(1+50s) + 600(1+s)}$$

$$50s^2 + 601s + 600.$$

$$K_0 = \underset{s \to 0}{Lt}\ F(s) = 0.$$

$$K_1 = d\underset{s \to 0}{Lt}\ \frac{d}{ds} F(s) = \underset{s \to 0}{Lt} \frac{d}{ds}\ \frac{50s^2 + s.}{50s^2 + 601s + 600} = \underset{s \to 0}{Lt}\ \frac{30000s^2 + 60,000s + 660}{(50s^2 + 601s + 600)^2}.$$

$$= \frac{1}{600}$$

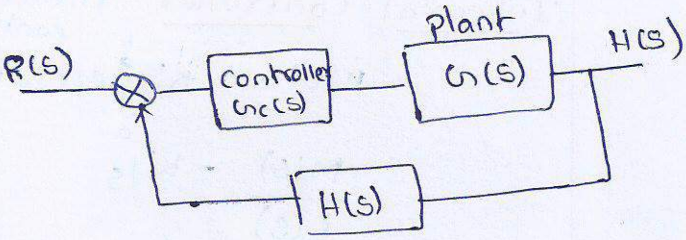
$$K_2 = \underset{s \to 0}{Lt}\ \frac{d^2}{ds^2} F(s) = \underset{s \to 0}{Lt}\ \frac{d}{ds} \left[\frac{30000s^2 + 60,000s + 600}{(50s^2 + 601s + 600)^2.} \right]$$

$$= \frac{58798}{(600)^2} = 0.163.$$

$$e_{ss}(t) = \frac{1}{600} r'(t) + \frac{1}{12.268} r''(t) + \text{———}$$

Controllers

The controllers are used in the system to produce a control signal necessary to reduce the error signal (deviation) to zero (or) to a small value. The control action may operate through mechanical, pneumatic hydraulic (or) electrical means.

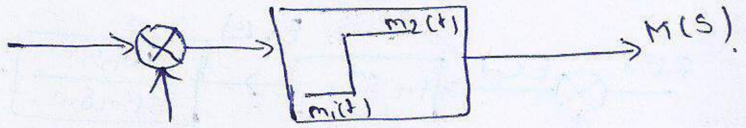


Classification of Controllers:-

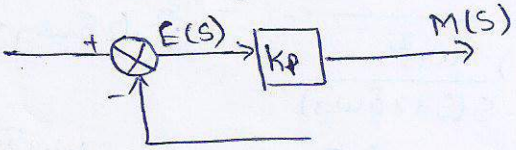
1. ON-OFF controller
2. Proportional Controller
3. Integral Controller
4. Proportional plus Integral controller.
5. " " derivative "
6. P-I-D Controller.

ON-OFF controller:- In ON-OFF controller, the actuating element is capable of assuming only two positions, with either zero (or) maximum i/p to the process.

eg:- An Electric switch is open (or) closed depending on whether the controlled variable is 6below or above set point.



Proportional Controller:- It is defined as the action of a controller in which the o/p signal m(t) is proportional to the measured actuating error signal e(t).



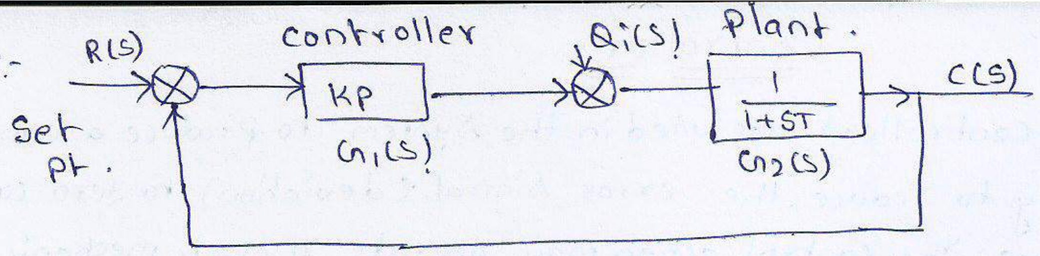
$m(t) \propto e(t)$. $\Rightarrow m(t) = K_p e(t)$ K_p -> Pro~~to~~portional Sensitivity (or) gain.

$M(S) = K_P E(S)$

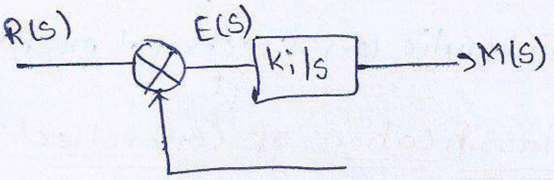
$$K_p = \frac{\text{change in the controller o/p}}{\text{change in deviation}}$$

-> It is essentially an amplifier with an adjustable gain.

eg:-



Integral Controller:- (Reset control)



$$m(t) = K_i \int_0^t e(t)\,dt$$

$$\frac{M(S)}{E(S)} = K_i/s$$

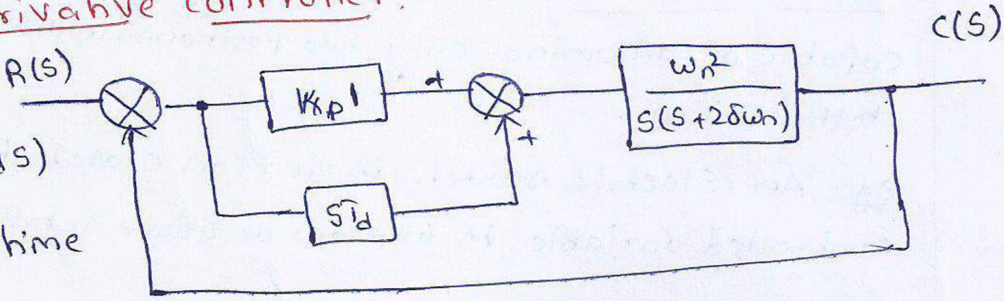
Dis adv:-

1. Response to error is slow. However it is capable of eliminating error completely.

Reset time $T_i = \frac{1}{K_I}$ $m(t) = \frac{1}{T_i}\int e\,dt$

$$M(S) = \frac{1}{T_i S} E(S)$$

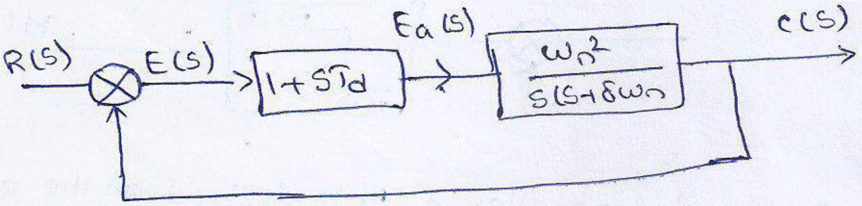
Proportional Derivative controller:-



$G_c(S) = K_P(1+T_d S)$

$T_d \rightarrow$ derivative time

→ for a



$$\frac{C(S)}{R(S)} = \frac{(1+ST_d)\frac{\omega_n^2}{S(S+2\delta\omega_n)}}{1+(1+ST_d)\frac{\omega_n^2}{S(S+2\delta\omega_n)}} = \frac{(1+ST_d)\,\omega_n^2}{S^2+(2\delta\omega_n+\omega_n^2 T_d)S+\omega_n^2=0}$$

$$\delta' = \frac{2\delta\omega_n+\omega_n^2 T_d}{2\omega_n} = \delta + \frac{\omega_n T_d}{2}$$

→ Effective damping is increased a

i.e., δ ↑ Mp ↓

$\rightarrow$

$$G(s) = (1+s\tau_d)\,\frac{\omega_n^2}{s(s+2\delta\omega_n)} \quad ; H(s)=1$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

$$= \frac{1}{1+\frac{(1+s\tau_d)\omega_n^2}{s(s+2\delta\omega_n)}\cdot 1} = \frac{s(s+2\delta\omega_n)}{s^2+2\delta\omega_n s+\omega_n^2\tau_d s+\omega_n^2}.$$

for unit ramp i/p $R(s)=1/s^2$.

$$E(s) = \frac{1}{s^2}\cdot\frac{s(s+2\delta\omega_n)}{s^2+2\delta\omega_n s+\omega_n^2\tau_d s+\omega_n^2}$$

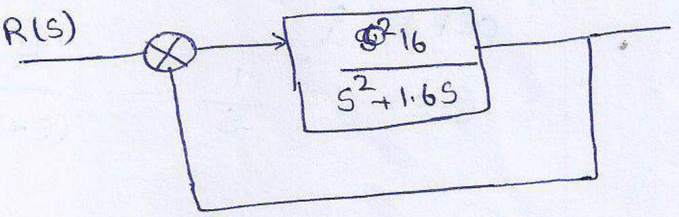
$$e_{ss} = \lim_{s\to 0} sE(s) \qquad e_{ss} = s\cdot\frac{1}{s^2}\cdot$$

$$e_{ss} = \frac{2\delta}{\omega_n}.$$

(Pr) A unity feedback system is shown in fig 6.9.3. By using derivative control the damping factor is to be made 0.8. Determine the value of τ_d and compare the rise time, t_p, M_p.

a) with out derivative control

b) with derivative control.



Sol!-

$$\frac{C(s)}{R(s)} = \frac{\frac{16}{s^2+1.6s}}{1+\frac{16}{s^2+1.6s}\cdot 1}$$

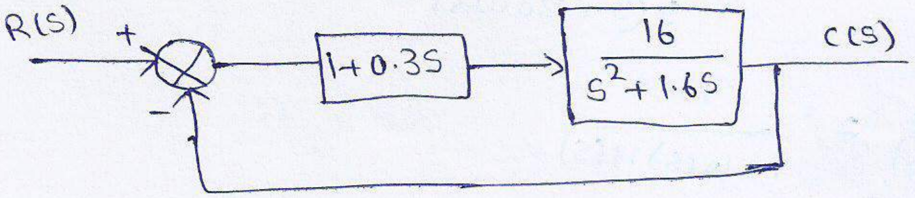
$$\frac{C(s)}{R(s)} = \frac{16}{s^2+1.6s+16}$$

$\omega_n = 4$ r/s.

$\delta = 0.2$.

$\delta' = \delta + \frac{\omega_n \tau_d}{2}$ $\Rightarrow 0.8 = 0.2 + \frac{4\cdot\tau_d}{2}$ $\Rightarrow \underline{\underline{\tau_d = 0.3}}$

The block diagram for system with derivative control



a) without derivative control.

$t_r = \frac{\pi - \phi}{\omega_d} = 0.45 \text{ sec}$ $\phi = \cos^{-1}\delta$.

$t_p = 0.8s.$

$M_p = 0.526.$

b) with derivative control.

$2\delta\omega_n = 6.4.$

$\frac{C(S)}{R(S)} = \frac{(1+0.3S)16}{S^2+6.4S+16}$ $R(S) = 1/S.$

$((S+\alpha)^2 + \omega_d^2.$

$\omega_d = \omega_n^2 - \alpha^2.$

$$C(S) = \frac{16}{S(S^2+6.4S+16)} + \frac{4.8}{S^2+6.4S+16.}$$

$$= \frac{A}{S} + \frac{BS+C}{S^2+6.4S+16} + \frac{4.8}{S^2+6.4S+16.}$$

$$= \frac{\cancel{A}1}{S} - \frac{S+6.4}{(S+3.2)^2+(16-3.2^2)^2} + \frac{4.8}{(S+3.2)^2+(16-3.2^2)^2}.$$

$c(t) =$

$$C(S) = \frac{1}{S} - \frac{S+3.2}{(S+3.2)^2+(16-3.2^2)^2} + \frac{1.6 \times \frac{2.4^2}{2.4^2}}{(S+3.2)^2+2.4^2}.$$

$= \frac{1}{}$

$c(t) = 1 - \bar{e}^{3.2t}\cos 2.4t + 0.66e^{-3.2t}\sin 2.4t.$

$t_r = t \,|\, c(t) = 1.$

$\cancel{1} = \cancel{1} - e^{-3.2t}\cos 2.4t_r + 0.66e^{-3.2t_r}\sin 2.4t_r.$

$\cos 2.4t_r = 0.44\sin 2.4t_r.$

$t_r = 0.41s.$

tp

$$c(t) = 1 - e^{-3.2t} \cos 2.4t + 0.66 e^{-3.2t} \sin 2.4t$$

$$= 1 - e^{-3.2t} (\cos 2.4t - 0.66 \sin 2.4t)$$

$$\frac{dc(t)}{dt} = e^{-3.2t}$$

- - ·1 ,!

$$\frac{dc(t)}{dt} = -e^{3.2t} (-0.28 \sin 2.4t - 4.78 \cos 2.4t)$$

tp $\frac{dc(t)}{dt} = 0$

$$-0.28 \sin 2.4t = -4.78 \cos 2.4t.$$

$$\tan 2.4t = -17.07$$

$$tp = \frac{\tan^{-1}(-17.07)}{2.4}$$

$$= 0.68 \text{ sec.}$$

$$c(t)\text{max.} = 1 - e^{-3.2(0.68)} [\cos 2.4(0.68) - 0.66 \sin 2.4(0.68)]$$

$$= 1.0812.$$

$$\% Mp = \frac{1.0812 - 1}{1} \times 100 = 8.12\%.$$

	without derivative	with derivative.
δ	0.2	0.8
tp	0.45	0.41
tr	0.85	0.68
Mp	52.6%	8.12%

G

⊕ Steady State Response:-

$$G(S)H(S) = (1+ST_d)\cdot\frac{\omega_n^2}{S(S+2\delta\omega_n)}$$

for unit step i/p.

$$E(S) = \frac{R(S)}{1+G(S)H(S)} =$$

$$\frac{E(S)}{R(S)} = \frac{1}{1+\frac{(1+ST_d)\cdot\omega_n^2}{S(S+2\delta\omega_n)}} = \frac{S(S+2\delta\omega_n)}{S^2+2\delta\omega_n S+ST_d\omega_n^2+\omega_n^2}$$

$$\frac{E(S)}{R(S)} = \frac{S(S+2\delta\omega_n)}{S^2+(2\delta\omega_n+\omega_n^2T_d)S+\omega_n^2}$$

unit step i/p $\quad e_{ss} = \underset{S\to0}{Lt}\, S\cdot E(S) = \underset{S\to0}{Lt}\, S\cdot\frac{1}{S}\cdot\frac{S(S+2\delta\omega_n)}{S^2+(2\delta\omega_n+\omega_n^2T_d)S+\omega_n^2}$

$$= 0.$$

unit ramp $\quad e_{ss} = \frac{2\delta\omega_n}{\omega_n^2} = \frac{2\delta}{\omega_n}$

unit Parabolic $e_{ss} = \infty$.

Summary:-

- → Steady state error remains unchanged with derivative control action.
- → It has Impact on Transient response.
- → δ is increased which reduces M_p.
- → t_p, t_r, t_s ↓, ↓.
- → It adds an zero at $S = -1/T_d$.

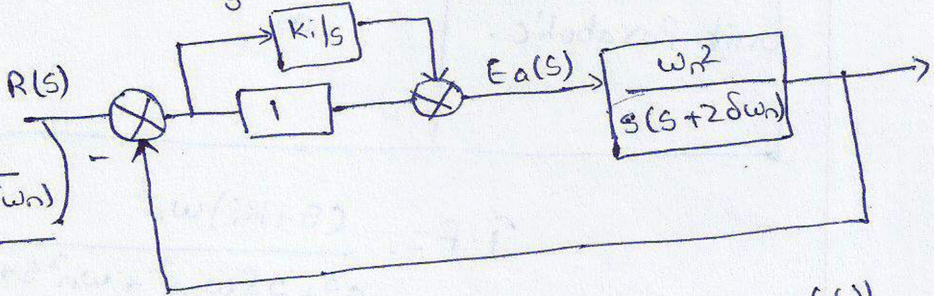
PI Controller:-

For Integral control action the actuating signal consists of Proportional error signal added with Integral of error signal.

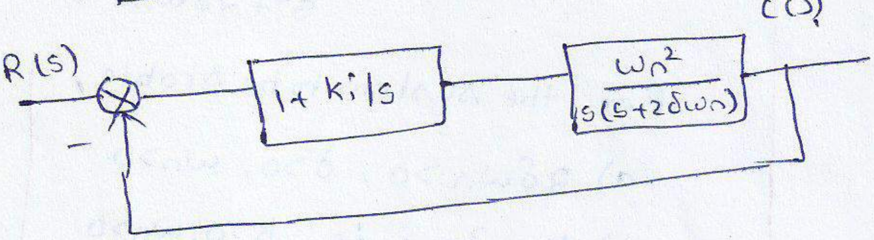
$$e_a(t) = e(t) + K_t \int e(t)\,dt.$$

L.T

$$E_a(s) = E(s) + K_i \frac{E(s)}{s}$$



$$\frac{C(s)}{R(s)} = \frac{\left(1+\frac{K_i}{s}\right)\left(\frac{\omega_n^2}{s(s+2\delta\omega_n)}\right)}{1+ \quad '' \quad (1)}$$



$$\frac{C(s)}{R(s)} = \frac{(s+K_i)\omega_n^2}{s^3+2\delta\omega_n s^2+\omega_n^2 s+K_i\omega_n^2}$$

$$\text{OLTF} = G(s)H(s) = \frac{(s+K_i)\omega_n^2}{s^2(s+2\delta\omega_n)}$$ Type-2-system

Ramp i/p $\quad K_v = \lim_{s\to 0} \cancel{s}\cdot\frac{(s+K_i)\omega_n^2}{\cancel{s^2}(s+2\delta\omega_n)} = \infty$; $\boxed{e_{ss} = 0.}$

Parabolic i/p $\quad K_A = \lim_{s\to 0} \frac{\cancel{s^2}(s+K_i)\omega_n^2}{\cancel{s^2}(s+2\delta\omega_n)} = \frac{K_i\omega_n^{\cancel{2}}}{2\delta\cancel{\omega_n}}$; $\boxed{e_{ss} = \frac{2\delta}{K_i\omega_n}}$

without Integral control

Ramp i/p : $K_v = \lim_{s\to 0} \cancel{s}\cdot\frac{\omega_n^2}{\cancel{s}(s+2\delta\omega_n)} = \frac{\omega_n^{\cancel{2}}}{2\delta\cancel{\omega_n}} = \frac{\omega_n}{2\delta.}$

$$\boxed{e_{ss} = \frac{2\delta}{\omega_n.}}$$

Parabolic i/p : $K_A = \lim_{s\to} s^2\cdot\frac{\omega_n^2}{s(s+2\delta\omega_n)} = \infty.$

$$\boxed{e_{ss} = \frac{1}{K_A} = 0}$$

i/p	ess, Steady State Error	
	without Integral Control	with Integral Contro
unit ramp	$\frac{2\delta}{\omega_n}$	0
unit Parabolic.	∞	$\frac{2\delta}{K_i\omega_n}$

$$T.F = \frac{(S+K_i)\omega_n^2}{s^3+2\delta\omega_n s^2+\omega_n^2 s+K\omega_n^2}$$

For the system to be stable,

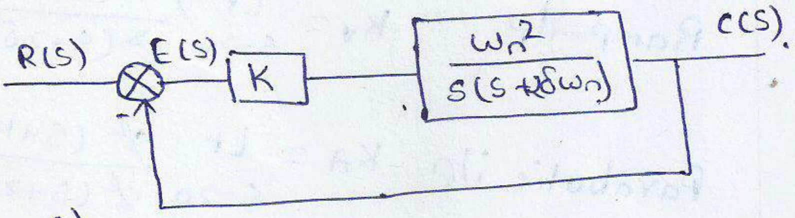
a) $2\delta\omega_n>0$, $\delta>0$, $\omega_n>0$.

b) $K\omega_n^2>0$ i.e, $K>0, \omega_n>0$

c) $2\delta\omega_n^3-K\omega_n^2$
i.e., $2\delta\omega_n>K$.

s^3	1	ω_n^2.
s^2	$2\delta\omega_n$	$K\omega_n^2$
s^1	$\frac{2\delta\omega_n^3-K\omega_n^2}{2\delta\omega_n}$	0.
s^0	$K\omega_n^2$.	

Steady state Error:-



$$E(S) = \frac{R(S)}{1+G(S)H(S)}$$

$$E(S) = \frac{R(S)\cdot(s^2+2\delta\omega_n s)}{s^2+2\delta\omega_n s+K\omega_n^2}$$

Unit step i/p.

$$e_{ss} = \lim_{s\to 0} sE(S) = 0.$$

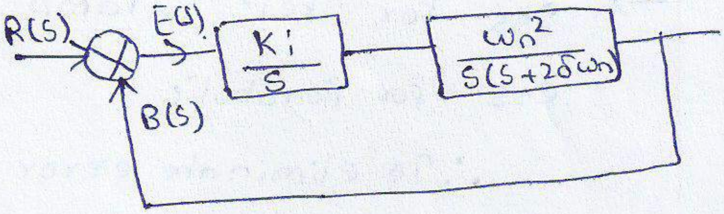
ramp i/p $\quad e_{ss} = \lim_{s\to 0.} s.\frac{1}{s^2}\cdot\frac{s(s+2\delta\omega_n)}{s^2+2\delta\omega_n s+K\omega_n^2}$

$$= \frac{2\delta}{K\omega_n}$$

for unit Parabolic.

$$e_{ss} = \underset{s \to 0}{Lt} \frac{s \cdot 1/s^3 \cdot s(s+2\delta\omega_n)}{s^2+2\delta\omega_n s + K\omega\omega_n^2} = \infty.$$

Steady state Error for ~~PI~~ Integral control :-



$$E(s) = \frac{R(s)}{1+\frac{K_i}{s} \cdot \frac{\omega_n^2}{s(s+2\delta\omega_n)}}$$

$$E(s) = \frac{s^2 \cdot R(s)(s+2\delta\omega_n)}{s^3+2\delta\omega_n s^2+K\omega_n^2}.$$

unit step i/p $R(s) = 1/s$.

$$e_{ss} = \underset{s \to 0}{Lt}\ s E(s) = 0$$

ramp $e_{ss} = \underset{s \to 0}{Lt} \frac{1}{s^2}\left[\frac{s^2(s+2\delta\omega_n) \times s}{s^3+2\delta\omega_n s^2+K_i\omega_n^2}\right] = 0.$

Parabolic i/p $e_{ss} = \underset{s \to 0}{Lt}\ s E(s) = \underset{s \to 0}{Lt} \frac{1}{s^3}\left[s \cdot \left(\frac{s^2 \cdot (s+2\delta\omega_n)}{s^3+2\delta\omega_n B^2+K_i\omega_n^2}\right)\right]$

$$= \frac{2\delta}{\omega_n}.$$

ess for PI control :- (refer)

$$E(s) = \frac{1}{1+\frac{(s+K_i)\omega_n^2}{s^2(s+2\delta\omega_n)}} = \frac{s^2(s+2\delta\omega_n)R(s)}{s^3+2\delta\omega_n s^2+\omega_n^2 s+K_i\omega_n^2}.$$

i/p.	PIontrol	ess. PI	P + I.
	0		
step	$\frac{7\delta}{K}$ 0	0	0
ramp	$\frac{2\delta}{K\omega_n}$	0	0
Parabolic.	∞	$\frac{2\delta}{K\omega_n}$	$\frac{2\delta}{K\omega_n}$.

Summary:-

-> It (P+I) increases the order and type of the system by one respectively. ∴ steady state performance is increased.

->. e_{ss} for step & ramp is zero

e_{ss} for parabolic is finit.

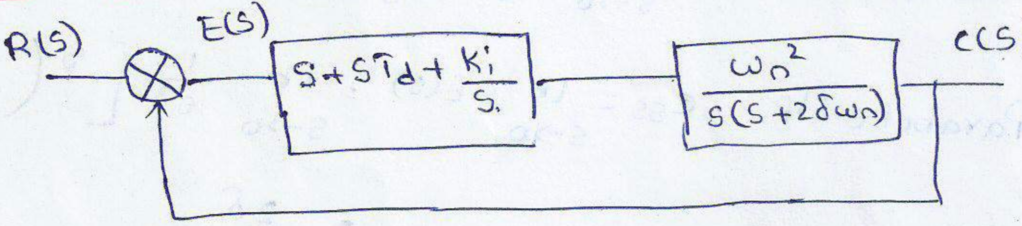
∴ To eliminate error, Integral control is can be used.

->. It adds zero at $s=-K_i$ & pole at origin. It will cause M_p to occur early and M_p will increase appreciably.

->. stability is reduced as 2nd order system becomes 3rd order system.

-> $t_r \uparrow$ $M_p \downarrow$, $BW \downarrow$,

PID control action:-



-> It is capable of controlling both transient and steady state Response.

-> It is BRF filter.

-> $t_r \downarrow$ $B.W \uparrow$, stability $\uparrow$, Eliminates Error.

Unit - III

Stability Analysis in S-Domain & Root Locus Technique.

Syllabus:-

(I) Stability Analysis :

→ The concept of stability.

→ Routh's stability criteria - qualitative stability & conditional stability - Limitations of Routh's stability.

(II) Root Locus Technique:-

→. The Root locus concept - construction of root/ locus - effect of adding poles and zero to G(S)H(S) on the root loci.

Introduction:-

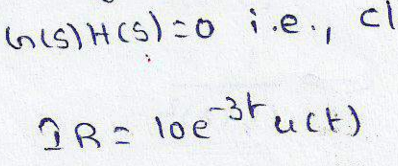
* The stability of LTI system may be defined as "when the system is subjected to bounded input, o/p should be bounded".

* It implies the impulse response of system should tend to zero as time 't' appraches to infinity.

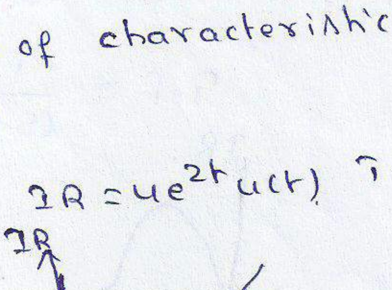
* Stability of system depends on roots of characteristic equation $1+G(S)H(S)=0$ i.e., closed loop poles.

eg:- $IR = 10e^{-3t}u(t)$ $\qquad$ $IR = 4e^{2t}u(t)$ $\quad T.F = \frac{4}{S-2}$.

$T.F = \frac{10}{S+3}$



Stable system
(Pole on leftside)



unstable system
(Pole on right half)

* Impulse response of a stable system must be absolutely Integrable between the limits (0 & ∞)

$$\int_0^{\infty} |IR| dt < \infty .$$

* $IR = 10e^{-3t}u(t)$ $\qquad$ $IR = 4e^{2t}u(t)$

$\int_0^{\infty} 10e^{-3t} dt = 10/3.$ $\qquad$ $\int_0^{\infty} 4e^{2t} dt = \infty.$

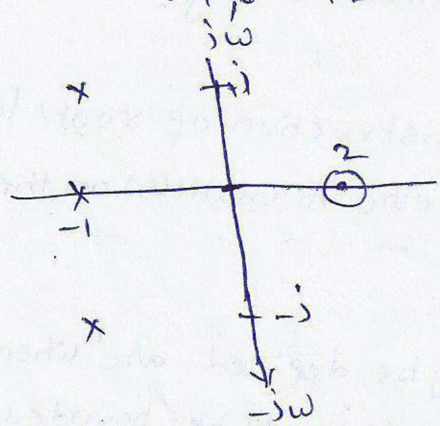
Absly Integrable. $\qquad$ Absly not Integrable.

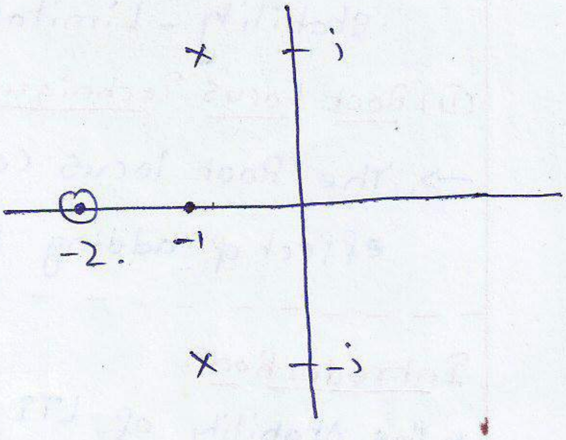
→ All the poles of a system should lies in the left half of s-plane i.e., poles should have -ve real parts.

eg:- $T.F = \frac{s-2}{(s+1)(s^2+2s+2)}$ $\qquad$ $T.F = \frac{10(s+2)}{(s-1)(s^2+2s+2)}$

CL poles $-1, \frac{-2 \pm \sqrt{4-4.2}}{2}$. $\qquad$ CL poles $= 1, -1 \pm j$

$-1, -1 \pm j$

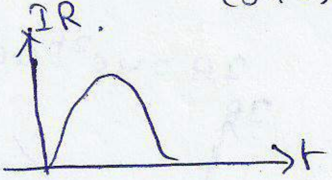




Stable System $\qquad$ Unstable System.

(Pb) IR of a certain system is $te^{-2t}u(t)$, the system is —

$T.F = \frac{1}{(s+2)^2}$. $\qquad$ $e^{-2t} \longleftrightarrow \frac{1}{(s+2)}$.

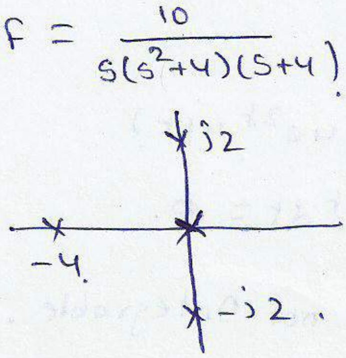


$te^{-2t} \longleftrightarrow \frac{1}{(s+2)^2}$.

Marginal (or) Limited (or) Critical Stability:-

* Impulse Response of a marginally (or) just stable system neither goes to '∞' nor approaches to '0' and will have fixed IR when $t \to \infty$.
* Marginally stable systems will have non repeated poles on the jω axis / at the origin, without any right hand pole.

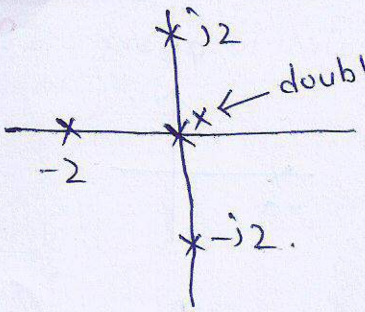
$T.F = \frac{10}{s(s^2+4)(s+4)}$



one pole at origin. $\qquad$ Stable.

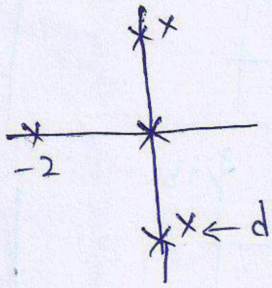
non repeated pole on jω axis.

* TF = $\frac{10}{s^2(s^2+4)(s+2)}$



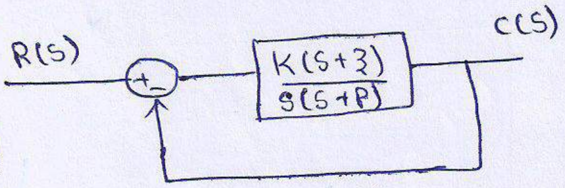
unstable. system.

* TF = $\frac{10}{s(s^2+4)^2(s+2)}$



unstable system

Conditional stability:-



K- gain of System.

* If a system is stable for an every value of gain 'K' it is said to be always stable system.

eg: range of K for stability is $0<K<\infty$.

* If it is stable ~~p~~ for only a certain value of gain 'K', it is called an conditionally stable System.

eg:- range of K for stability is $0<K<40$.

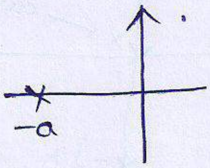
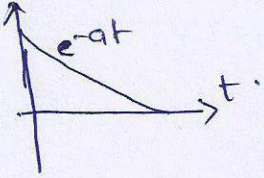
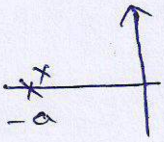
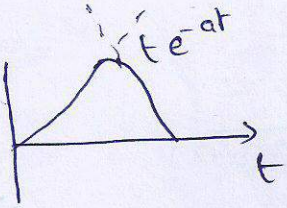
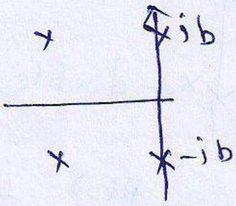
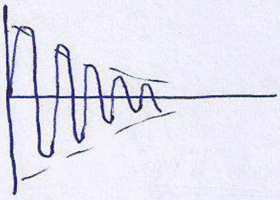
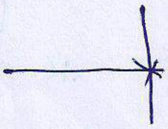
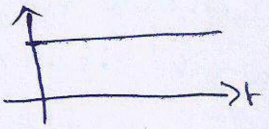
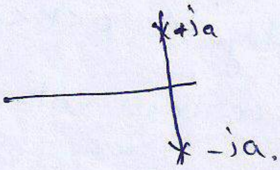
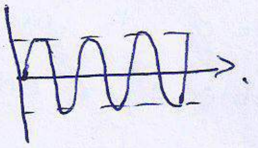
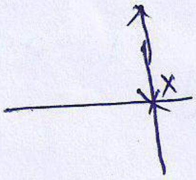
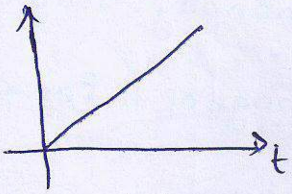
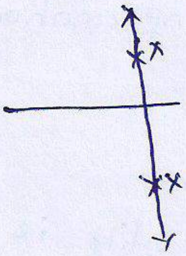
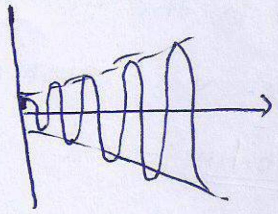
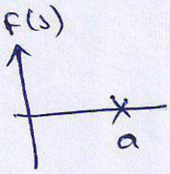
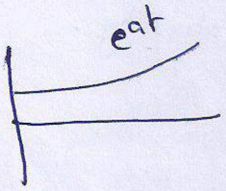
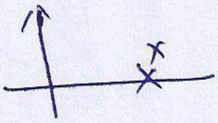
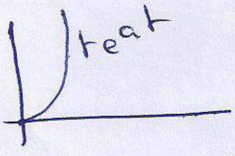
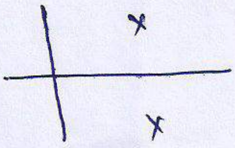
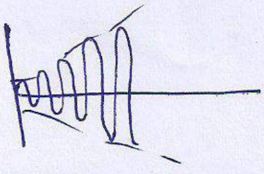
Absolute stability:-

It is the yes or NO information about the stability

eg:- Yes = stable.

Relative stability:-

It is the degree of the stability ie., complete information - about the stability.

	T.F	CL poles-locations	Impulse Response	Stability Criteria
1.	$F(S) = \frac{1}{S+a}$			Absolutely Integrable.
2.	$F(S) = \frac{1}{(S+a)^2}$.			"
3.	$F(S) = \frac{1}{(S+a)^2+b^2}$			"
4.	$F(S) = \frac{1}{S}$			Marginally stable (or) Critically "
5.	$F(S) = \frac{1}{S^2+a^2}$			"
6.	$F(S) = 1/s^2$			unstable.
7.	$F(S) = \frac{1}{(S^2+a^2)^2}$			"
8.	$F(S) = \frac{1}{S-a}$			"
9.	$F(S) = \frac{1}{(S-a)^2}$.			"
10.	$F(S) = \frac{1}{(S-a)^2+b^2}$.			"

Routh Hurwitz (RH) Stability criteria:-

$$CLTF = \frac{G(S)}{1+G(S)H(S)} = \frac{(S+Z_1)(S+Z_2)---}{(S+P_1)(S+P_2)---}$$

$1+G(S)H(S)=0$ is called characteristic Eqn.

→ Roots of CE = Poles of CLTF.

→ T.F is strictly Proper if the no. of poles are more than no. of zeroes.

→ without solving C.E, R-H criteria gives the no. of right hand roots (Poles).

→ Let us consider the characteristic eqn $1+G(S)H(S)=0$

$$a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0.$$

For stability:-

* All coefficients a_0, a_1, a_2, a_3 & a_4 must be of same sign.

* There shouldl not be any missing coefficients in the characteristic Equation.

* If the above two conditions are satisfied the stability is tested by the following RH criteria. (Necessary & sufficient condition).

RH tabulation:-

$$\begin{array}{c|cccc}
s^4 & a_0 & \overset{1^{st}}{\nearrow} & a_2 & a_4 \\
s^3 & a_1 & & a_3 & 0 \\
s^2 & \frac{a_1a_2 - a_0a_3}{a_1} = A & & \frac{a_1a_4 - 0}{a_1} = a_4 & 0 \\
s^1 & \frac{A \cdot a_3 - a_1a_4}{A} = B. & & 0 & 0 \\
s^0 & \frac{B \cdot a_4 - A \cdot 0}{B} = a_4 & & &
\end{array}$$

→ for stability all the 1st column elements must be +ve i.e., greater than zero.

→ No. of sign changes in the 1st column is equal to no. of Right hand poles/roots.

(Pb) $T.F = \frac{10}{s^3+6s^2+11s+6}$. Test the stability

s^3	1	11
s^2	6	6
s^1	$\frac{6\times11-6}{6}=10$	0
s^0	$\frac{10\times6}{6}=6$	

No.of Sign changes = 0

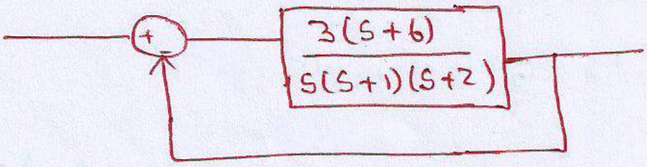
" " − RHP = 0

" " jω poles (jωP) = 0.

" " Left hand Poles LHP = 3.

∴ System is stable.

(Pb)



$C.E = 1+G(s)H(s)=0 \quad 1+\frac{3(s+6)}{s(s+1)(s+2)}=0$

$s^3+3s^2+5s+18=0$.

s^3	1	5
s^2	3	18
s^1	$\frac{15-18}{3}=-1$.	0
s^0	$\frac{-18-0}{-1}=18$.	

No.of sign changes = 2.

No.of RHP = 2.

jωP = 0

LHP = 1

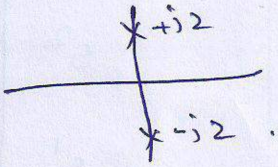
(Pb) $G(s)H(s)=4/s^2$

$1+G(s)H(s)=0 \quad 1+4/s^2=0$.

$s^2+4=0$

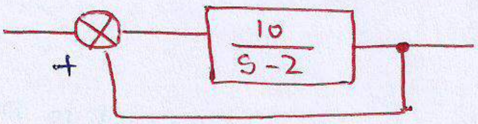
$s=\pm j2$.

$\omega_n=2$ rad/s.



→ System Response oscillates with $\omega_n=2$ rad/s.

(Pb)



∴ System is unstable system.

$1-G(s)H(s)=0$

$1-\frac{10}{s-2}=0 \Rightarrow s-12=0$.

RH Criteria: Special Cases:-

Case ①:- In tabulation, if one of the 1st column element becomes zero then tabulation can't be proceeded further.

Remedy:- Replace '0' by ϵ $(\epsilon \to 0)$ and continue tabulation as usual.

(Pb) characteristic Equation is $s^3 - 3s + 2 = 0$. Find the no. of Right hand poles.

+	s^3	1	−3
+	s^2	$0(\epsilon)$	2
−	s^1	$\frac{-3\epsilon-2}{\epsilon}$	0
+	s^0	2	

No. of sign changes = 2

RHP = 2

jωP = 0

LHP = 1

(Pb) $T.F = \frac{10}{s^5+2s^4+3s^3+6s^2+5s+3}$. Test the stability of the System.

+	s^5	1	3	5.
+	s^4	2	6	3
+	s^3	$\frac{6-6}{2} = 0(\epsilon)$	$7/2$	0
−	s^2	$\frac{6\epsilon-7}{\epsilon} = \frac{-7}{\epsilon}$	3	0
(+) or (−1)	s^1	$6\cdot\frac{7}{2}\cdot\epsilon \div \frac{7}{2}\cdot\frac{7}{2}$	0	0
+	s^0	3.		

No. of sign changes = 2

No. of RHP = 2

jωP = 0

LHP = 3

$$\frac{-\frac{7}{\epsilon}\cdot\frac{7}{2} - 3\epsilon}{-7/\epsilon}$$

Sperial Case 2:- In this case, the entire row becomes zero $(s^1, s^3, s^5, \ldots)$ and tabulation stops.

This indicates non availabily of coefficient in that row.

Procedure to eliminate this difficulty:-

Form an equation by using coefficients of a row which is just above the row of zeroes. Such an equation is called an Auxillary Equation denoted as A(s)

-> Take derivative Auxillary Eqn with respect to S.

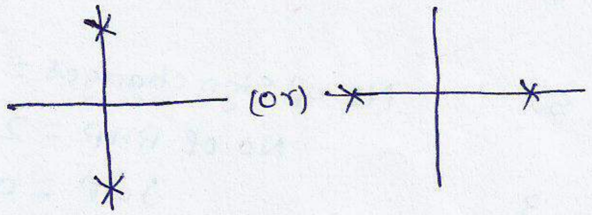
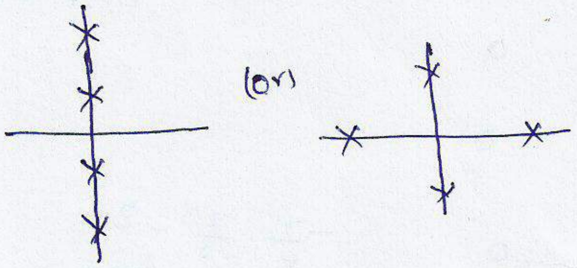
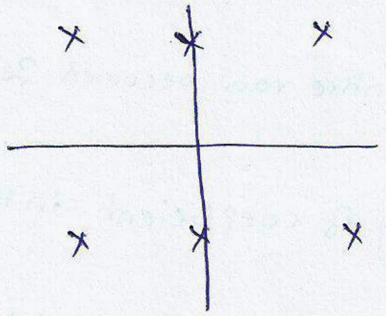
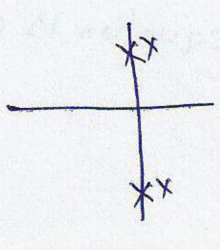
-> Replace row of zeroes by the coefficients of $\frac{dA(s)}{ds}$

-> Complete the array interms of these new coefficients.

s^5			
s^4	1	2	1
s^3	0 (4)	0 (4)	0
s^2			
s^1			
s^0			

$A.E = s^4 + 2s^2 + 1.$

$\frac{dA.E}{ds} = 4s^3 + 4s + 0.$

Symmetrical roots w.r.t. origin.	RH tabulation. Row of zero	Row of AE.
① 	s^1.	s^2
② 	s^3	s^4.
③ 	s^5	s^6
④ 	s^3 & s^1	s^4 & s^2.

(Pb) $T.F = \dfrac{4}{s^5+4s^4+8s^3+8s^2+7s+4}$. Test the stability.

s^5	1	8	7	
s^4	4	8	4	
s^3	$\frac{32-8}{4} = 6.$	$\frac{24}{4} = 6$	0.	
s^2	$\frac{48-24}{6} = 4$	$\frac{24-0}{6} = 4.$	0	→ Row of A.E.
s^1	0 (8)	0	0	→ Row of zero
s^0	4.			

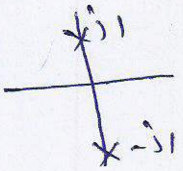
$A.E = 4s^2+4$

$\frac{dAE}{ds} = 8s$

Roots of $A.E = 4s^2+4=0$

$s = \pm j = \pm j\omega_n$

$\omega_n = 1$ rad/s.



No. of RHP = 0 , jωP = 2 , LHP = 3.

∴ system is marginally stable.

(Pb) $C.E = 2s^3+2s^2+4s+4 = 0$. Find the natural frequency of the system

s^3	2	4	
s^2	2	4	– Row of A.E.
s^1	0 (4)	0	– Row of zero
s^0	4		

RHP = 0
jωP = 2
LHP = 1

$2s^2+4=0 \Rightarrow s^2+2=0$

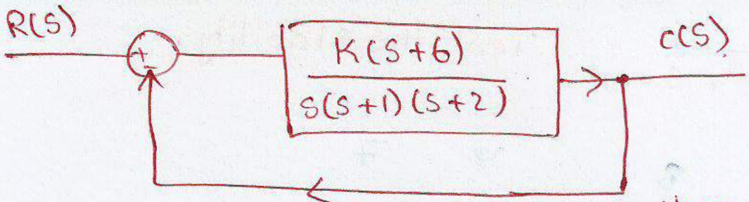
$s = \pm j\sqrt{2}$.

$\omega_n = \sqrt{2}$ rad/s.

$4s = 0$

Q.5

(Pb)



1) Find the range of K for the system to be stable
2) Find the range of 'K' for the system to oscillate with fixed amplitude
3) corresponding oscillating frequency.

$$C.E = 1+\frac{K(S+6)}{S(S+1)(S+2)}=0$$

$$\Rightarrow S^3+3S^2+2S+KS+6K=0$$

$$S^3+3S^2+(2+K)S+6K=0$$

$$\begin{array}{c|cc} S^3 & 1 & (2+K) \\ S^2 & 3 & 6K \\ S^1 & \frac{3(2+K)-6K}{3} & 0. \\ S^0 & 6K. & \end{array}$$

→ Row of zero.

① → For stability all first column elements must be +ve,

$$\frac{3(2+K)-6K}{3}>0 \Rightarrow 6+3K-6K>0 \qquad 6-3K>0.$$

$$-K<-2. \qquad K<2$$

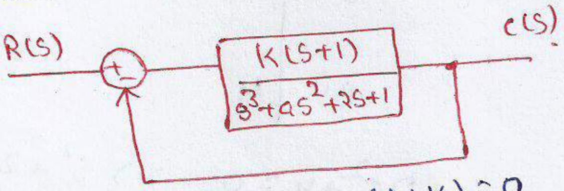
$$6K>0 \qquad K>0. \qquad \boxed{0<K<2}$$

② $K=2.$

$3S^2+\cancel{6}12=0.$ $\quad S=\pm j2.$

③. $\omega_n=2\text{rad/sec}.$

(Pb) If $\omega_n=2\text{r/s}$. Find 'K' & 'a'



$$1+\frac{K(S+1)}{S^3+aS^2+2S+1}=0.$$

$$S^3+aS^2+2S+1+SK+K=0. \qquad S^3+aS^2+(2+K)S+(1+K)=0$$

$$\begin{array}{c|cc} S^3 & 1 & (2+K) \\ S^2 & a & (1+K) \\ S^1 & \frac{a(2+K)-(1+K)}{a} & 0. \\ S^0 & 1+K. & \end{array}$$

$a(2+K)=1+K.$

$2a+aK=1+K. \Rightarrow a=\frac{1+K}{2+K}.$

$A.E=aS^2+K+1=0.$

$$A\cdot E = \frac{(1+K)s^2}{2+K} + K+1 = 0.$$

$$(1+K)s^2 + (K+1)(K+2) = 0.$$

$$s^2+K+2 = 0. \quad \Rightarrow s = \pm j\sqrt{K+2}.$$

$$= \pm j\,2$$

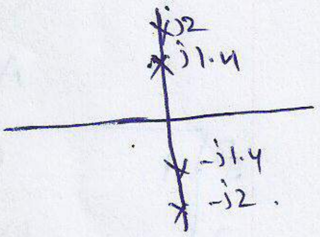
$$\sqrt{K+2} = 2 \quad \Rightarrow K = 2.$$

$a = 3/4.$

(Pb) Test the stability for the given system with C.E

$$C.E = s^6+2s^5+8s^4+12s^3+20s^2+16s+16.$$

s^6	1	8	20	16
s^5	2	12	16.	0
s^4	2	12	16	0
s^3	0(8)	0 (24)	0	0
s^2	6	16.	0	0.
s^1	2.6.	0	0	0.
s^0	16	0	0	0



$$A\cdot E = 2s^4+12s^2+16$$

$$\frac{d}{ds}A\cdot E = 8s^3+24s+$$

Roots of A.E = C.L Poles lying on jω axis.

$$2s^4+12s^2+16 = 0. \qquad s^4+6s^2+8 = 0.$$

$$2s^4+4s^2+8s^2+16 = 0 \qquad s^4+2s^2+4s^2+8 = 0$$

$$s^2(\qquad s^2(s^2+2)+4(s^2+2) = 0$$

$$s = \pm j1.4, \pm 2j$$

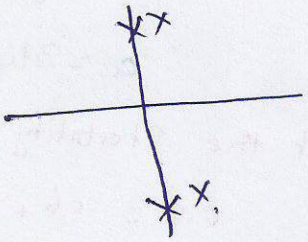
a.6

Pb $C \cdot E = 2S^5 + S^4 + 4S^3 + 2S^2 + 2S + 1$

S^5	2	4	2
S^4	1	2	1
S^3	0 (4)	0 (4)	0.
S^2	1	1	0
S^1	0 (2)	0	
S^0	2.		

$\longrightarrow A \cdot E.$

– Row of zero



$A \cdot E_1 = S^4 + 2S^2 + 1$

$\frac{d}{dS} A(S) = \cancel{8S} \, 4S^3 + 4S$

$A \cdot E_2 = S^2 + 1$

$\frac{d}{dS} A(S) = 2S.$

$S^4 + S^2 + S^2 + 1 = 0.$

$S^2(S^2+1) + 1(S^2+1) = 0.$

$S = \pm j1 , \pm j1$

"Unstable"

Pb $G(S) = \frac{K}{(S^2+2S+2)(S+2)}$

$1 + G(S)H(S) = 0.$ $1 + \frac{K}{(S^2+2S+2)(S+2)} = 0$

$S^3 + 4S^2 + 6S + (4+K) = 0$

$A(S) =$

S^3	1	6
S^2	4	(4+K)
S^1	$\frac{24-4-K}{4}$	0.
S^0	4+K.	

$\frac{20-K}{4} = 0.$

$K_{max} = 20.$

$A \cdot E = 4S^2 + 24 = 0$

$S^2 + 6 = 0$

$S = \pm j\sqrt{6}.$

$\omega = \sqrt{6}$ rad/s.

Difficulties & limitations of R-H Criteria :-

1. All the Coefficients of Polynomial must be real and +ve (Limitation)

2. The no. of Sign Changes in the first column gives the no. of Poles in R.H.S of S-plane. but not their locations. (L)

3. When the System is marginally stable with Oscillatory response to find the frequency of osc., the A.E should be an even Polynomial of order 2 (L).

4. Relative stability Analysis using Routh Array is not - feasible for higher order Polynomial because it involves shifting of S-plane more -vely. (L)

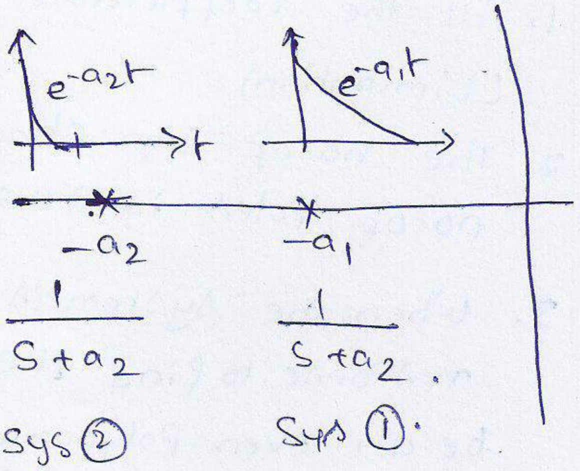
5. When the first element of any row is Zero while the rest of row has at least one non zero term then in such cases Substitute small +ve ϵ in place of zero and evaluate the rest of routh array interms of ϵ, checking for sign changes by taking Lt ϵ tends to zero for the 1st column element to coment on stability. (D)

⑥ When Routh Array ends abruptly then construct an A.E, Differentiate it to get new coefficient and evaluate rest of Routh Array. Check for Repeatability of A.E roots (or) Imaginary axis to coment on stability. (Difficulty)

Relative Stability Analysis:-

→ Both systems are absolutely stable.

→ System ② is said to ~~more~~ relatively more stable than System① because $t_2 << t_1$

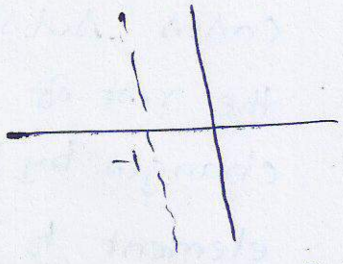


→

$t_2 << t_1$.

Eg C.E = $s^3+7s^2+25s+39=0$.

To check ~~more~~ ~~yet~~ whether the roots are lying more -vely w.r.t -1



$(s+1)=z$.

$s=z-1$

$P(z)=(z-1)^3+7(z-1)^2+25(z-1)+39=0$.

$=z^3+3z-3z^2-1+7z^2+7-14z+25z-25+39=0$

$=z^3+4z^2+14z+20=0$.

$-1+7-25+39$

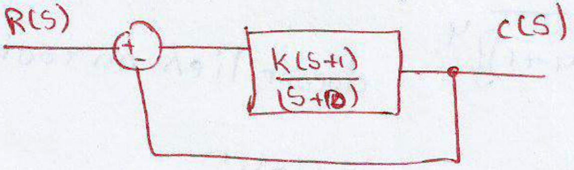
$\underbrace{\quad}$ 6 14

z^3	1	14
z^2	4	20.
z^1	$\frac{56-20}{4\cancel{0}}=9$	0.
z^0	20.	

Unit-III Part-2 Root Locus Technique

The Root Locus is defined as Locus of CL poles obtained when system gain 'K' is varied from zero to ∞.

(Pb)



Sketch RLD.

C.E : $1+G(S)H(S)=0$ $\quad 1+\frac{K(S+1)}{S+10}=0 \Rightarrow S+10+SK+K=0.$

$$S=-\frac{(K+10)}{K+1}$$

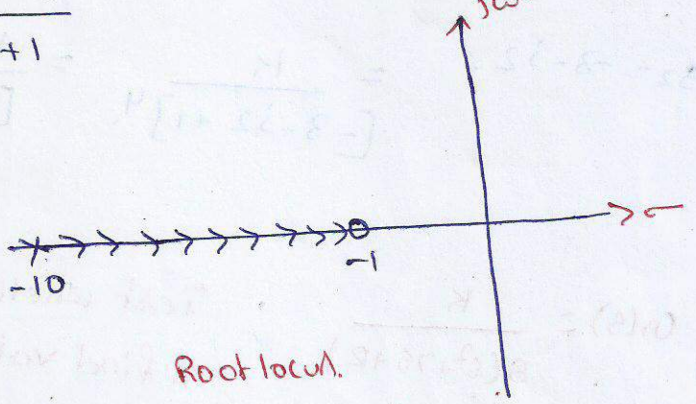
$K=0$ $\quad S=-10$

$K=1$ $\quad S=-11/2$

$K=2$ $\quad S=-4$

$K=3$ $\quad S=-13/4$

$K=\infty$ $\quad S=-1.$



Angle & Magnitude Condition:-

→ Angle condition is used for checking whether certain points lie on root locus or not and also validity of root locus for. closed loop poles.

Magnitudes of $G(S)H(S) = |G(S)H(S)|=1$

$$\Rightarrow K|G_1(S)H_2(S)|=1.$$

$$K=\frac{1}{|G_1(S)H_1(S)|}$$

Phase of $G(S)H(S)$

from C.E $\quad 1+G(S)H(S)=0$

$$G(S)H(S)=-1+j0.$$

$$\angle G(S)H(S)=180^\circ-\tan^{-1}\left(\frac{0}{1}\right)$$

$$=(2q+1)\pi$$

$$q=0,1,2,----$$

→ Angle condition may be stated as " for a point to lie on root locus the angle evaluated at that point must be odd multiple of ±180°.

→ The Magnitude condition is used for finding the value system gain 'K' at any point on root locus.

(Pb) $G(s)H(s) = \frac{K.}{(s+1)^4}$. : Test whether points $s_1 = -3+j4$; $s_2 = -3-j2$ lie on root locus.

Angle condition.

$$G(s)H(s) = \frac{K+j0}{[-3+j4+1]^4}.$$ doesnt lies on root locus.

$$\angle G(s)H(s) = \frac{0}{[\tan^{-1}(\frac{4}{-2})]^4} - = -464'$$

$$s_2 = -3-j2. \quad = \frac{K}{[-3-j2+1]^4}. = \frac{K+j0}{[-2-j2]^4} = \frac{0'}{-135' \times 4} = +540'$$

lies on to root locus.

(Pb) $G(s) = \frac{K}{s(s^2+7s+R)}$; Test whether point $s = -1+j$ lie on root locus & find value of K at that point.

$$~~G~~ \angle G(s)H(s) = \frac{K+j0}{(-1+j)((-1+j)^2+7(-1+j)+12))} = \frac{K+j0}{(-1+j)(5+5j)}$$

$$= \frac{0'}{[135'][45']} = -180'$$

Point lies on root locus.

Value of K at $s = -1+j$

$$K = \frac{1}{|G(s)H(s)|} = \frac{1}{\quad}$$

$$\frac{K}{\sqrt{1+1}\ \sqrt{5^2+5^2}} \Rightarrow \frac{K}{\sqrt{2}\ \sqrt{50}} \Rightarrow K = \sqrt{100}.$$

$$\boxed{K=10}$$

Construction Rules of Root locus:-

Rule:① Every branch of RLD starts at the poles. and terminates at the zeroes $(K=\infty)$ of the OLTF.

Let $G(S)H(S) = \frac{K(S+Z_1)(S+Z_2)(}{(S+P_1)(S+P_2)(S+P_3)}\Big|$ at $S=P_1, -P_2, -P_3$, K becomes ∞.

$\therefore K=0 \rightarrow$ poles of $G(S)H(S)$

$$K = \frac{+1}{|G_1(S)H_1(S)|}\Big|_{S=-Z_1,Z_2} = \infty.$$

$\therefore K=\infty \rightarrow$ zeroes of $G(S)H(S)$

Rule:-② RLD is symmetrical w.r.t real axis.

Rule ③:- Let P = no. of OL poles and $P \geq Z$.

Z = " " zeroes.

No. of branches of Root locus $= P$

" " terminating at zeroe $= Z$.

" " " at Infinity $= P-Z$.

Rule:-④ A point on real axis is said to be on root locus if to the right side of the point, the sum of openloop poles and zeroes is odd.

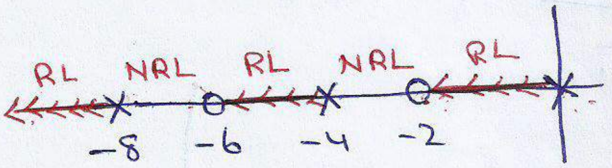
eg:- $G(S)H(S) = \frac{K(S+2)(S+6)}{S(S+4)(S+8)}$

CL poles $= 0, -4, -8$.

zeroes $= -2, -6$.

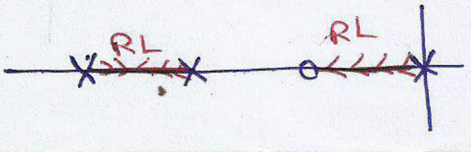
$\rightarrow P=3, \; Z=2., \; P-Z=1$

$\rightarrow$



eg:- $G(S)H(S) = \frac{K(S+1)}{S(S+3)(S+4)}$

$P=3, \; Z=1, \; P-Z=2$



Rule 5:- Centroid (σ):-

It is intersection of asymptotes and lies on the real axis.

$$\sigma = \frac{(\Sigma \text{ real part of openloop pole} - \Sigma \text{ real part of openloop zero}}{P-Z}.$$

→ It may or may not be part of root locus.

Rule 6:- Angle of asystem asymptotes

The P-Z branches terminate at "∞" along certain straight lines known as asymptotes of root locus. Therefore no. of asymptotes (P-Z)

Angle of asymptotes is given by relation

$$\theta = \frac{(2q+1)\,180^\circ}{P-Z} \;;\; q = 0,1,2,3,\dots(P-Z-1)$$

$P-Z=2$, asymptote angle will $\theta_1 = \frac{2(0)+1}{2}180, \frac{2(1)+1}{2}180^\circ$

$= 90^\circ, 270^\circ$

$P-Z=3$, $\theta_1 = \frac{2(0)+1}{3}180^\circ, \frac{2(1)+1}{3}180^\circ, \frac{2(2)+1}{3}$

$= 60^\circ, 180^\circ, 300^\circ.$

Angle between asymptotes is given by $\frac{2\pi}{P-Z}$.

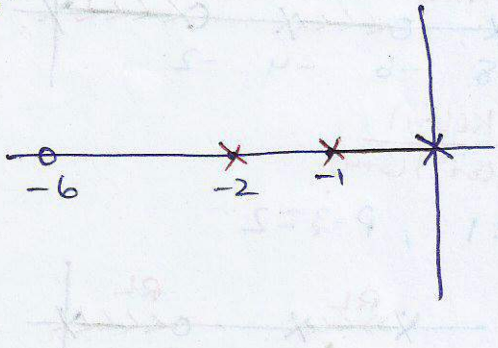
(Pb) $G(s)H(s) = \frac{K(s+6)}{s(s+1)(s+2)}$. Find the centroid and angle of asymptotes.

$$\sigma = \frac{\Sigma P - \Sigma Z}{P-Z} = \frac{(0-1-2)-(6)}{3-1} = \frac{-3+6}{2} = 1.5.$$

$P-Z = 3-1 = 2.$

$\theta_1 = \frac{2(0)+1}{2}\pi,\quad \theta_2 = 270\ \&\ (-90^\circ)$

$= 90^\circ$



(Pb) Find centroid & asymptotes of RLD, if characteristic Equation is $s^3+5s^2+(K+6)s+k=0$

$$s^3+5s^2+6s+K(s+1)=0.$$

$$1+K\frac{(s+1)}{s^3+5s^2+6s}=0.$$

$$G(s)H(s)=\frac{K(s+1)}{s(s^2+5s+6)}=\frac{K(s+1)}{s(s+2)(s+3)}$$

$P=3$

$Z=1$

$P-Z=2$

$\theta_{1,2}=90^\circ, -90^\circ$

Zeroes $=-1+j0$

Poles $=0+j0$

$-2+j0$

$-3+j0$

$\overline{-5}$.

$$\sigma=\frac{-5-(-1)}{2}.$$

$\sigma=-2$

$=(-2,0)$

Rule 7:- Break away Points.

These are Points at which the two branches of root locus meet (or) from which they separate out.

The breakaway Points can be obtained as roots of C.E after making $\frac{dK}{ds}=0$.

Importance of Break away Points:-

① It is found when two Poles (or) two zeroes lie adjacent to each other.

② It is to be selected such that the no. of poles and zeroes to right of break away point should be odd.

Procedure:-

1. Construct $1+G(s)H(s)=0$
2. write 'K' interms of s
3. find $\frac{dK}{ds}=0$
4. the roots of $\frac{dK}{ds}=0$ will give Break away points.
5. To test valid Breakaway point substitute in step (2) if K = +ve. valid Break away Point.

Rule 8:- Intersection of Root locus with Imaginary axis.

The roots of auxillary Eqn A(S) at K=Kmax from Routh array gives the intersection of Root locus with Imaginary axis.

Rule 9:- The value of K on the RLD at S=S₁ | operating point |

$$K|_{s=s_1} = \frac{\text{Product of distances from various poles of } G(S)H(S) \text{ to } S=S_1}{\text{Product " " " zeroes "}}$$

(or)

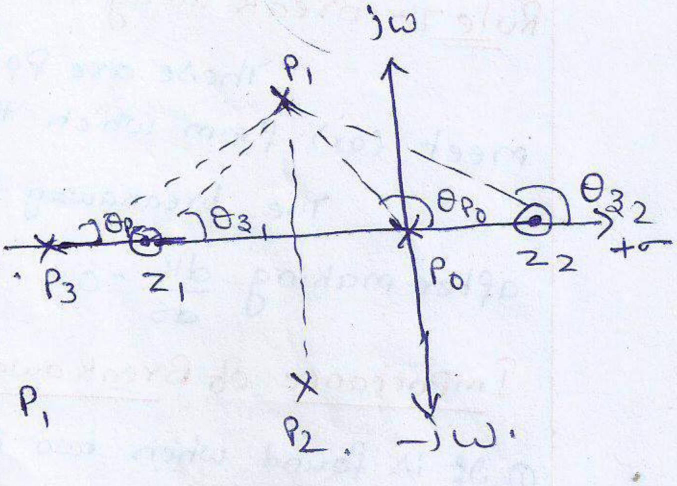
from Magnitude condition.

$$K|_{s=s_1} = \frac{1}{|G_1(S)\,H_1(S)|_{s=s_1}}$$

Rule 10:- If there is a complex pole then determine angle of departure.

$$\phi_D = 180° - \sum \phi_p + \sum \phi_z .$$

$\sum \phi_p$ = Sum of all pole angle w.r.t. P_1



$\sum \phi_z$ = " " zero " " P_1

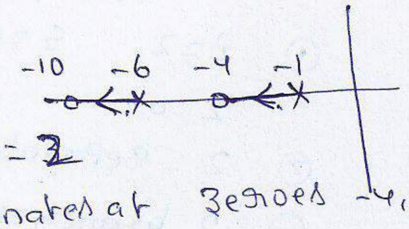
$$\phi_D \text{ for } P_1 = \phi_{D_1} = 180° - (\theta_{P_2} + \theta_{P_3} + \theta_{P_0}) - (\theta_{z_1} + \theta_{z_2})$$

Rule 11:- Angle of Arrival at complex zero.

$$\phi_A = 180° - \sum \phi_z + \sum \phi_p .$$

$\sum \phi_z$, & $\sum \phi_p$ - Same as above.

① $G(s)H(s) = \frac{K(s+4)(s+10)}{(s+1)(s+6)}$ K at $s = -2$, $s = -8$



1) $P = 2$, $s = -1, -6$

$Z = 2$, $s = -4, -10$

2) No. of separate root locii = No. of pole = 2

3) Starting point of RL −1, −6 and terminates at zeroes −4, −10.

4) No. of asymptotes $P - Z = 0$.

σ, θ not required.

Real axis RLD.

5) $$K\Big|_{s=-2} = \frac{(\text{Distance from } -2 \text{ to } -1)(\text{Dist. from } -2 \text{ to } -6)}{(\text{Dist from } -2 \text{ to } -4)(\text{Dist. from } -2 \text{ to } -10)}$$

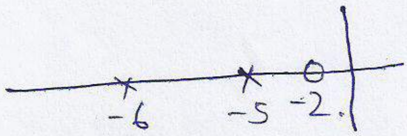
$= 1/4.$

(or)

$$K\Big|_{s=-2} = \frac{1}{|G_1(s)H_1(s)|\,|_{s=2}} = \left|\frac{1}{\frac{2 \times 8}{(-1)(-4)}}\right| = 1/4$$

$$K\Big|_{s=-8} = \frac{(7)(2)}{(4)(2)} = 7/4.$$

② $G(s)H(s) = \frac{K(s+2)}{(s+5)(s+6)}$ K at $s = -3$, $s = -10$



1) $P = 2$, $s = -5, -6$

$Z = 1$ $s = -2$.

2) No. of separate root locii = 2.

3) Starting point of RL branches −5, −6 and one branch terminate at −2

other ~~reaches~~ goto to ∞ along asymptote

4) No. of asymptotes $N = P - Z = 3 - 1 = 2$.

σ = not required.

$$\theta = \frac{(2l+1)\pi}{P - Z} \qquad l = 0, 1 \dots (P-Z) - 1$$

$= 180°$

5) Real axis RLD.

6) $$K\Big|_{s=-3} = \frac{(-3+5)(-3+6)}{-3+2} = \frac{(2)(3)}{1} = 6$$

$$K\Big|_{s=-10} = \frac{(5)(4)}{8} = 5/2$$

③ $G(S)H(S) = \frac{K}{S(S+4)}$. Find K at $S = -2$.

Sol:- ① $P = 2$, $S = 0, -4$

$Z = 0$

② 2 separate root locii

③ 2 branches starts at poles and go to infinity via asymptotes

④ No. of asymptotes $= P - Z = 2$.

$$\sigma = \frac{-4}{2} = -2.$$

$$\theta = \frac{(2l+1)\pi}{P-Z}.$$ 90° & 270

90° & -90°

⑤ B.A =

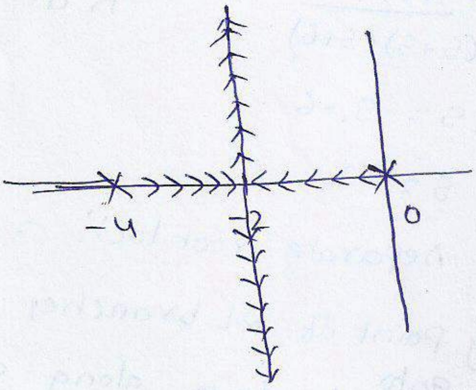
$$1 + \frac{K}{S(S+4)} = 0 \Rightarrow S^2 + 4S + K = 0.$$

$$2S + 4 + \frac{dK}{dS} = 0$$

$$K = -S^2 - 4S$$

$$\frac{dK}{dS} = -2S - 4 = 0.$$ $2S + 4 = 0$

$\boxed{S = -2}$



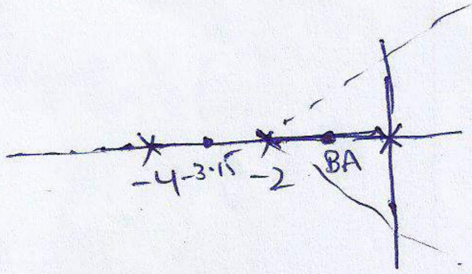
① Sketch the RLD for OLTF of UFB CS $G(S)H(S) = \frac{K}{S(S+2)(S+4)}$

Sol:-

1) No. of Poles $P = 3$.

$S = 0, -2, -4$.

$Z = 0$.



2) No. of separate locii $= P - Z = 3$

3) Starting of root locus are from poles at 0, -2, -4. End point of root locus is at zero. As we have no zeroes, end points is ∞.

4) No. of Asymptotes $N = P - Z$.

$= \underline{3}$

5) Centroid $(\sigma) = \frac{\Sigma P - \Sigma Z}{P - Z} = \frac{-6-0}{3} = \underline{-2}$.

6) Angle of asymptotes ; $\theta_0 = 60$, $\theta_1 = 180°$, $\theta_2 = 300°$

7) Break away Points $1 + \frac{K}{S(S+2)(S+4)} = 0$

$\Rightarrow S(S^2 + 6S + 8) + K = 0$.

$K = -(S^3 + 6S^2 + 8S)$

To find Break away, differentiate 'K' w.r.t. S & equate it to zero.

$\frac{dK}{dS} = 0 \Rightarrow 3S^2 + 12S + 8 = 0$. $S = -0.84, -3.15$.

Note:- $S = -3.15$, cannot be taken as Breakaway Point since the right side of Break away pt has even no. of Poles & zeroes.

⑧ Intersection on Imaginary axis.

$C.E = S^3 + 6S^2 + 8S + K = 0$.

S^3	1	8
S^2	6	K.
S^1	$\frac{48-K}{6}$	0
S^0	K.	

$K > 0$

$48 - K = 0$ $K_{mar} = 48$.

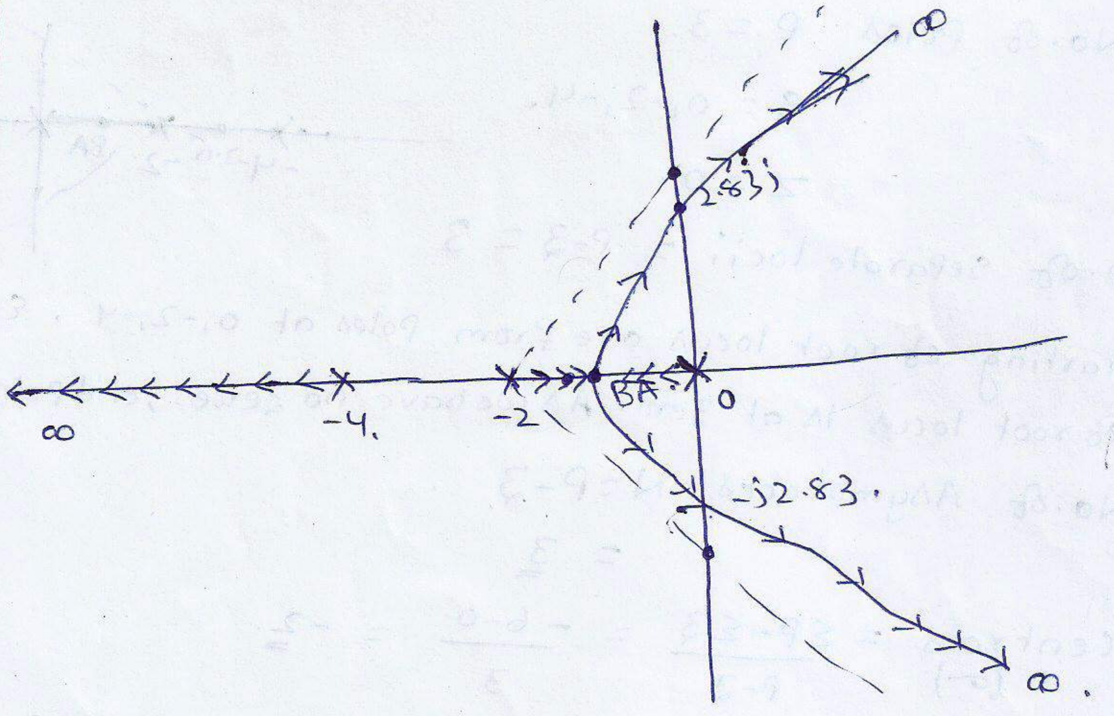
$K = 48 \Rightarrow$

$6S^2 + 48 = 0$.

$S^2 + 8 = 0$

$S = \pm\sqrt{8}\, j$

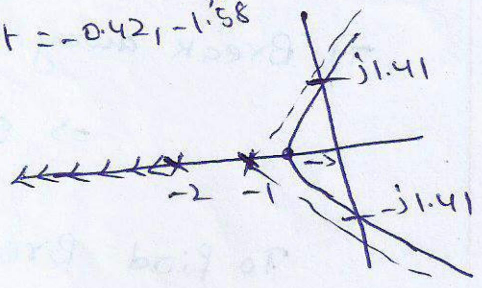
$= \pm 2.83 j$



② Sketch the Root LD, $G(s) = \frac{K}{s(s+1)(s+2)}$. Also find K at B.A.pt.

$\sigma = -1.$,

B.A.Pt = -0.42, -1.58



$K = 0.42 \times 0.58 \times 1.58.$

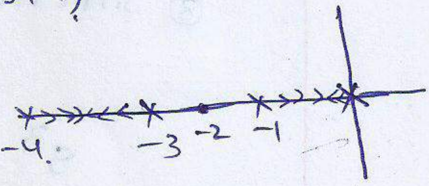
$= \underline{0.385.}$

③ Sketch RLD for UFB T.F $= \frac{K}{s(s+1)(s+3)(s+4)+K}$.

$$\frac{G(s)}{1+G(s)} = \frac{K}{s(s+1)(s+3)(s+4)+K}$$

$$G(s)\{s(s+1)(s+3)(s+4) + \cancel{K\,G(s)}\} = K \cancel{+ K\,G(s)}$$

$$, \quad G(s) = \frac{K}{s(s+1)(s+3)(s+4)}$$



① No of Poles = 4 at s = 0, -1, -3, -4.
No. of Zeroes = 0

② No. of root locii = No. of Poles = 4.

③ Starting point of root locus is from Poles at 0, -1, -3, -4.
End Point of root locus at ∞ as zeroes are absent.

④ No. of asymptotes $N = 4$.

$$\text{centroid} = \frac{\Sigma P - \Sigma Z}{P - Z} = \frac{(-1, -3 -4) - 0}{4} = -8/4 = -2.$$

⑤ Angle of asymptote $P - Z = 4$

$$\theta = 45', \; 135', \; 225', \; 315'.$$

⑥ Break away pt.

$$C.E \Rightarrow 1 + G(S)H(S) = 0.$$

$$1 + \frac{K}{S(S+1)(S+3)(S+4)} = 0$$

$$K = -(S^4 + 8S^3 + 19S^2 + 12S)$$

$$\frac{dK}{dS} = 0 \Rightarrow S^3 + 6S^2 + 9.5S + 3 = 0.$$

$$-3.58, \; -0.418, \; -2.1$$

✓ ✓ X

⑦ Intersection with Imaginary axis

$$C.E = 1 + G(S)H(S) = 0.$$

$$= S^4 + 8S^3 + 19S^2 + 12S + K = 0.$$

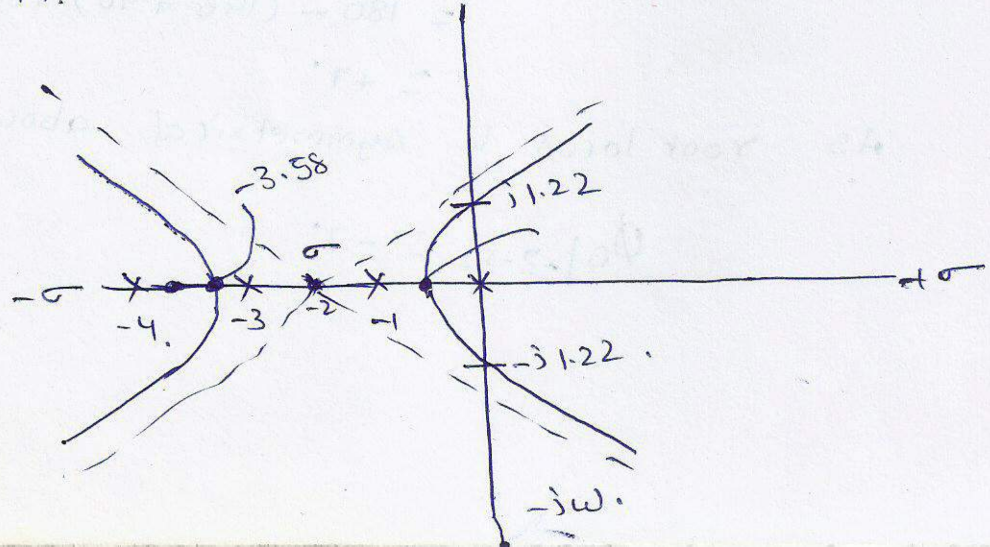
S^4	1	19	K
S^3	8	12.	
S^2	17.5	K	
S^1	$\frac{210 - 8K}{17.5}$	0	
S^0	K.		

$$\frac{210 - 8K}{17.5} = 0$$

$$K = 26.25.$$

$$17.5S^2 + 26.25 = 0$$

$$S = \pm j1.22$$



④ Sketch RLD $G(S)H(S) = \frac{K(S+4)}{S(S^2+6S+13)}$

$\frac{-6 \pm \sqrt{36-52}}{2a}$

$-3 \pm j2$.

Sol:-

① No. of Poles $P = 3, S = 0, -3+j2, -3-j2$.

No of zeroes $Z = 1$ $S, -4$.

② No. of separate locii = No. of Pole = 3.

③ starting point root locus are from poles at $0, -3+j2, -3-j2$, one Poles terminates at zero, $S=-4$. The remaining two root locii goto infinity along asymptotes.

④ No. of asymptotes $N = P - Z = 3 - 1 = 2$.

$$\sigma = \frac{-6+4}{2} = -1.$$

angle of asymptotes = 90°, 270°

⑤ Breakaway pts. No adjacent poles ∴ no. BA pts.

④ complex poles move to infinity along 90° & 270° asymptotes.

⑥ Intersection of Imaginary axis

$$C.E = 1 + \frac{K(S+4)}{S(S^2+6S+13)}$$

$$C.E = S^3 + 6S^2 + (13+K)S + 4K = 0.$$

$$\begin{array}{c|cc} S^3 & 1 & (13+K) \\ S^2 & 6 & 4K \\ S^1 & \frac{78+2K}{6} & 0 \\ S^0 & 4K. & \end{array}$$

$78 + 2K = 0 \Rightarrow K = -39$

$6S^2 + 4K = 0$

$6S^2 - 156 = 0$

$S = \pm 5.0.$

real number.

if K = -ve no intersection with jω axis.

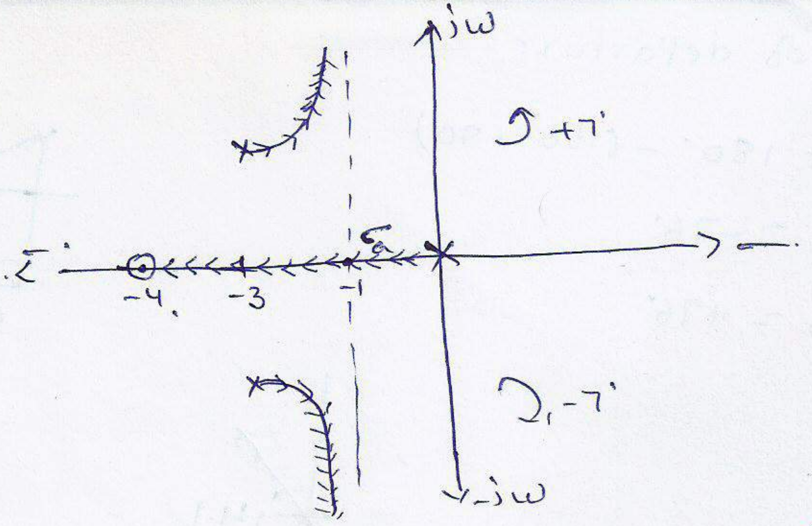
⑦ Angle of departure $\phi_D = 180 - \sum\phi_P + \sum\phi_Z$.

$= 180 - (146° + 90°) + 63°$

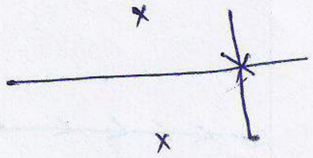
$= +7°$

As root locus is symmetrical about the real axis.

$\phi_D|_{-3-j2} = -7°$



⑤ Sketch the root locus of the UFB system with $G(s)H(s) = \frac{K}{s(s^2+8s+17)}$ find K for damping ratio of 0.5. Also determine the corresponding CLTF..



Sol:- ① No.of Poles = 3, $s = 0, -4 \pm j$

" zeroes = 0

② No.of Root loci = No.of poles = 3

③ Starting point of root locus are from poles $0, -4 \pm j$, . End point of root locus is zero. As there are no ~~poles~~ zeroe end point is infinity.

④ No. of asymptotes $N = P - Z = 3 - 0 = 3$.

$$\text{centroid } (\sigma) = \frac{-8}{3} = -2.67.$$

⑤ Angle of asymptotes = 60°, 180°, 300°

⑥ Break away pts :- In this problem, no poles are adjacent to each other. The pole at origin moves to ∞ along 180° asymptote and complex poles move to ∞ along 60° & 300° asymptotes. So no B.A pts.

⑦. Intersection with jω axis

$$C.E = 1 + G(s)H(s) = 0$$

$$= 1 + \frac{K}{s(s^2+8s+17)} = s^3 + 8s^2 + 17s + K = 0.$$

s^3	1	~~8~~17
s^2	8	K.
s^1	$\frac{136-K}{8}$	0.
s^0	K.	

$\boxed{K = 136}$

$8s^2 + K = 0$

$s^2 = -17.$

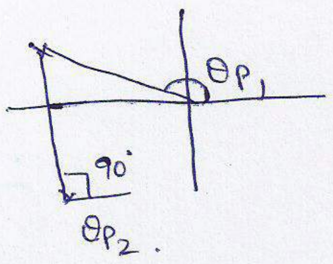
$s = \pm j4.1$

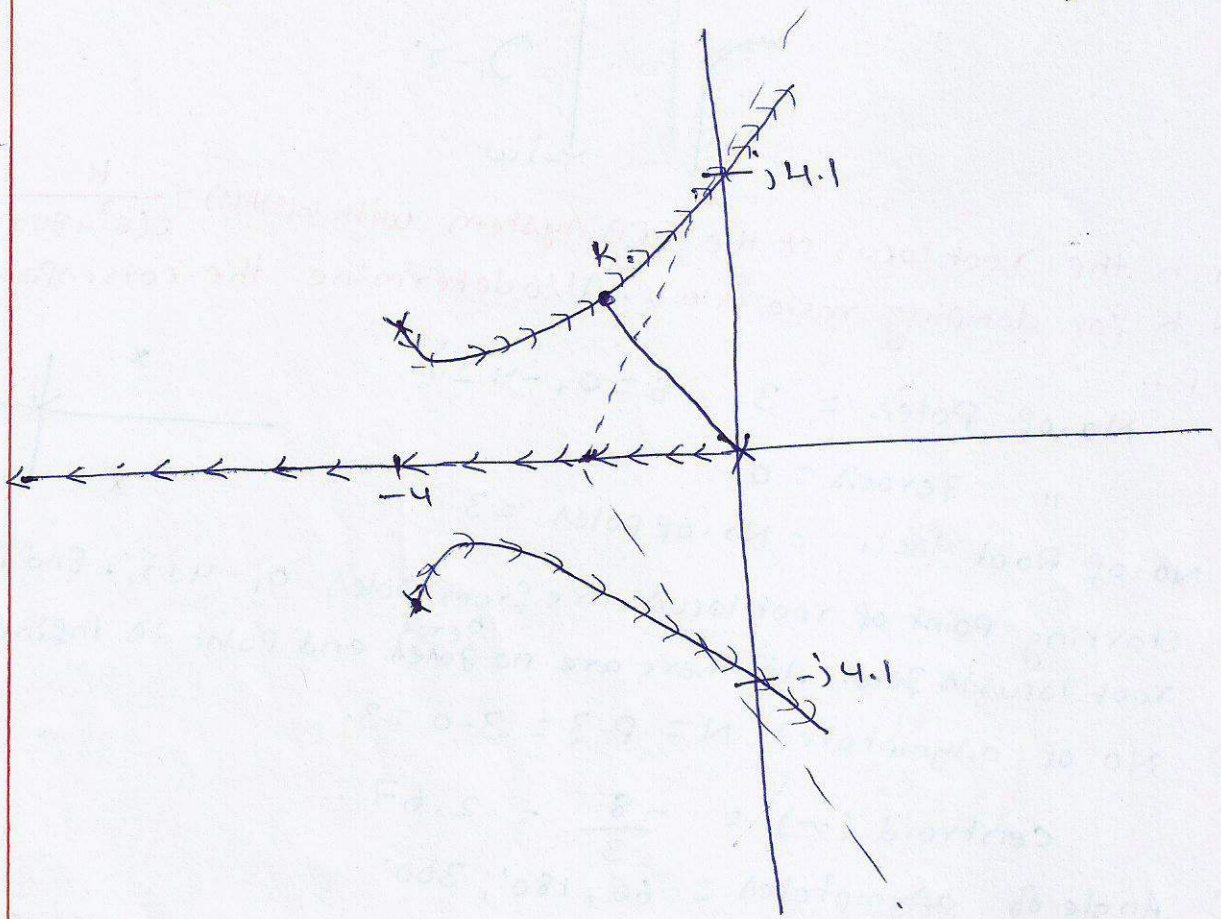
⑤ Angle of departure.

$$\phi_{D_3} = 180^\circ - (166^\circ + 90^\circ)$$

$$= -76^\circ$$

$$\phi_{D_2} = +76^\circ$$





⑨ Calculation of K.

$$s_1 = -1.5 + 2.6j$$

$$\delta = 0.5 \Rightarrow \phi\delta = \cos^{-1}\delta \Rightarrow 6\phi = \underline{\underline{60^\circ}}$$

$$K = |s(s^2 + 8s + 17)|.$$

$$= |(-1.5+2.6j)|\ ((-1.5+2.6j)^2 + 8(-1.5+2.6j) + 17)|$$

$$= \sqrt{2.25 + 6.76}.$$

$$= |3|(\ 2.25 - 6.76 - 7.83j) + (-12 + 20.6j) + 17)$$

$$= |3|\ (-4.51 - 7.83j - 12 + 20.6j + 17)$$

$$= |3|\ (0.5. + 13.27j)$$

$$= |3| \sqrt{0.25 + 176.09} = 13.27 \times 3$$

$$= 39.83 \approx \boxed{40.5}$$

$$K \simeq 40.$$

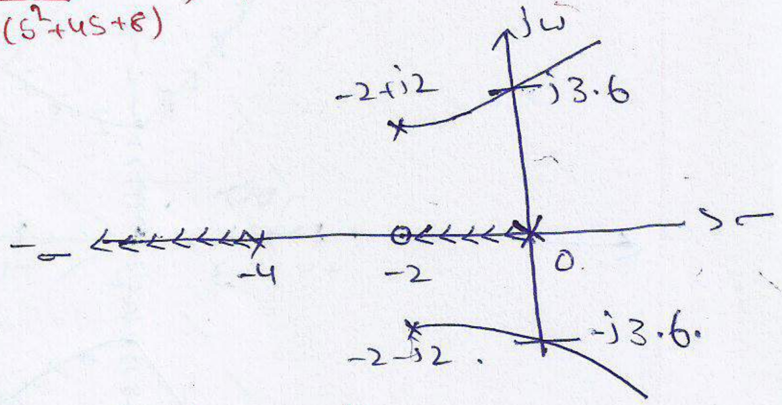
(90) $G(s)H(s)$ for $(\delta = 0.5) = \dfrac{40.5}{s(s^2+8s+17)}$

$$\frac{C(s)}{R(s)} = \frac{40.5}{s(s^2+8s+17)+40.5}.$$

⑥ Sketch RLD, $G(s)H(s) = \dfrac{K(s+2)}{s(s+4)(s^2+4s+8)}$; find the range of K.

$K_{max} = 71.55.$

$0 < K < 71.55.$

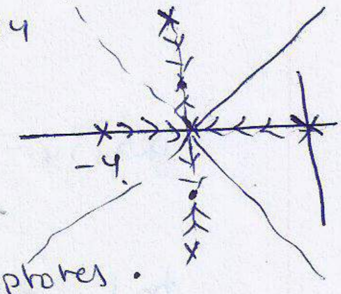


⑦ Sketch the root locus for OLTF $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+20)}$

(i) find K for $s = -3$

(ii) find δ at $s = -2 + j3$.

① No. of poles = 4 , at $s = 0, -4, -2 \pm j4$
zeroes = 0



② No. of separate root locii = 4.

③ No. of asymptotes = 4
All four root locii move along asymptotes.
angle of asymptotes = 45°, 135°, 225°, 315°

$$\text{Centroid}(\sigma) = \frac{-4-4}{4} = -2.$$

④ $C.E = 1 + \dfrac{K}{s(s+4)(s^2+4s+20)} = 0.$

$K = -(s^4 + 8s^3 + 36s^2 + 80s)$

$\dfrac{dK}{ds} = 4s^3 + 24s^2 + 72s + 80 = 0.$

$s^3 + 6s^2 + 18s + 20 = 0.$

valid B.A pts $-2 \pm j2.45.$, -2

⑤ Intersection jω axi.

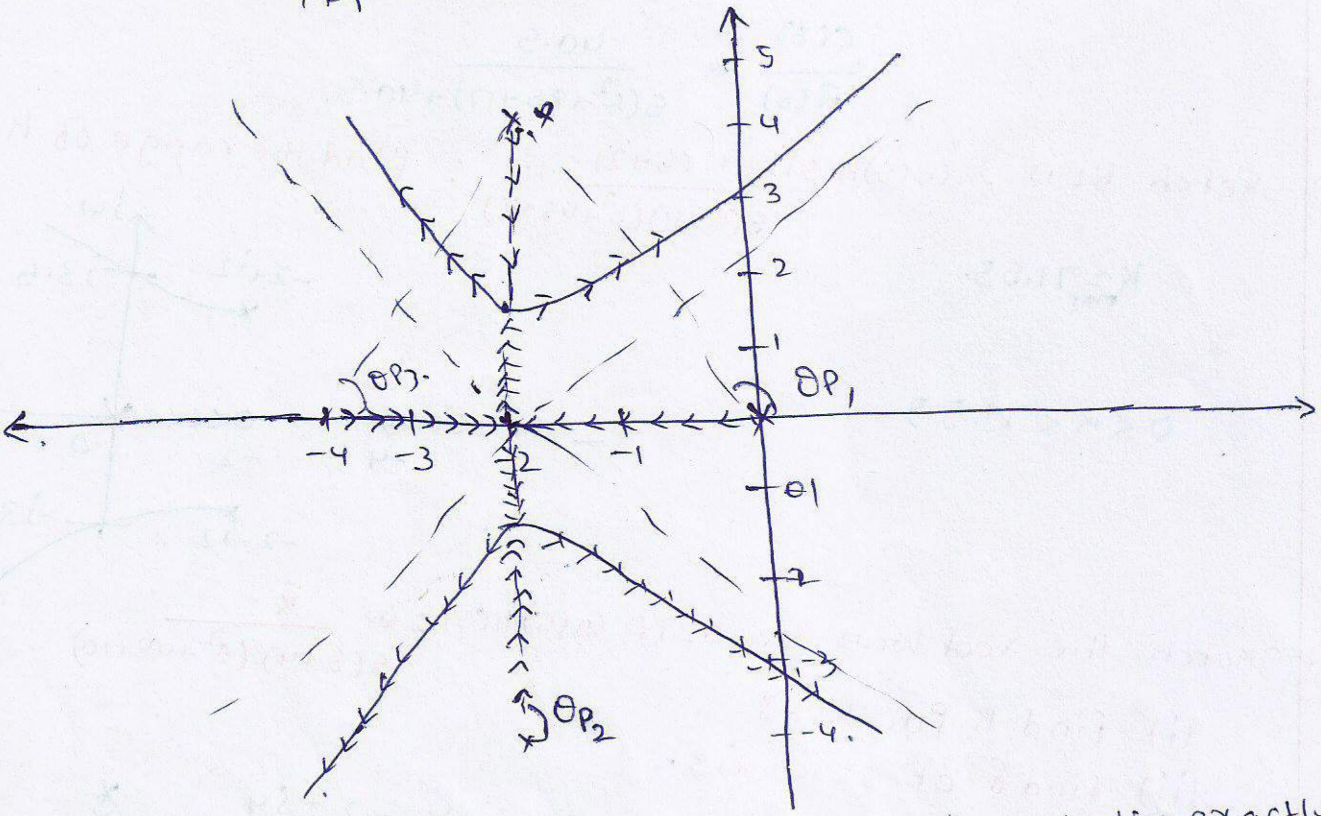
s^1 row $= 0$ $\quad 8K = 2080$
$K = 260$

$26s^2 + K = 0.$
$s^2 + 10 = 0 \quad s \pm j3.12.$

$$\begin{array}{c|ccc} s^4 & 1 & 36 & K. \\ s^3 & 8 & 80. & \\ s^2 & 26 & K. & \\ s^1 & \frac{2080-8K}{26} & 0. & \\ s^0 & K & & \end{array}$$

⑥ Angle of departure.

$$\phi_{D_1} = 180° - \Sigma \phi_P$$



$$\phi_{D-2+j4} = 180° - (90° + 120° + 60°)$$

$$= -90°$$

Complex poles lies exactly above and below B.A.pt. So, they form adjacent poles..

$$\phi_D|_{-2-j4} = +90°$$

⑦ K for $s=-3$ $K = 52.92.$

$$\delta = \cos 55° = 0.57$$

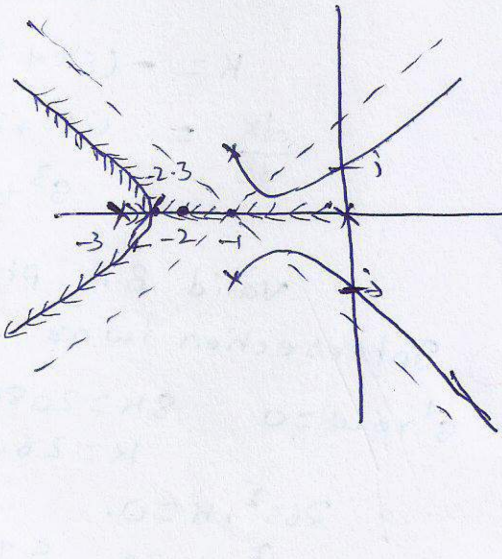
⑧ Sketch the RLD, $G(s)H(s) = \frac{K}{s(s+3)(s^2+2s+2)}$

Two Poles 0, −3 are adjacent to eachother. B.A pts b/n (0, −3)

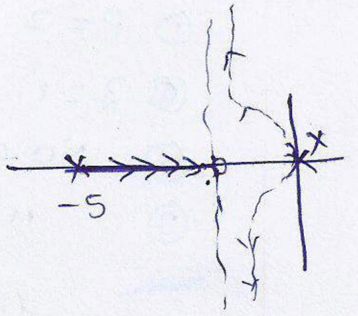
$K = 8.16$

$\phi_D = -76°$

$\phi_{D_1} = +76°$



⑨ Draw the root locus for $G(s)H(s) = \frac{K(S+1)}{S^2(S+5)}$



Sol:- ① $P = 3, \quad S = 0, 0, -5$

$Z = 1 \quad S = -1$

② No. of Root locii branches $= P = \cancel{4}\ 3$

③ No. of asymptotes $= P - Z = 3 - 1 = 2$.

angle of asymptotes $= \frac{-5+1}{2} = -2$.

$\theta = 90°, 270°$

④ starting ~~Pole terminate~~ Point of root locus are from origin and -5. End Point of one of the root locus is at zero at -1 and two root loci will wfollow asymptotes towards infinity.

⑤ Break away Point : Two Poles lie at origin are overlapping. These can be assumed as a Pole at 0^- and Pole at 0^+.

$s = 0$ is break away point.

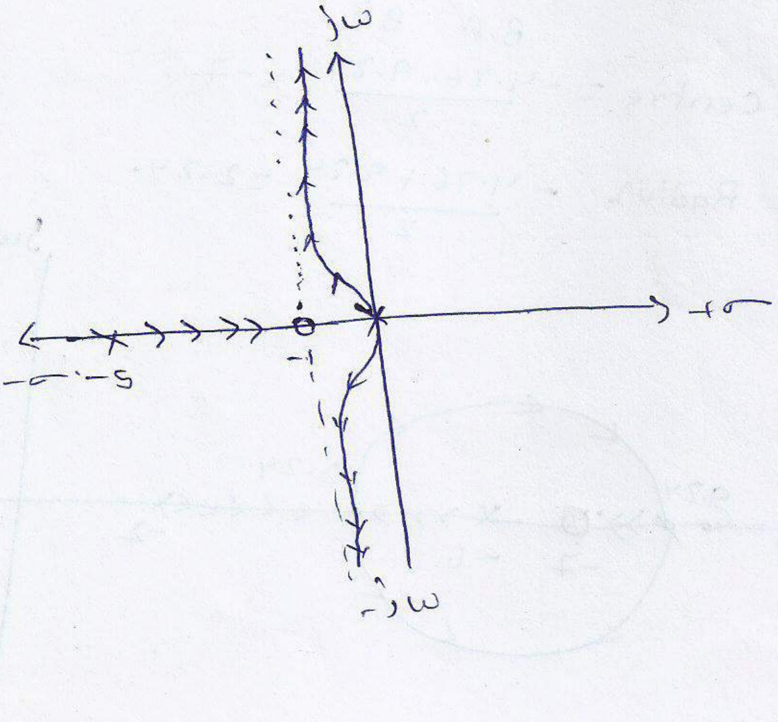
⑥ Intersection of Imaginary axis.

$C.E = s^3 + 5s^2 + Ks + K = 0.$

s^3	1	K
s^2	5	K
s^1	$\frac{5K-K}{5}$	0
s^0	K	

$K = 0.$

So, Intersection on imaginary axis is also at origin



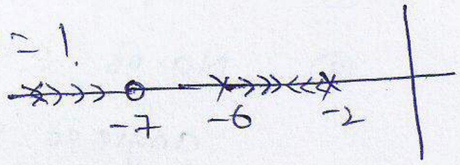
⑪ Sketch root locus for the system having. $G(S)\,H(S) = \frac{K(S+7)}{(S+2)(S+6)}$

① $P = 2$ $S = -2, 6$

② $Z = 1$ $S = -7$

② No. of Root locii branches $= P = 2$.

③ " asymptotes $= P - Z = 2 - 1 = 1$.



angle of asym = 180°

one branch start. Pole

④ B.A. Pts.

$$C \quad 1 + \frac{K(S+7)}{(S+2)(S+6)} = 0.$$

$$K(8+7) = -\frac{(S+2)(S+6)}{(S+7)}$$

$$\frac{dK}{ds} = S^2 + 14S + 144 = 0$$

$S = -4.8$ B.A Point

$S = -9.2$ B.I Point.

⑤ Intersection on jω axis.

$$C.E' = S^2 + S(8+K) + (12+7K) = 0$$

S^2	1	$12+7K$
S^1	$8+K$	0
S^0	$12+7K$	

$8 + K = 0$

$K = -\infty$.

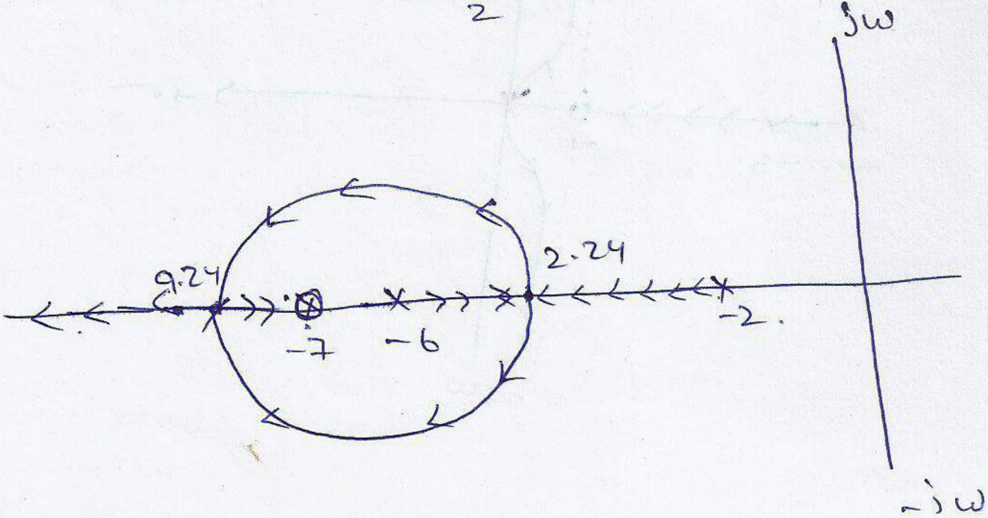
∴ no intersection on jω axis.

B.A B.I

$$\text{Centre} = \frac{-4.76 - 9.24}{2} = -7.$$

$$\text{Radius} = \frac{-4.76 + 9.24}{2} = 2.24.$$

$$G(S) = \frac{K(S+3)}{(S+1)(S+2)}$$



⑫ Sketch RLD, $G(S)H(S) = \frac{K(S+1)}{S^2(S+9)}$. find Range of values of 'K'.

Sol:- $G(S) = \frac{K(S+1)}{S^2(S+9)}$

① $P = 3, \quad S = 0, 0, -9$

$Z = 1, \quad S = -1.$

② No. of RL branches $= 3$

③ Starting Points of RL branches $= 0, 0, -9.$

④ No. of branches terminating at zeroes $= Z = 1$

" " $\infty = P - 3 = 2.$

④ No. of asymptotes $= P - 3, = 2, \quad \sigma = \frac{-9+1}{2} = -4., \theta = 90° \& 270°$

⑤ B.A. pts:-

$S^3 + 9S^2 + KS + K = 0 \Rightarrow K = -\left[\frac{S^2(S+9)}{S+1}\right]$

$\frac{dK}{dS} = \frac{2S(S^2+6S+9)}{(S+1)^2}, \quad S = 0, -3, -3.$

if $S = 0, \quad K = 0$ valid Break away pt.

if $S = -3, \quad K = -\left[\frac{(-3)^2(-3+9)}{-3+1}\right] = +\left[\frac{9(6)}{+2}\right] = 27 = +ve.$

As K is +ve, $S = -3$ is also valid B.A. pt.

⑥ Intersection with jω axis

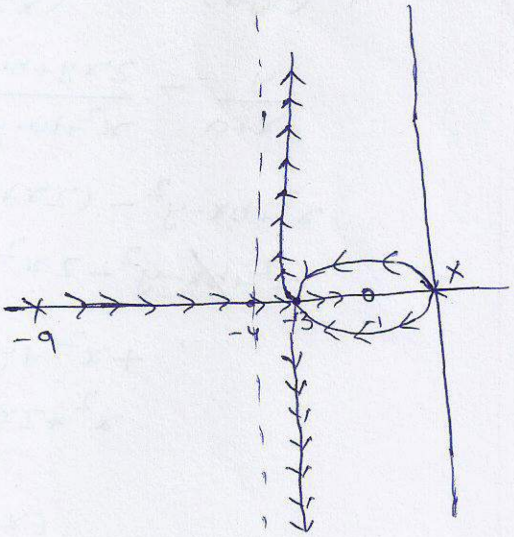
S^3	1	K
S^2	9	K
S^1	$\frac{9K-K}{9}$	0
S^0	K	

Intersection of jω axis is at $S=0$.

$K = 0$

$S = 0$

$K > 0.$



⑬ OLTF of UFB system is $G(S)=\frac{K(S+a)}{S(S+b)}$

a) Prove that Break away & Breakin point will exist only when $|a|>|b|$

b) Prove that complex points on root locus form a circle with centre $(-a,0)$ and radius $\sqrt{a^2-ab}$.

Sol:- CLTF $=\frac{G(S)}{1+G(S)}$ C.E $= 1+G(S)H(S)=0$

$$1+\frac{K(S+a)}{S(S+b)}=0 \Rightarrow S^2+bS+KS+aK=0.$$

$$K=\frac{-(S^2+bS)}{S+a}.$$

$$\frac{dK}{dS}=\frac{(S+a)(2S+b)-(S^2+bS)}{(S+a)^2}=\frac{2S^2+\cancel{bS}+2aS+ab-S^2-\cancel{bS}}{(S+a)^2}$$

$$\frac{dK}{dS}=0 \Rightarrow S^2+2aS+ab=0 \qquad S=\frac{-2a\pm\sqrt{4a^2-4ab}}{2}$$

$$=-a\pm\sqrt{a(a-b)}$$

To test the validity of B.A pt Sub pt in $K=-\frac{(S^2+bS)}{S+a}$.

$$K=\frac{(-a+\sqrt{(a(a-b)})^2+b.(-a\pm\sqrt{a(a-b)}}{-a+\sqrt{a(a-b)}+a}$$

② From angle condition $G(S)H(S)=\frac{K(S+a)}{S(S+b)}$

$$=\frac{x+jy+a}{(x+jy)(x+jy+b)}=\frac{x+jy+a}{(x+jy)(x+jy+b)}=\frac{x+jy+a}{x^2+bx-y^2+j(2xy+by)}.$$

$$\tan^{-1}\left(\frac{y}{x+a}\right)-\tan^{-1}\left(\frac{2xy+by}{x^2+bx-y^2}\right)=180^\circ$$

$$\frac{y}{x+a}-\frac{2xy+by}{x^2+bx-y^2}=0$$

$$x^2+bx-y^2-(2x+b)(x+a)=0$$

$$x^2+\cancel{bx}-y^2-2x^2-\overset{2xa}{\cancel{ax}}-\cancel{bx}-ab=0$$

$$+x^2+y^2+2xa=ab$$

$$x^2+2xa-\cancel{b^2}+y^2=ab-\cancel{b^2}a^2.$$

$$-a^2$$

$$(x+a)^2+y^2=a(a-b).$$

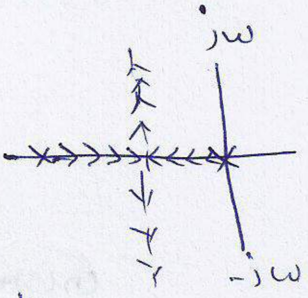
centre $=(-a,0)$

radius $=\sqrt{a(a-b)}$

Effect of ADDITION of OL Poles on RLD:-

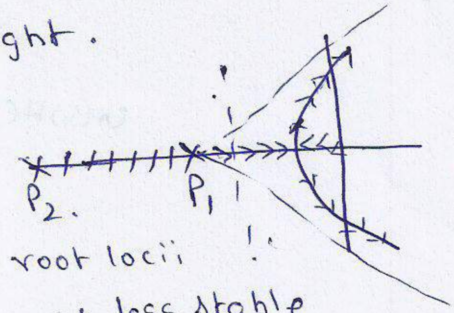
→ Relative stability of CL system is reduced due to addition of Poles to G(S)H(S).

→ $G(S)H(S) = \frac{K}{S(S+P_1)}$

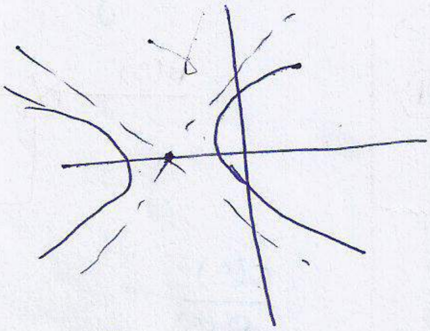


→ If we add Pole at $S=-P$, root locus will change. It will be observed that due to an addition of Pole.

1. Root loci have beant toward right.
2. B.A pt shifted to right.



→ If one more real pole is added at $S=-P_3$, root locii is further pushed to the right, the system becomes less stable.



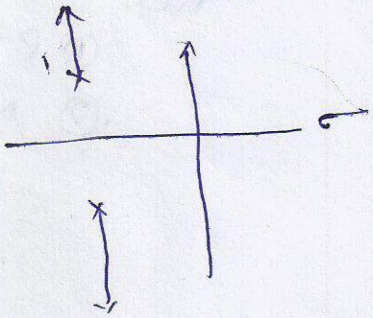
→ Let us consider one more example.

$G(S)H(S) = \frac{K}{S^2+2S+2}$

Complex poles at $S=-1+j1$

System is always stable.

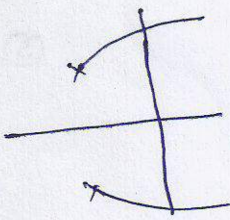
It is not crossing Imaginary axis



→ If real pole $S=-P_1$ is added,

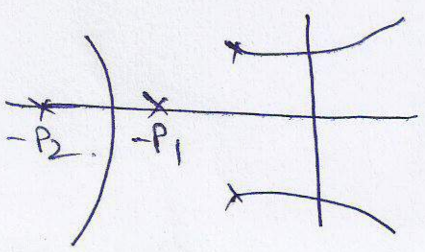
Root locus shift to right.

becomes less stable.



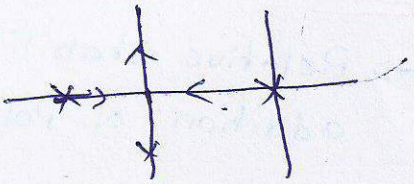
→ If one more pole is added at $S=-P_2$.

becomes less stable.

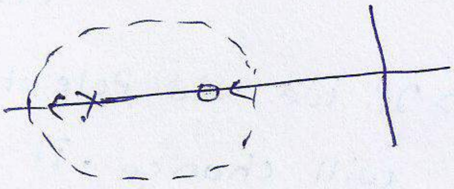


Effect of addition of OL zeroes to G(S)H(S) on Root locus:-

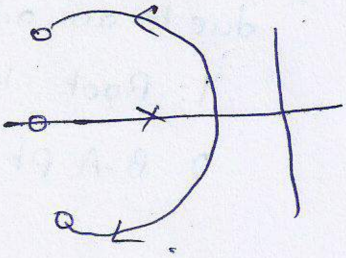
$$G(S)H(S) = \frac{K}{S(S+P_1)}$$



$$G(S)H(S) = \frac{K(S+Z_1)}{S(S+P_1)}$$

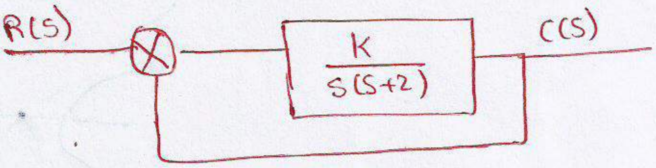


$$G(S)H(S) = \frac{K(S+Z_1+j_1)(S+Z_1-j_1)}{S(S+P_1)}$$



Adding Complex zero also result in stabilizing more.

(Pb)

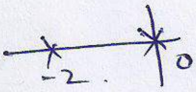


$\frac{C(S)}{R(S)}$ $\quad G(S) = \frac{K}{S(S+2)}$

① P = 2 , S = 0, -2 , Z = 0.

② No. of asymptotes = P - Z = 2 - 0 = 2.

No. of Root locus branches = 2

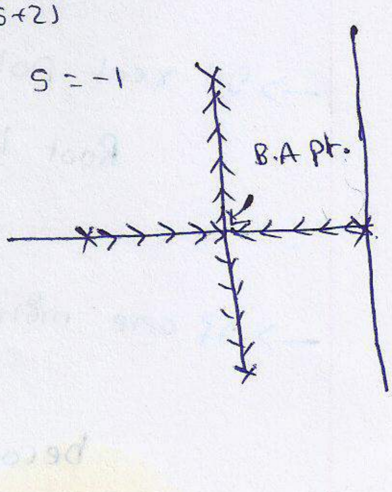


$\theta_1 = 90°, \quad \theta_2 = 270°$

$\sigma = \frac{-2}{2} = -1.$

③ B.A = $1 + G(S)H(S) = 0. \Rightarrow 1 + \frac{K}{S(S+2)} = 0.$

K = $\quad S = -1$

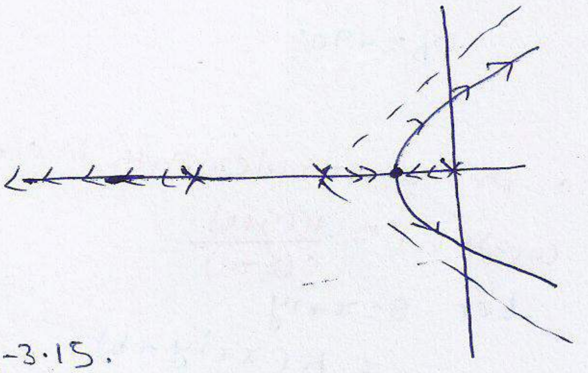


Effect of adding poles $G(S)H(S) = \frac{K}{S(S+2)(S+4)}$

1) $P=3$, $Z=0$

2) $\theta_1 = 60, 180°, 300°$

$$\sigma = -\frac{6}{3} = -2.$$



3) B.A pts.

$$s^3+6s^2+8s+K=0$$

$$\frac{dK}{ds} = 0 \Rightarrow -0.8, -3.15.$$

stability decrease.

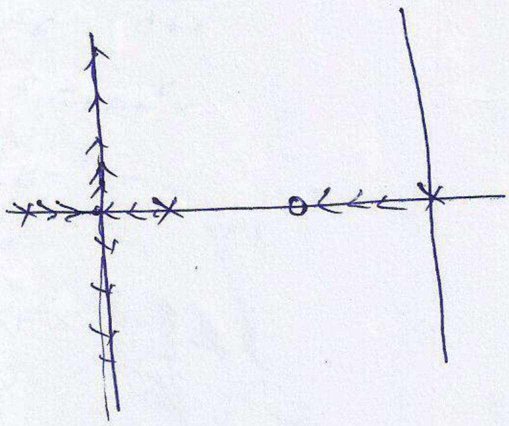
Effect of adding zero $G(S)H(S) = \frac{K(S+1)}{S(S+2)(S+4)}$

① $P=3$, $Z=1$, $P-Z=2$.

② $\theta = 90°, 270°$

$$\sigma = \frac{-6+1}{2} = -2.5$$

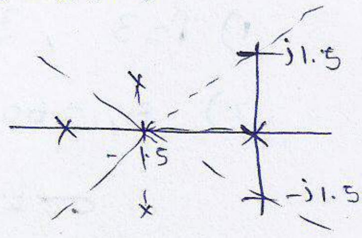
③ B.A. pts. $S=-2.8$



$G(s) = \frac{K(\text{[illegible]})}{s(s+3)(s^2+3s+4.5)}$

1) $\sigma = -1.5$, B.A pt $= -1.5$, $K_{mar} = 25.3$, $s = \pm j1.5$.

$\phi_D = -90°$



→ To prove complex path is circle.

$G(s)H(s) = \frac{K(s+b)}{s(s+a)}$

Sol:- Let $s = x+jy$

$$= \frac{K(x+jy+b)}{(x+jy)(x+jy+a)} = \frac{K(x+b+jy)}{x^2+ax-y^2+j(2xy+ay)}$$

$$\tan^{-1}\left(\frac{y}{x+b}\right) - \tan^{-1}\left(\frac{2xy+ay}{x^2+ax+y^2}\right) = 180°$$

Taking tan on both sides

$$\frac{y}{x+b} - \frac{2xy+ay}{x^2+ax+y^2} = 0.$$

$$x^2+ax\mp y^2 - [(2x+a)(x+b)] = 0$$

$$-x^2-y^2-2xb-ab = 0.$$

$$x^2+2xb+y^2 = -ab.$$

$$x^2+2xb+b^2+y^2 = -ab+b^2.$$

$$(x+b)^2+y^2 = b(b-a)$$

centre. $= -b, 0$

radius $= \sqrt{b(b-a)}$

→ whenever there is zero on real axis and to the left of the zero there are no poles or zero on real axis ~~with~~, then entire section of real axis to the side of zero as apart of root locus and there. exists a B.A.pt to the left side of zero.

→ whenever, there are two adiacently placed zeroes on real axis with section of real axis. b/n them as a part of root locus there exists B.A pt b/n adiacently placed zeroes.

→ whenever there are 2 adiacently placed poles on real axis when the section of real axis b/n them as a part of root locus then their exists B.A pt b/n the adjacently placed poles.

→ The complex conjugate path for the branches of root locus approaching or leaving B.A.pt is a circle.

Unit - IV

Frequency Response Analysis

Syllabus :-

→ Introduction, frequency domain specifications.

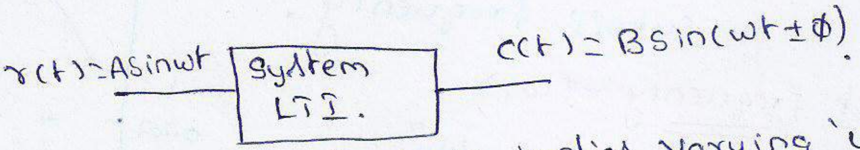
→ Bode diagrams - Determination of frequency domain specification and Phase Margin and Gain Margin - Stability Analysis - Bodeplots.

→ Polar plots - Nyquist plots - Stability Analysis

→ Compensators.

Introduction :-

Frequency response is the steady state o/p of a system - to the sinusoidal i/p. Since i/p is sinusoidal, $s = j\omega$ is substituted in transfer function.



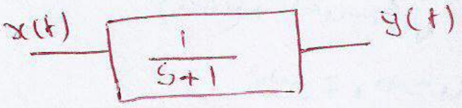
→ Frequency response Analysis implies varying 'ω' from 0 to ∞ and observing corresponding variations in Magnitude and phase angle of response.

$$T.F = \frac{C(S)}{R(S)} = F(S)$$

Put $S = j\omega$, $F(j\omega)$ = Sinusoidal transfer function.

$$F(j\omega) = |F(j\omega)| \angle F(j\omega)$$

eg:- $x(t) = \sin t$



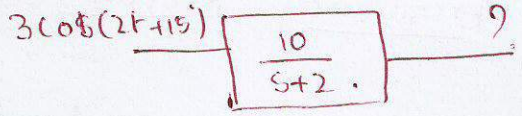
$$F(S) = \frac{1}{S+1} \Rightarrow F(j\omega) = \frac{1}{j\omega+1}$$

$$F(j\omega) = \frac{1}{\sqrt{\omega^2+1}} \angle -\tan^{-1}(\omega)$$

Since $\omega = 1$ $F(j\omega) = \frac{1}{\sqrt{2}} \angle -45^\circ$

$$y(t) = \frac{1}{\sqrt{2}} \sin(t - 45^\circ)$$

eg:- $T.F = \frac{10}{s+2}$, i/p = $3\cos(2t+15°)$. Find steady state o/p of the system.



$$F(j\omega) = \frac{10}{j\omega+2}$$

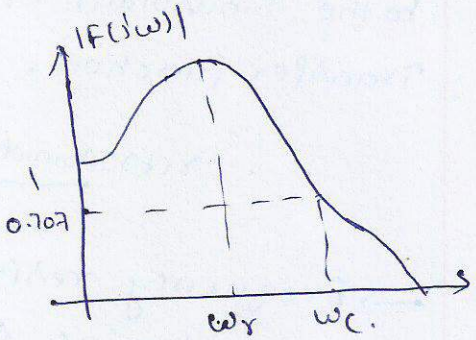
$$F(j\omega) = \frac{10}{\sqrt{\omega^2+4}} \angle -\tan^{-1}\frac{\omega}{2}$$

$\omega = 2$ r/s $\quad F(j\omega) = \frac{10}{2\sqrt{2}} \angle -45°$

$\therefore$ steady state o/p = $\frac{10\times3}{2\sqrt{2}}\cos(2t+15°-45°)$

* Frequency Domain specifications

1. Resonant frequency (ω_r)
2. Resonant Peak (or) Peak Magnitude.
3. Bandwidth
4. Cutoff frequency.



1. Resonant frequency :- (ω_r)

$$F(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$$

$$F(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\delta s}{\omega_n} + 1}$$

Let $u = \frac{\omega}{\omega_n}$

Put $s = j\omega$, $F(j\omega) = \frac{1}{\frac{(j\omega)^2}{\omega_n^2} + 2\delta j\cdot\frac{\omega}{\omega_n} + 1}$

$$|F(j\omega)| = \frac{1}{\sqrt{(1-u^2)^2+(2\delta u)^2}} \quad ; u =$$

At $\omega = \omega_r$, $u = u_r = \frac{\omega_r}{\omega_n}$.

$$\frac{d}{du_r}\left[(1-u_r^2)^2 + (2\delta u_r)^2\right]^{-1/2} = 0$$

$$-\frac{1}{2}\left[(1-u_r^2)^2+(2\delta u_r)^2\right]^{-3/2} \cdot \frac{d}{du_r}\left[(1-u_r^2)^2+(2\delta u_r)^2\right] = 0.$$

$$2\overset{-2u_r}{(1-u_r^2)} + 4\delta^2 u_r \cdot 2 = 0$$

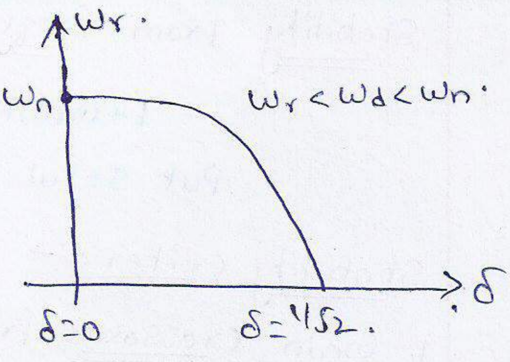
$$[2-2u_r^2][-2u_r] + 8\delta^2 u_r = 0$$

$$u_r^2 = 1-2\delta^2 .$$

$$\boxed{\omega_r = \omega_n\sqrt{1-2\delta^2}} \text{ r/s.}$$

for ω_r to be real & +ve,

$$2\delta^2<1 \Rightarrow \delta<\frac{1}{\sqrt{2}}.$$



② Resonant Peak (or) Peak Magnitude :-

It is maximum value of magnitude occurs at 'ωr'..

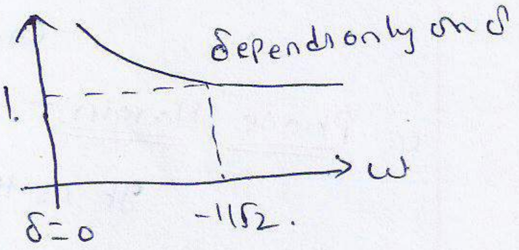
$$M_r = |M(ju)|_{u=u_r} = \frac{1}{\sqrt{(1-u_r^2)^2+(2\delta u_r)^2}}$$

$$u_r^2 = 1-2\delta^2 \Rightarrow 2\delta^2 = 1-u_r^2$$

$$\therefore M_r = \frac{1}{\sqrt{4\delta^4+4\delta^2(1-2\delta^2)}} = \frac{1}{\sqrt{4\delta^2-4\delta^4}}$$

$$M_r = \frac{1}{2\delta\sqrt{1-\delta^2}} \qquad 0 \le \delta \le 1/\sqrt{2}$$

$$M_r = 1 \quad \text{—} \quad \delta \ge 1/\sqrt{2}$$



③ Calculation of B.W :-

$$|M(j\omega)|_{\omega=B\omega} = 1/\sqrt{2}.$$

$$\frac{1}{\sqrt{(1-u^2)+(2\delta u)^2}} = 1/\sqrt{2}.$$

$$1+u^4-2u^4+4\delta^2u^2=2.$$

$$u^4-2(1-2\delta^2)u^2-1=0$$

$$u^2=\sqrt{(1-2\delta^2)+\sqrt{(1-2\delta^2)^2+1}}.$$

$$u=\frac{\omega}{\omega_n} \; ; \quad \underset{\omega_c}{BW} = \omega_n\sqrt{(1-2\delta^2)+\sqrt{(1-2\delta^2)^2+1}}$$

→ It indicates the speed of response of the system.

wider B.W = faster response.

$$B.W \propto \frac{1}{t_r}.$$

t_r - rise time.

Stability from frequency Response plots:-

$$1+G(S)H(S)=0 \Rightarrow G(S)H(S)=-1+j0.$$

Put $S=j\omega$, $G(j\omega)H(j\omega)=-1+j0$ (critical point)

Stability criteria:-

1. **Gain crossover frequency** (ωgc):-

$$|G(j\omega)H(j\omega)|_{\omega=\omega gc}=1 \text{ or } 0dB.$$

It is gain ~~which~~ can be ~~varied~~ ~~before system becomes just stable~~.

2. **Phase crossover frequency** (ωpc):-

$$\angle G(j\omega)H(j\omega)|_{\omega=\omega pc}=-180^\circ$$

3. **Gain Margin**:- (G.M)

It is the "allowable Gain"

$$|G(j\omega)H(j\omega)|_{\omega=\omega pc}=X$$

$$G.M=\frac{1}{X} \quad ; \quad G.M(dB)=20\log\frac{1}{X}.$$

4. **Phase Margin**:- (P.M)

It is the "allowable Phase lag".

$$\angle G(j\omega)H(j\omega)|_{\omega=\omega gc}=\phi \quad ; \quad P.M=180+\phi.$$

* System is stable : $\underset{20dB}{GM}$ & $\underset{30^\circ}{PM}$ = +ve $\Rightarrow \omega gc<\omega pc$.

* Unstable : $\overset{-20dB}{GM}$ & $\overset{-30^\circ}{PM}$ = -ve. $\Rightarrow \omega gc>\omega pc$.

* Marginally stable : $\overset{0dB}{GM}=\overset{0^\circ}{PM}=0 \Rightarrow \omega gc=\omega pc$.

Correlation b/n frequency & Time Response:-

① M_p & M_r , both are function of δ and independent of ω_n.

② As $\delta\uparrow$ $M_p\downarrow$

③ Smaller M_p corresponds to a system with better stability.

④ B.w is fn of both δ & ω_n.

⑤ As $\delta\uparrow$ B.w$\downarrow$

⑥ Smaller B.w indicates unstable system.

⑦ " " gives better filtering of noise.

BODE PLOTS

Frequency Response plots.

1. Polar plot : Absolute values of $F(j\omega)$ i.e., $|F(j\omega)|$ vs ω, $\angle F(j\omega)$ vs ω.

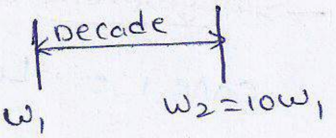
2. Bode Plots : deciBel values $[20\log|F(j\omega)|]$ vs $\log\omega$.

Mag. plot   Phase plot.

Mag. plot — Mag. (M) in dB vs log ω

Phase Angle plot — Phase angle (ϕ) vs log ω.

→ The range of frequencies on a semilog sheet between ω_1 & ω_2 where $\omega_2 = 10\omega_1$ is called one decade.



1 decade $\omega_2 = 10\omega_1$

→ Standard form of OLTF fore BODE plot :-

$$G(S)H(S) = \frac{K(1+ST_{z_1})(1+ST_{z_2})---}{s^n(1+ST_{p_1})(1+ST_{p_2})---}$$

K - Bode gain ; $T_{z_1}, T_{z_2}, T_{p_1}, T_{p_2}$ = Time constants of different poles & zeroes

→ Slope and Phase angle of standard functions :-

① System gain (±K) :-

$G(S)H(S) = \pm K$.

Man $M = 20\log|K|$ where is constant

$M = 0$ if $K = 1$.

$\phi = 0^\circ$ $K > 0$

$= \pm 180$ $K < 0$.

② Integral & derivative factors :- (Poles & zeroes at origin).

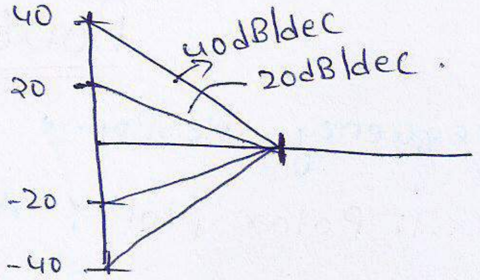
$F(j\omega) = (j\omega)^{\pm n} = (0 \pm j\omega)^{\pm n}$.

$|F(j\omega)| = \omega^{\pm n}$; It's dB value $= 20\log[\omega]^{\pm n}$

$= \pm 20 \times n \times \log\omega$.

Slope $M = \pm 20n$ dB/dec.

Phase angle(ϕ) = $\pm 90°$



→ First order factors:- $(1+sT)^{\pm 1}$

$$F(j\omega) = (1 \pm j\omega T)^{\pm 1}$$

$$|F(j\omega)| = \left[\sqrt{1+\omega^2T^2}\right]^{\pm 1}$$

It's dB value $= 20\log\left[\sqrt{1+\omega^2T^2}\right]^{\pm 1}$.

$$= \pm 20\log\sqrt{1+\omega^2T^2}.$$

→ Asymptotic approximation:-

Case 1:- Low freq. $1 >> (\omega T)^2$.

$$\pm 20 \log \sqrt{1} = 0dB.$$

Case 2:- High freq $(\omega T)^2 >> 1$ $\pm 20\log\omega T$.

$$\underset{M}{M} = \pm 20 \log \underset{x}{\omega} \underset{c.}{\pm 20 \log T}$$

slope (M) = ± 20 dB/dec.

$0 = \pm 20\log\omega T \Rightarrow \log\omega T = 0 \Rightarrow \omega T = \log^{-1} 0 = 1.$

$$\omega_{cf} = 1/T \text{ r/s}.$$

eg:- $(s \pm 2)^{\pm 1} = 2^{\pm 1}\left(1 \pm \frac{s}{2}\right)^{\pm 1}$

$$T = \frac{1}{2} \Rightarrow \omega_{cf} = 2 \text{ r/s}.$$

→ Quadratic factors:- $(s^2 + 2\delta\omega_n s + \omega_n^2)^{\pm 1}$

$$\left(\frac{s^2}{\omega_n^2} + \frac{2\delta s}{\omega_n} + 1\right)^{\pm 1}$$

Put $s = j\omega$, $\left[\frac{(j\omega)^2}{\omega_n^2} + \frac{2\delta . j\omega}{\omega_n} + 1\right]^{\pm 1} = \left[1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\xi\frac{\omega}{\omega_n}\right]$

$$M = \pm 20\log . \sqrt{\left[1 + \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\delta\omega}{\omega_n}\right)^2}.$$

0dB $\qquad\qquad$ $\pm 40\log\left(\frac{\omega}{\omega_n}\right)$

LFR $\qquad\qquad$ HFR.

slope (M) = ±40dB/dec.

$\omega_{cf} = \omega_n$ r/s

eg:- $(s^2+4s+25)^{\pm 1}$. $\qquad \omega_{cf} = \omega_n = 5$ r/s.

Summary of Magnitudes & phase angle.

Function	Slope in dB/de.	Phase angle.	ω_{cf}.
1. K	zero	$0°$ if $K>0$ $-180°$ if $K<0$	— —
2. $\frac{1}{s^n}$	$-20*n$	$-90°*n$	—
3. s^m	$+20*m$	$90°*m$	—
4. $\frac{1}{1+T_a s}$	-20	$-\tan^{-1}\omega T_a$	$\frac{1}{T_a}$
5. $1+T_1 s$	$+20$	$+\tan^{-1}\omega T_1$	$\frac{1}{T_1}$
6. $\frac{1}{s^2+2\delta\omega_n s+\omega_n^2}$	-40	ϕ	ω_n.
7. $s^2+2\delta\omega_n s+\omega_n^2$	$+40$	ϕ	ω_n.

Starting Point of Magnitude plot :-

Case (i) :- If $1/s$ term and K is Present.

$$SP = 20\log(K) + 20\log\frac{1}{j\omega} = 20\log K - 20\log j\omega$$

Case (ii) :- If s term and K is Present.

$$SP = 20\log K + 20\log\omega.$$

Case (iii) If s and $1/s$ is absent.

$$SP = 20\log K.$$

In general starting frequency is assumed as 0.1 rad/s.

Problems :-

1\. Sketch the Bodeplot for given OLTF $G(s) = \frac{5}{s(1+0.2s)(1+0.02s)}$ Determine ω_{gc} & ω_{pc}.

Sol :- ① Given $G(s)H(s)$ is in standard form of OLTF fore Bode plot.

② Corner frequency or Break frequency

$\frac{1}{1+0.2s}$, $C.F = \frac{1}{0.2} = 5$ rad/s

$\frac{1}{1+0.02s}$, $C.F = \frac{1}{0.02} = 50$ rad/s.

③ Substitute $s = j\omega$ $G(j\omega) = \frac{5}{j\omega(1+0.2j\omega)(1+0.02j\omega)}$

Factor.	ω_{cf}	slope.	net slope.	Phase angle ϕ
$K=5$	—	0	0 (slope ~~at starting~~)	0°
$\frac{1}{s}$	—	-20	-20 (slope from ~~ω_{c1} to ω_{c2}~~ SP)	$-90°$
$\frac{1}{1+0.2s}$	5	-20	-40 ω_{c1} to ω_{c2}.	$-\tan^{-1}0.2\omega$
$\frac{1}{1+0.02s}$	50.	$-$~~+~~20.	~~-20~~ -60. ω_{c2} to EP.	$-\tan^{-1}0.02\omega$.

④ To find points on Magnitude plot.

starting point $\omega_s = 0.1$ rad/sec Let.

$$SP = 20\log K - 20\log 0.1 = 34dB.$$

Magnitude at $\omega_{c_1} = 5$ rad/s $\quad M_1 = 20\log K - 20\log\omega_{c_1}$

$$= 20\log 5 - 20\log 5 = 0dB.$$

Magnitude at $\omega_{c2} = 5$ rad/s $\quad M_2 = \left\{ \begin{matrix} \text{Net slope} \\ \text{from} \\ \omega_{c_1} \text{ to } \omega_{c2} \end{matrix} \right\} \times \log\left(\frac{\omega_{c2}}{\omega_{c4}}\right) + M_1 \}.$

$$= -40\log\left(\frac{50}{5}\right) + 0.$$

$$M_2 = -40dB$$

Magnitude at end point. $\quad EP = \left\{ \begin{matrix} \text{Net slope} \\ \text{from } \omega_{c2} \text{ to} \\ EP \end{matrix} \right\} \times \log\frac{\omega_e}{\omega_{c2}} + M_2.$

let $\omega_e = 100$ r/s.

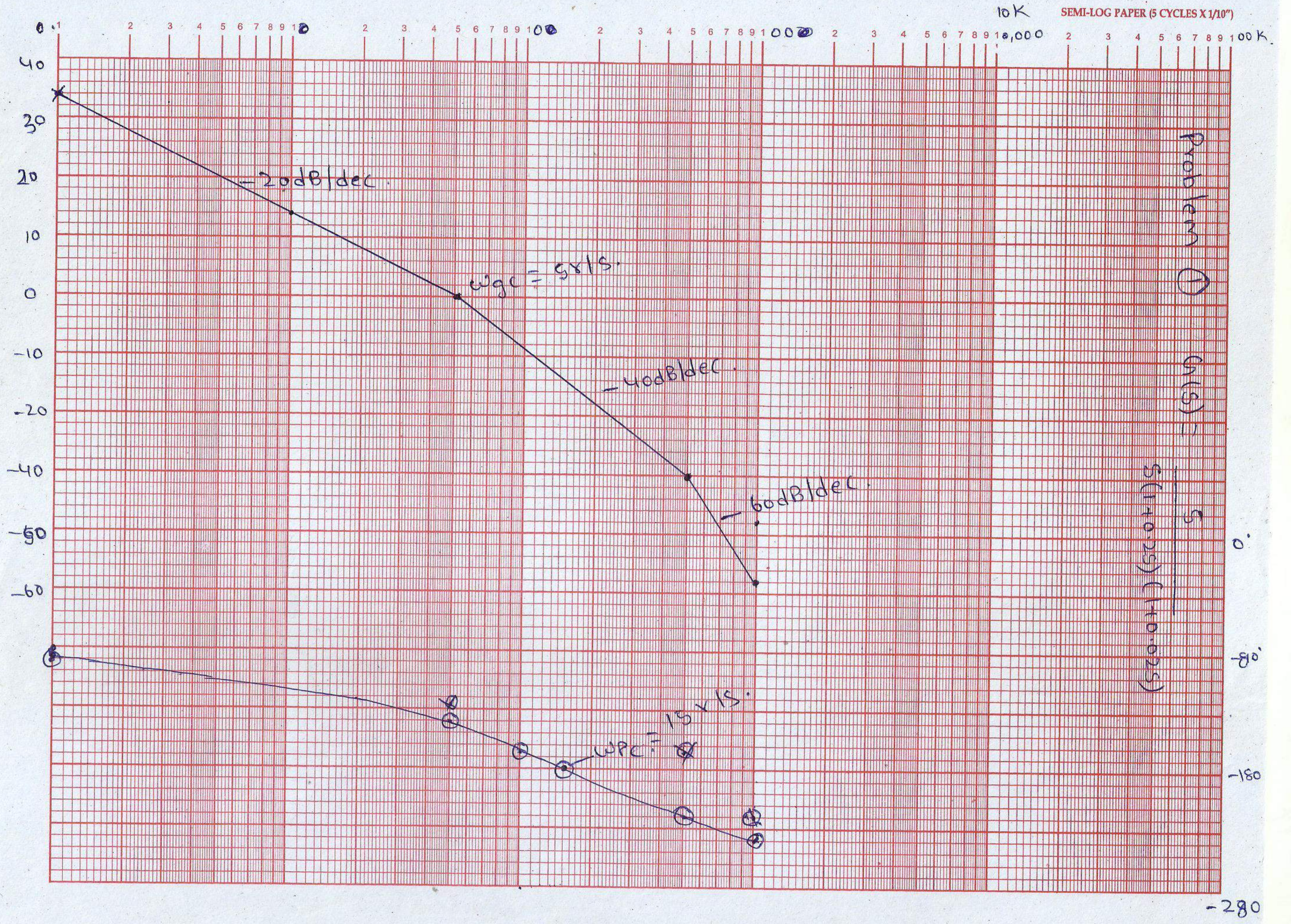
$$EP = -60\log\left(\frac{100}{50}\right) - 40 = -50db.$$

⑤ phase plot

ω in rad/s	$\phi = -90' - \tan^{-1}0.2\omega - \tan^{-1}0.02\omega.$
0.1	$-91.26'$
5	-140.7°
10	-164.7°
50	-219.3°
100	-240.57°

⑥ $\omega_{gc} = 5$ r/s

$\omega_{pc} = 16$ r/s.



Problem ① $G(s) = \frac{5}{s(1+0.2s)(1+0.02s)}$

② $G(s)H(s) = \frac{20s^2}{(1+0.2s)(1+0.02s)}$. Sketch Bode plot & determine ω_{gc} & ω_{pc}.

Sol :-

① Standard form

② $\omega_{c1} = 5\,r/s$ $\omega_{c2} = 50\,r/s$.

③

Factor.	ω_{cf}	Slope	net slope	ϕ
$K=20$	—	0	0	0°
s^2	—	$+20\times2$	+40 $S.P \rightarrow \omega_{c1}$.	+180°
$(1+0.2s)$	5	−20	~~0~~ +20° $\omega_{c1} - \omega_{c2}$.	$-\tan^{-1}0.2\omega$
$(1+0.02s)$	50	−20	+20° $\omega_{c2} - \omega_{c}$.	$-\tan^{-1}0.02\omega$.

④ $\omega_s = 0.1\,r/s$, $\omega_e = 100\,r/s$.

$M_{S.P} = 20\log K + 40\log\omega_s = 20\log 20 + 40\log 0.1 = -\overset{14}{H}\,dB$

$M_1 = 20\log 20 + 40\log 5 = 54\,dB$.

$M_2 = 20\log\frac{(50)}{5} + M_1 = 74\,dB$

$MEP = 20\log\frac{100}{50} + M2. = 74\,dB$.

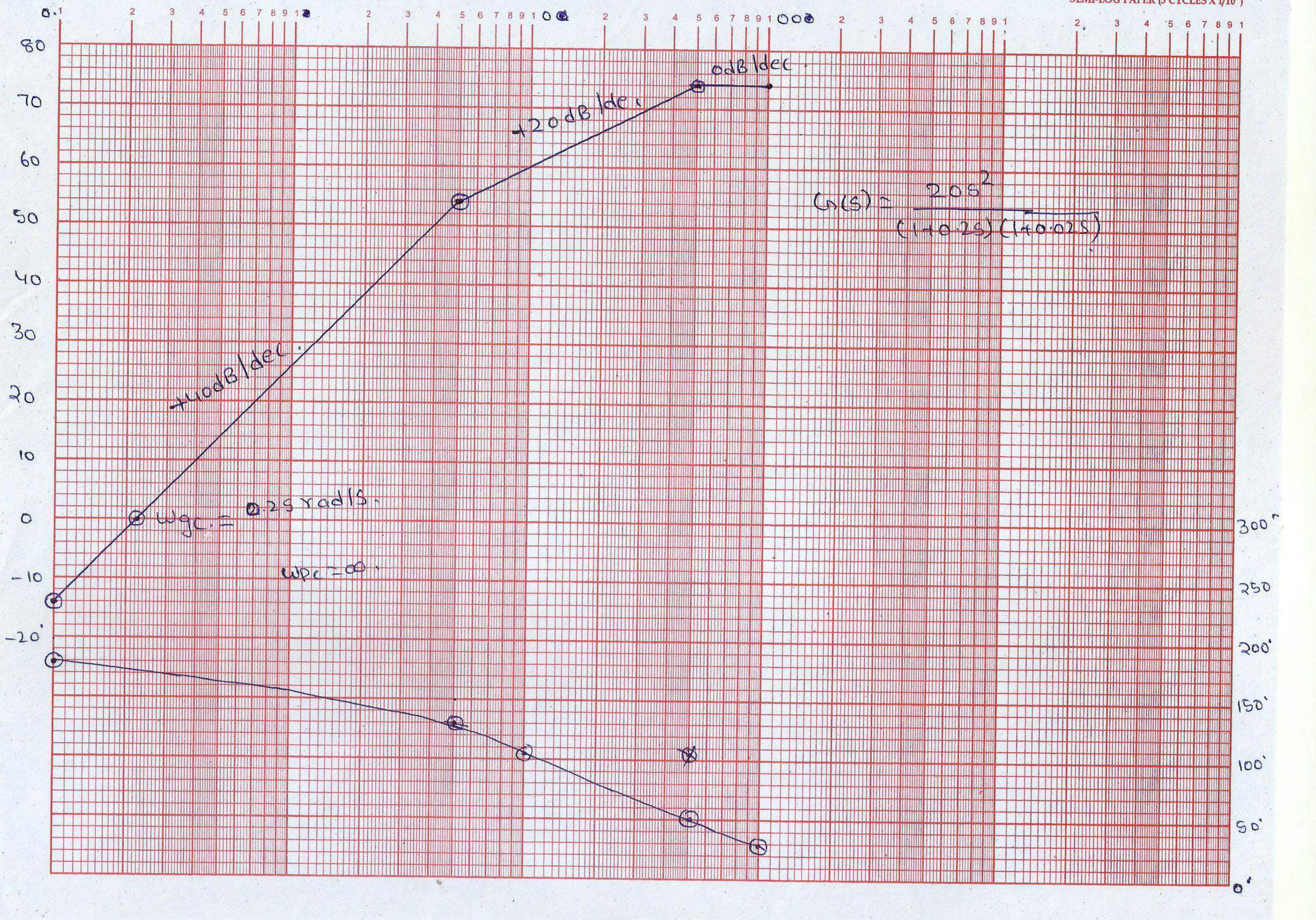
⑤

ω in rad/sec.	$\phi = 180° - \tan^{-1}0.2\omega - \tan^{-1}0.02\omega$.
0.1	178.74.
5	129.3
10	105.23
50	50.71
100	29.43.

⑥ $\omega_{gc} = 0.24$ rad/s

$\omega_{pc} = \infty$, as phase plot cannot cut −180°

②



$$G(s) = \frac{20s^2}{(1+0.2s)(1+0.02s)}$$

③ $G(s)H(s) = \frac{K(1+0.2s)(1+0.025s)}{s^3(1+0.001s)(1+0.005s)}$ Determine. G.M & P.M find Range of K.

① standard form

② $\omega_{c_1} = 5$ r/s , $\omega_{c2} = 40$ r/s , $\omega_{c3} = 200$ r/s & $\omega_{c4} = 1000$ r/s.

③

factor	ω_{cf}	slope	Net slope.	ϕ.
K	—	0	0	0°
$\frac{1}{s^3}$	—	-60	-60	$-270°$
$1+0.2s$.	5	$+20 \leftarrow -60$	-40 S.P to ω_{c1}	$\tan^{-1} 0.2\omega$
$1+0.025s$	40	$+20 \leftarrow -40$	-20 ω_{c1} to ω_{c2}	$\tan^{-1} 0.025\omega$
$1+0.005s$	200	$-20 \leftarrow -20$	-40 ω_{c2} to ω_{c3}	$-\tan^{-1} 0.005\omega$
$1+0.001s$	1000	$-20 \leftarrow -40$	-60. ω_{c3} to ω_{c4}. $\leftarrow -60$	$-\tan^{-1} 0.001\omega$.

④ Let $K=1$ $SP = 20\log K - 60\log\omega_s = 20\log 1 - 60\log 0.1 = 60dB$.

$M|_{at\,\omega_{c1}}$: $M_1 = 20\log K - 60\log\omega_{c_1} = -60\log 5 = -42dB$.

$M.|_{at\,\omega_{c2}}$: $M_2 = -40\log\left(\frac{40}{5}\right) + M_1 = -78dB$.

$M|_{at\,\omega_{c3}}$: $M_3 = -20\log\left(\frac{200}{40}\right) + M_2 = -92dB$

$M|_{at\,\omega_{c4}}$: $M_4 = -40\log\left(\frac{1000}{200}\right) + M_3 = -120dB$.

$M|_{at\,\omega_{c5}}$: $M_5 = -60\log\left(\frac{2000}{1000}\right) + M_4 = -138dB$.

⑤

ω	$\phi = \tan^{-1}0.2\omega + \tan^{-1}0.025\omega - \tan^{-1}0.05\omega - \tan^{-1}0.001\omega - 270$
0.1	-268.74
5	-210.6.
40	-155.72.
200	-159.0
1000	-216.27
2000	-239.0.
10	-195.96
100	-146.9.

⑥ $\omega_{gc} = 1$ r/s $\omega_{pc} = 14$ r/s 540 r/s.

G.M = 62dB , 104dB.

PM = 70°

⑦ $20\log K = 62$

$K = 1259$.

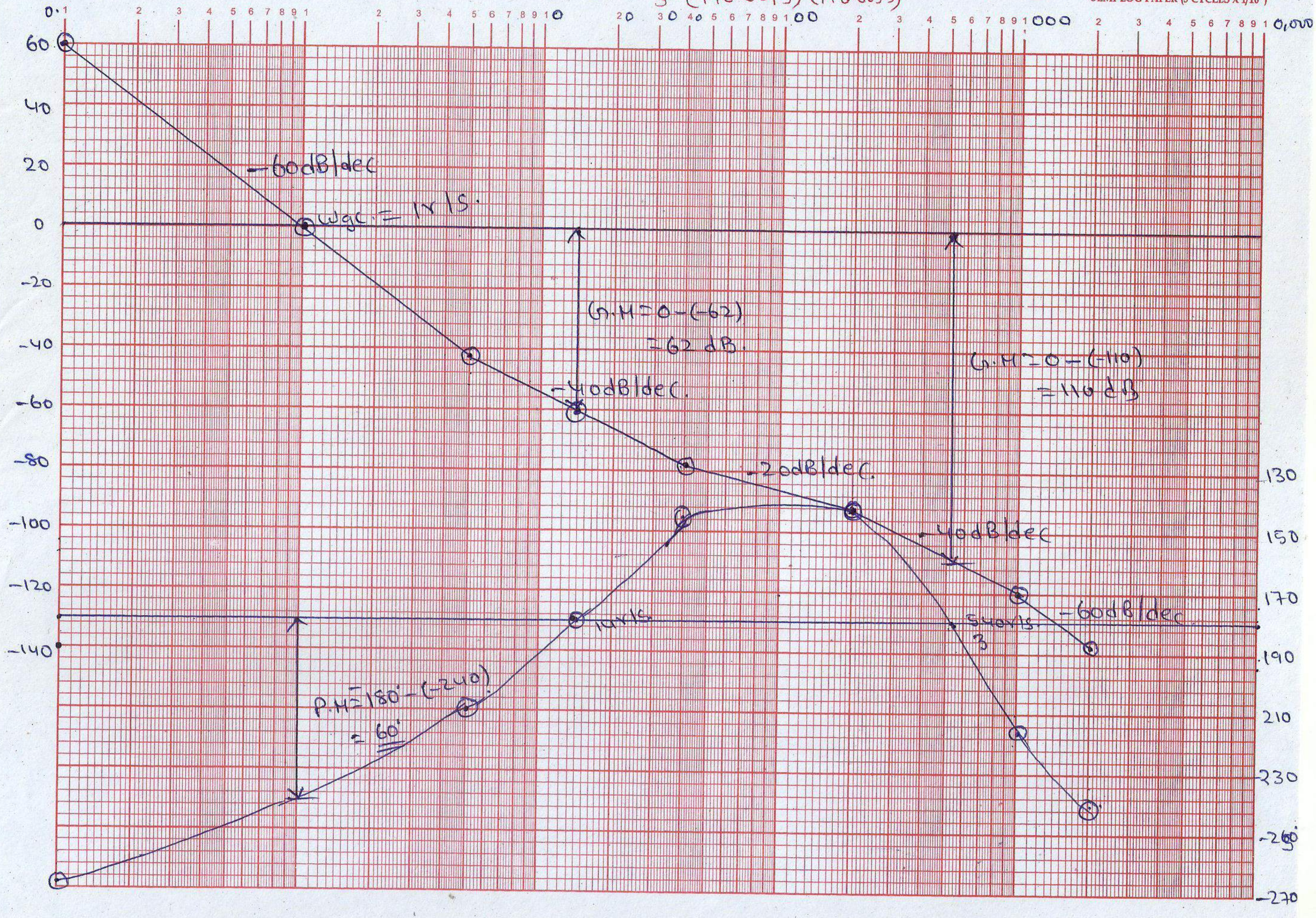
$GM_2 = 104$

$20\log K = 104$

$K = 158489$

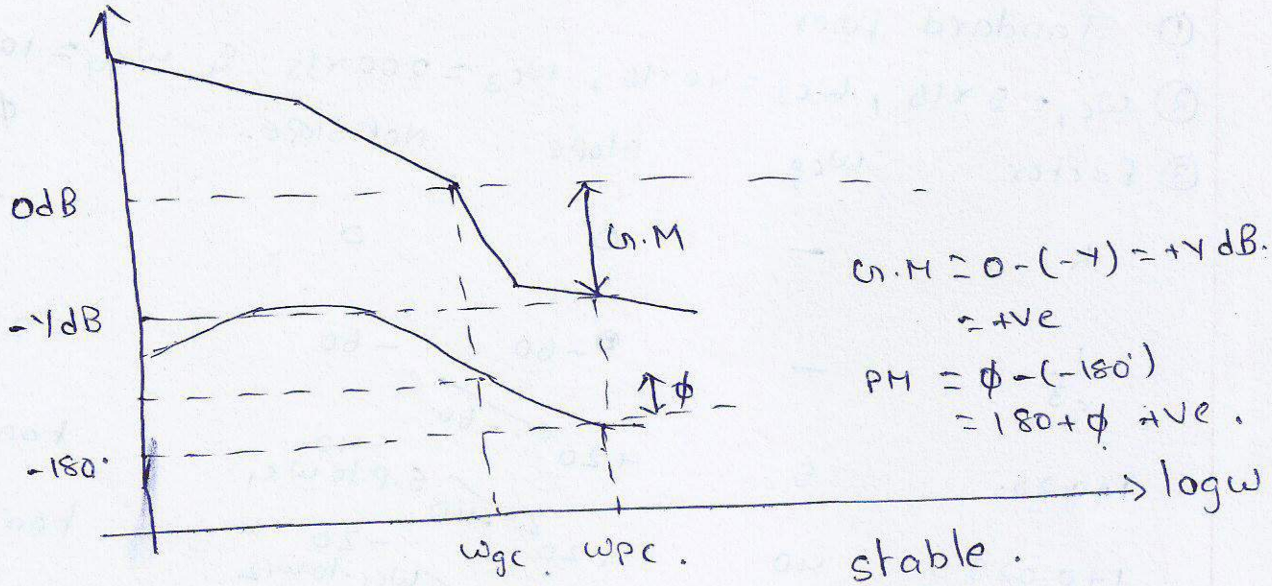
$104 < K < 158489$

$$G(s)H(s) = \frac{K(1+0.25S)(1+0.0025S)}{S^3(1+0.001S)(1+0.005S)}$$ Problem ③



To find G.M & P.M from Bode plot:-

①

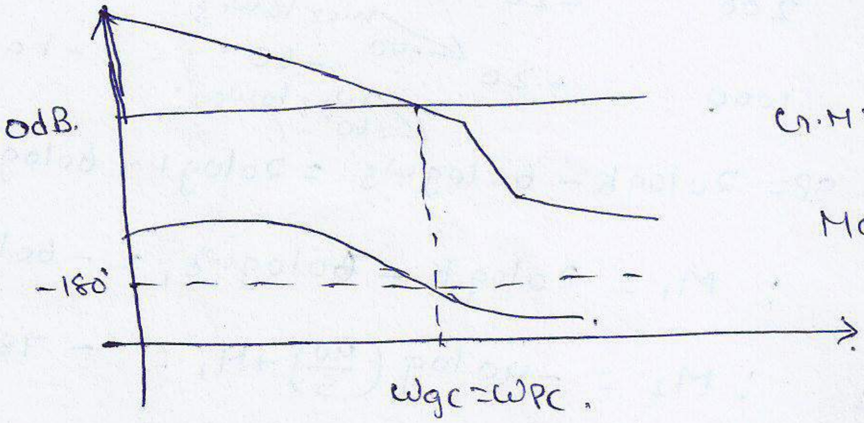


$G.M = 0-(-Y) = +Y$ dB.
= +ve

$PM = \phi-(-180°)$
$= 180+\phi$ +ve.

stable.

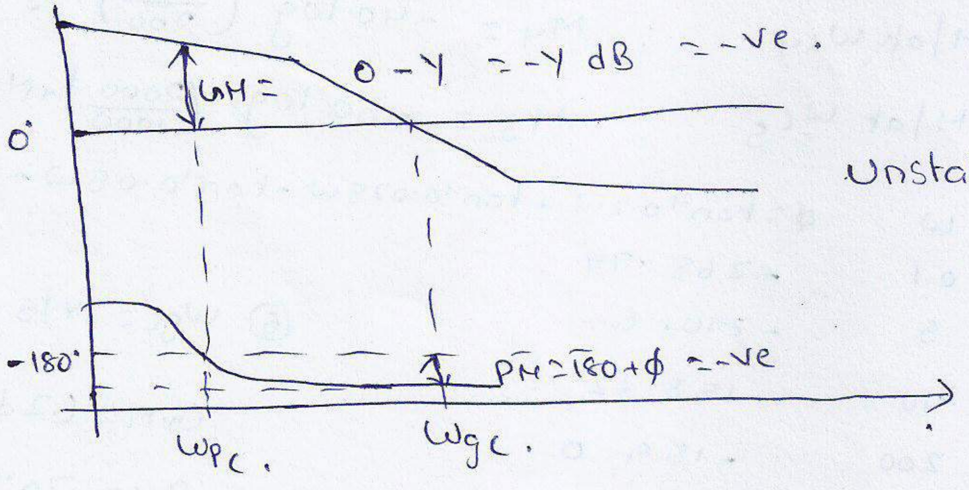
②



G.M = P.M = 0.

Marginally stable.

③



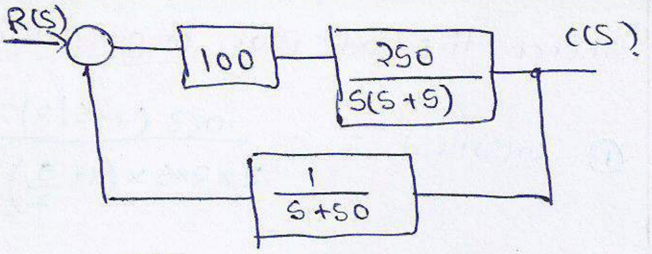
Unstable System

→ ωpc > ωgc. ∴ Stable System G.M & P.M = +ve.

→ ωpc = ωgc ⇒ GM = PM = 0 Marginally stable.

→ ωpc < ωgc ⇒ GM & PM = -ve Unstable.

④ Sketch the Bode plot and find stability of system shown below.



Sol:- $G(S)H(S) = \frac{25000}{S(S+5)(S+50)}$

standard time constant form $G(S)H(S) = \frac{100}{S(1+0.2S)(1+0.02S)}$

② $\omega_{c_1} = 5\,r/s$, $\omega_{c_2} = 50\,r/s$.

③ K=100, 1/s term,

Factor	ω_c	slope	Net slope	ϕ
K=100	–	0	0	0°
$\frac{1}{S}$	–	-20	-20	-90°
1+0.2S	5	-20	-40	$-\tan^{-1}0.2\omega$
1+0.02S.	50	-20	-60	$-\tan^{-1}0.02\omega$.

④ To find the Points of Magnitude plot.

$M_{s.p} = 20\log K - 20\log\omega_s = 60\,dB$. at

$M_1|_{\omega_{c_1}=5r/s} = 20\log K - 20\log\omega_{c_1} = 26\,dB$

$M_2|_{\omega_{c_2}=50r/s} = -40\log\left(\frac{50}{.5}\right) + M_1 = -14\,dB$.

$M_3|_{\omega e} = -60\log\left(\frac{100}{50}\right) + M_2 = -32\,dB$

⑤

ω in rad/se.	$\phi = -90° - \tan^{-1}0.2\omega - \tan^{-1}0.02\omega$
0.1	-91.26
5	-140.7
10	-164.7
50	-219.29
100	-240.57.

⑥ $\omega gc = 23\,r/s$ $\qquad$ $\omega pc = 19\,r/s$.

G.M = -3 dB.

~~G.M~~
P.M = -8°

⑤ Sketch the Bode plot for given OLTF $G(S)H(S) = \frac{10(S+3)}{S(S+2)(S^2+S+2)}$.

① $G(S)H(S) = \frac{10\times 3\,(1+S/3)}{2\times 2\times S\times(1+\frac{S}{2})(1+\frac{S}{2}+\frac{S^2}{2})}$.

② $\omega_{c_1} = \sqrt{2}$, $\omega_{c_2} = 2$, $\omega_{c3} = 3$ r/s.

③

factor.	ω_{cf}	slope	net slope	ϕ
$K = 7.5$	—	0	0	0°
$\frac{1}{S}$	—	-20	-20 ω_S	$-90°$
$1+\frac{S}{2}+\frac{S^2}{2}$	1.414	-40	ω_{c1}. -60 ω_{c1}	$-\tan^{-1}\left[\frac{0.5\omega}{1-\frac{\omega^2}{2}}\right]$
$1+0.5S$.	2	-20	ω_{c2}. -80	$-\tan^{-1}0.5\omega$.
$1+0.33S$	3	$+20$	-60	$+\tan^{-1}0.33\omega$.

④ $\omega_S = .1$ r/s $\omega_e = 10$ r/s.

$M_{S.P} = 20\log K - 20\log\omega_S = 37.5$. dB

$M_1|_{at\,\omega_{c_1}} = 20\log K - 20\log\omega_{c_1} = 14.5$ dB

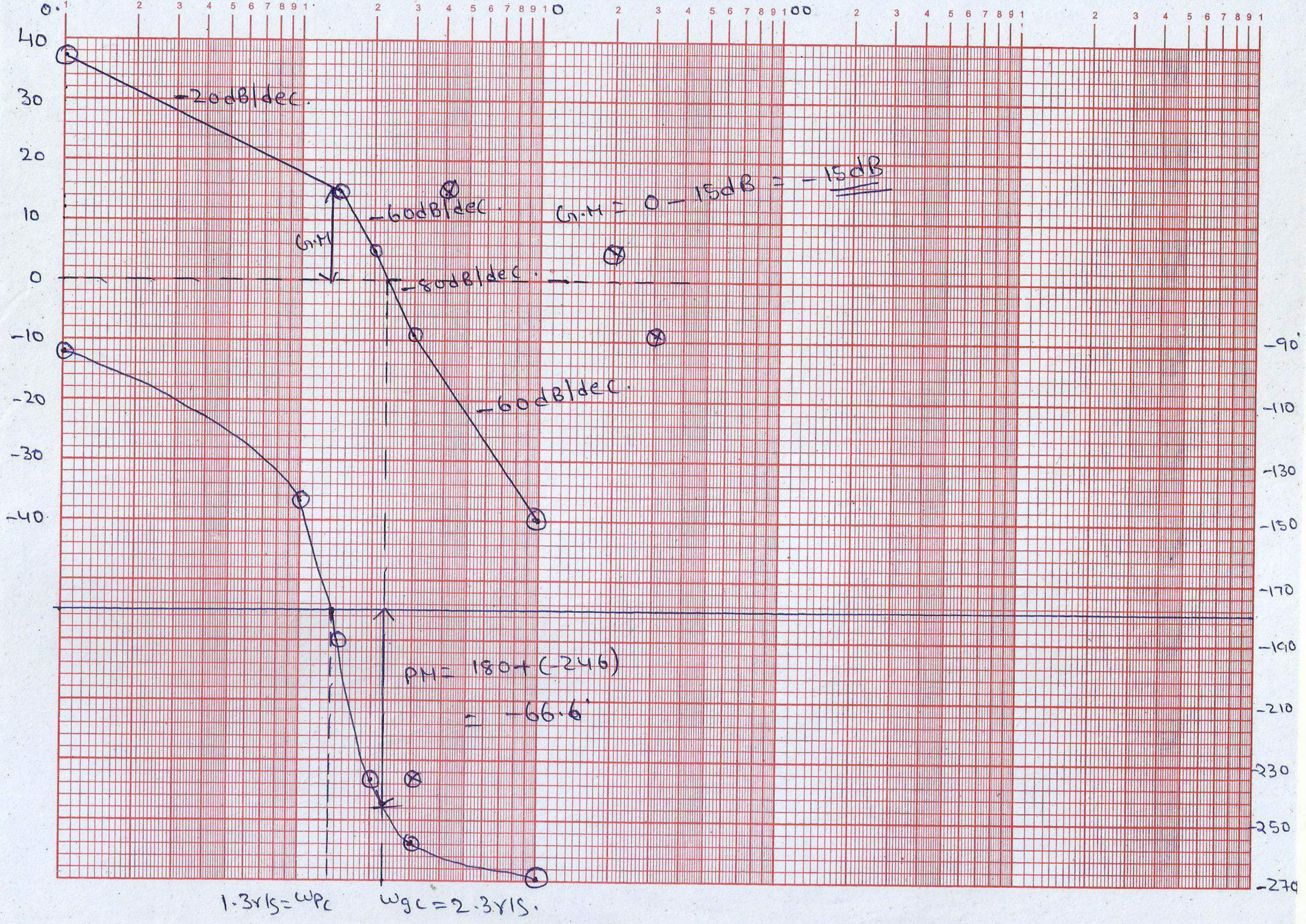
$M_2|_{at\,\omega_{c2}} = -60\log\left(\frac{2}{1.414}\right) + M_1 = 5$ dB

$M_3|_{at\,\omega_{c3}} = -80\log\left(\frac{3}{.2}\right) + M_2. = -9$ dB

$M_e.|_{at\,\omega_e} = -60\log\left(\frac{10}{.3}\right) + M_3 = -40$ dB.

P (37.5 dB, 0.1 r/s) (14.5 dB, 1.414) (5, 2) (−9, 3)
(−40, 10)

SEMI-LOG PAPER (5 CYCLES X 1/10")



⑤ Phase plot.

w in rad/sec	ϕ
0.1	$-94°$
1	$-143°$
1.41	$-190°$
2	$-236.5°$
3	$-258.4°$
10	$-269.7°$

⑥ $\omega_{gc} = 2.3$ r/s $\omega_{pc} = 1.3$ r/s.

$P.M = -66°$ $GM = -15dB$.

System is unstable.

(Pb) construct Bodeplot $G(s)H(s) = \frac{100}{s(1+\frac{s}{20})^2(1+\frac{s}{40})^2}$. find GM, & PM

Ans $\omega_{gc} = 43$ r/s $\omega_{pc} = 30$ r/s $P.M = -36°$ $GM = -7dB$

(Pb) construct Bodeplot if $C.E = s(1+0.1s)(1+0.2s)+50=0$

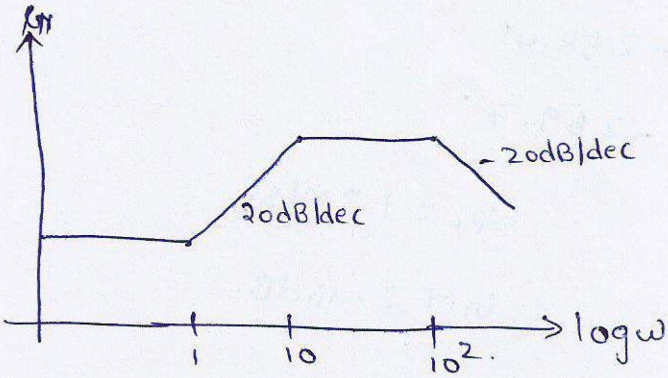
$C.E = 1+G(s)H(s)=0$ $\omega_{gc} = 14$ r/s $\omega_{pc} = 8$ r/s

$PM = -36°$ $GM = -12d$.

Inverse Bode Plot:-

1. observe the starting slope, this will give information of poles or zeroes occuring at origin.
2. for starting slope write the Eqn $y=mx+c$, $c=20\log K$.
3. At Every corner frequency obsesve the change in slope.
4. This will give information of first 8der & higher 8der system.

①



$Y = Mx + c$.

At $\omega=1$

$-20 = 0 \times \log(1) + c$.

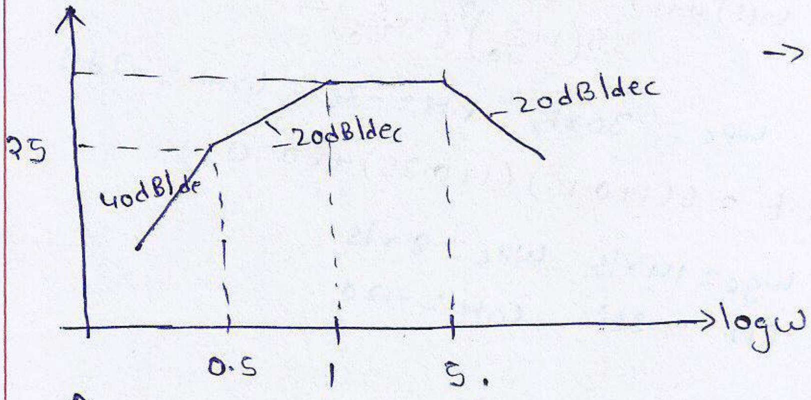
$c = -20$.

$20\log K = -20$

$K = \log^{-1}(-1) = 0.1$

$$G(s) = \frac{0.1\,(1+s)}{(1+0.1s)\,(1+0.01s)}$$

②



$\rightarrow Y = mx\,c$.

$25 = 40\log 0.5 + c$.

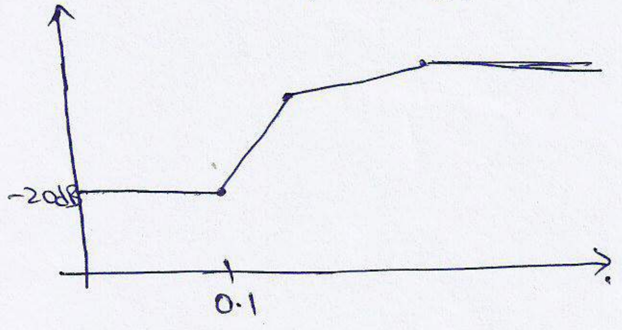
$c = 37$.

$20\log K = 37 \Rightarrow K = \log^{-1}(37/20)$

$= 70$.

$$G(s)H(s) = \frac{\overset{2}{5}.70\,\cancel{(1+0.2s)}\,(1+2(s))}{(1+2s)(1+s)\,(1+0.2s)}$$

③



④

Polar plot

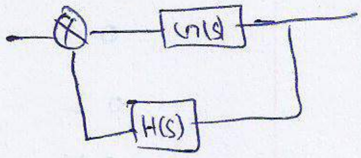
→ It is a plot of magnitude of OLTF vs phase angle of OLTF on polar co-ordinates as ω is varied from zero to infinity.

Procedure to draw polar plot:-

① Only OLTF of a system must be considered. $[G(S)H(S)]$

② If system has unity FB consider only $G(s)$

③ If Block diagram is given, multiply with forward block gain with feedback path gain to get OLTF.



④ Substitute $s = j\omega$ in OLTF.

⑤ write Magnitude & phase equation for OLTF.

eg:- $G(S) = \frac{1}{S+a}$ $G(j\omega) = \frac{1}{j\omega+a}$.

$\angle G(j\omega) = -\tan^{-1}\frac{\omega}{a}$

$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+a^2}}$.

⑥ Calculate value of M & Phase angle ϕ for different frequency. In general select $\omega = 0, 0.2, 0.4, 0.6, 0.8, 1.0, 10, 100, 1000, \infty$.

⑦ Draw the Polar Plot. It is usually drawn on a Polar graph sheet. It has concecentric circle which indicates constant Magnitude and radial lines which indicate phase angle.

+angle = ACLK

-ve angle = CLK.

⑧ Gain cross over:- The point where the plot touches unit circle is called gain cross over. The angle measured at this point is gain cross over angle ϕ_{gco}.

Phase cross over:- The point where the plot touches -180° line is called phase cross over. The magnitude measured at this point is called Phase cross over Magnitude $|G(j\omega_{pco})|$

Gain Margin:- $K_g = \frac{1}{|G(j\omega_{pco})|}$

phase Margin:- $P.M = 180° + \phi_{gco}$

⑨ Stability.

P.M & G.M = +ve

stable.

P.M &(or) G.M = -ve

unstable.

Pb ① OLTF of UFB is given by $G(s) = \frac{10}{s+1}$. Sketch Polar Plot & Test stability.

Sol:- ① $G(j\omega) = \frac{10}{j\omega+1}$. $|G(j\omega)| = \frac{10}{\sqrt{\omega^2+1}}$; $\angle G(j\omega) = -\tan^{-1}\omega$.

② Generaly w is varied from 0 to ∞ . But for simple calculation, select w from zero, cornerfrequencies, and w has to be taken greater than the highest cornerfrequency.

$w_{cf} = 1$ r/s. So, take $w = 0, \underbrace{0.2, 0.6, 0.8, 1.0}_{\text{added to get smooth plot.}}, 10$

③

ω	$M = \frac{10}{\sqrt{\omega^2+1}}$	$\phi = -\tan^{-1}\omega$.	$M/0.2$
0	10	0°	$\frac{10}{0.2}$ 50 circle.
0.2	9.8	-11°	49
0.4	9.28	-22°	46
0.6	8.6	-31°	43
0.8	7.8	-39°	39
1.0	7	-45°	35
10	3	-84°	15th

③ Scale Selection :- Highest magnitude circle cannot exceed 50 circle in X-axis & 35 circle in Y-axis.

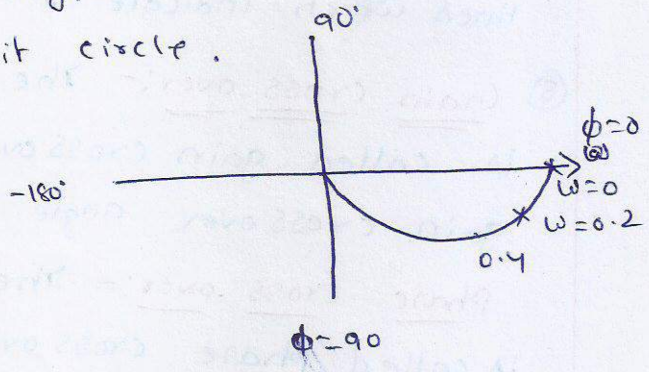
1 circle = 0.2 magnitude

M=10 which lies on 50th circle

Unity circle :- the magnitude of circle which give one (or) unity is called as unity circle. unity.circle = 0.2×5 = 1

∴ 5th circle acts as unit circle.

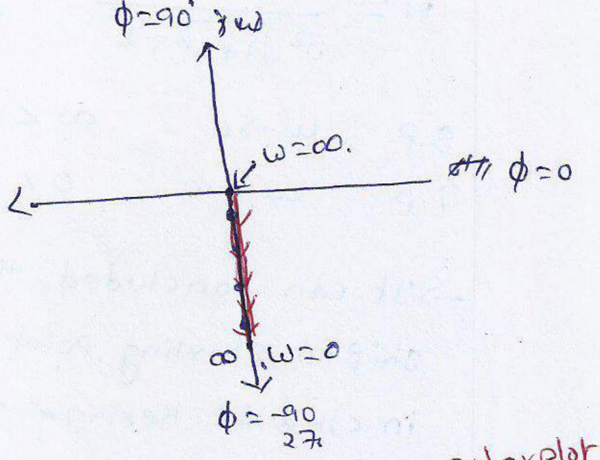
④ Calculation of GM & PM.



② Given OLTF $G(s)H(s) = \frac{10}{s}$, obtain Polar Plot.

Sol: ① $|G(j\omega)| = \frac{10}{\omega}$ $\angle|G(j\omega)H(j\omega)| = \phi = \frac{\tan^{-1}\left(\frac{0}{10}\right)}{\tan^{-1}(\omega/0)} = -90°$

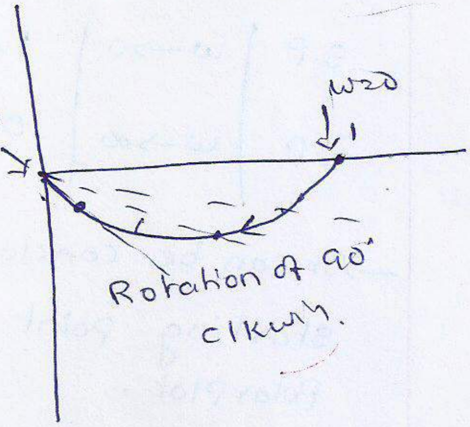
ω	M	φ
0	∞	-90
10	1	-90°
100	0.1	-90°
⋮	⋮	
∞	0	-90°



③ Consider a Type 0 system with OLTF $G(s)H(s) = \frac{1}{1+sT}$. obtain polar plot

Sol: $M = \left|\frac{1}{1+j\omega T}\right| = \frac{1}{\sqrt{1+\omega^2T^2}}$; $\phi = -\tan^{-1}\omega T$

ω	M	φ
0	1	0
$\frac{1}{T}$	$1/\sqrt{2}$	-45°
$\frac{10}{T}$	$\frac{1}{\sqrt{101}}$	-84.2°
⋮		
∞	0	-90°



④ Type-1-system $G(s)H(s) = \frac{1}{s(1+sT)}$

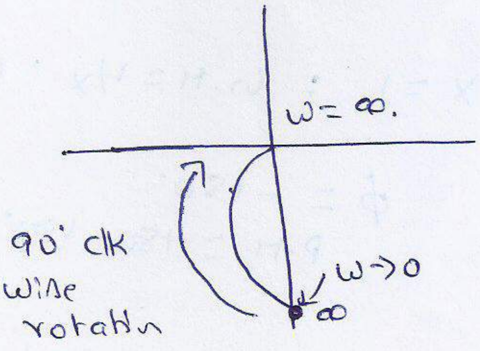
$M = \frac{1}{\omega\sqrt{1+\omega^2T^2}}$; $\phi = -90° - \tan^{-1}\omega T$

Starting point $\omega \to 0$ $\infty\angle-90°$

Terminating point $\omega\to\infty$ $0\angle-180°$

rotation of plot $-180° - (-90°) = -90°$

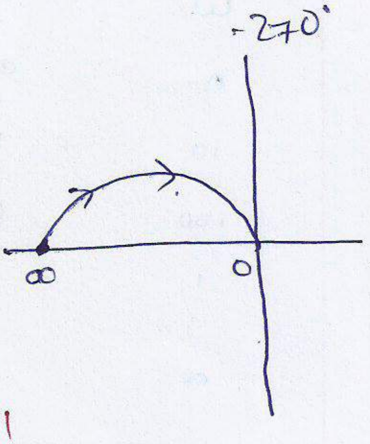
Pole at origin shifts.
Starting point through 90°
in clkwise direction.



⑤ Type-2 system with transfer fn. $G(S)\,H(S) = \frac{1}{S^2(1+TS)}$

$M = \frac{1}{\omega^2\sqrt{1+\omega^2T^2}}$; $\phi = -180° - \tan^{-1}\omega T$.

			rotation of plot
S.P	$\omega \to 0$	$\infty \angle -180°$	$-270° - (-180°)$
T.P	$\omega \to \infty$	$0 \angle -270°$	$= -90°$

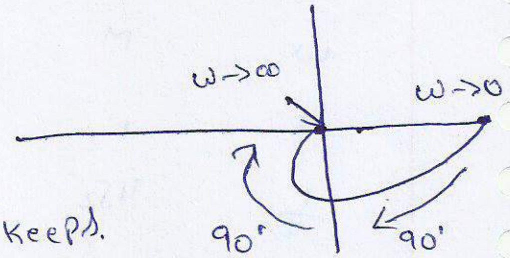


→It can concluded that a pole at origin shifts starting point of polar plot by 90°(180° in clkwise keeping rotation of plot same.

⑥ Effect of adding Simple pole:- $G(s) = \frac{1}{(1+T_1S)(1+T_2S)}$

$M = \frac{1}{\sqrt{1+\omega^2T_1^2}\ \sqrt{1+\omega^2T_2^2}}$; $\phi = -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2$.

			Rotation of plot
S.P	$\omega \to 0$	$1\angle 0°$	$-180 - 0°$
T.P	$\omega \to \infty$	$0\angle -180°$	$= -180°$



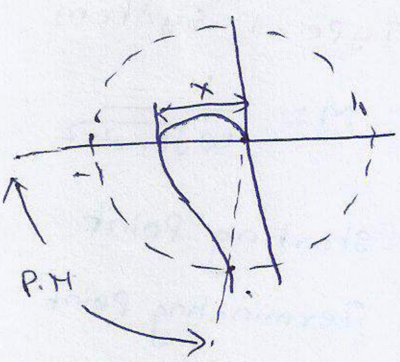
→It can be concluded that a simple pole keeps starting point same but adds a 90° clk wise rotation to the polar plot.

Determination of GM & PM from Polar plot :-

① $G.M = \frac{1}{x}$; $G.M(dB) = 20\log 1/x$
$= +ve$.

$P.M = \phi - (-180°) = 180 + \phi = +ve$.

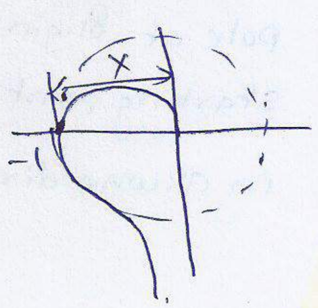
Stable.



② $x = 1$: $G.M = 1/x$; $G.M$ in $dB = 20\log 1$
$= 0\,dB$.

$\phi = -180°$
$P.M = 180 - 180°$

Marginally stable



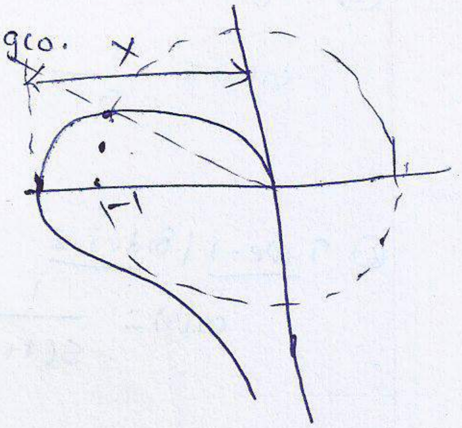
③ $G.M = 1/x$

$G.M\,(dB) = 20 \log 1/x = -ve.$

$P.M = 180°. + \phi_{gco}$

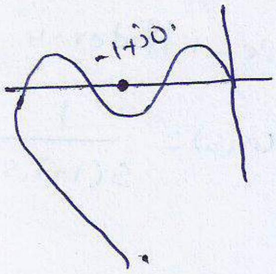
$= -ve.$

Unstable.



Limitation of Polar Plot :-

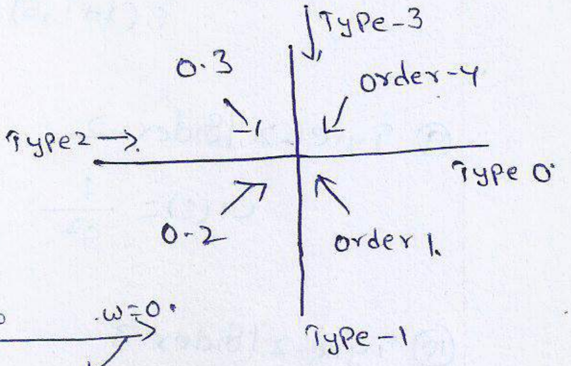
→ It is very difficult to determine G.M & P.M & stability for given polar plots.



→ Such systems are difficult to judge it's stability from polar plot.

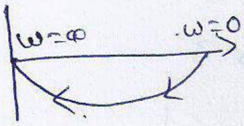
→ Nyquist plot provides more clear answer about the stability, hence in practice Nyquist criterion is used for stability and not the polar plot.

General shapes of Polar plots :-



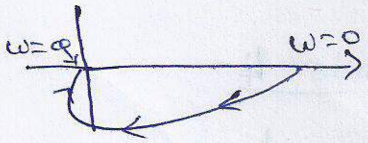
1. Type-0 | order-1

$$G(s) = \frac{1}{1+ST}$$



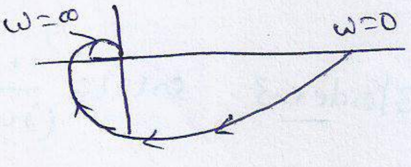
2. Type 0 | order-2.

$$G(s) = \frac{1}{(1+T_1S)(1+T_2S)}$$



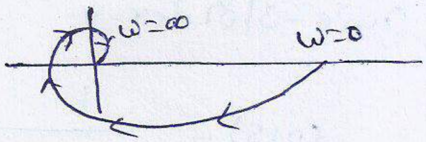
3. Type-0 | order-3

$$G(s) = \frac{1}{(1+T_1S)(1+T_2S)(1+T_3S)}$$



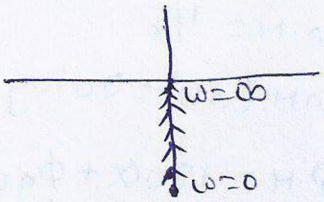
4. Type-0 | order-4

$$G(s) = \frac{1}{(1+T_1S)(1+T_2S)(1+T_3S)(1+T_4S)}$$



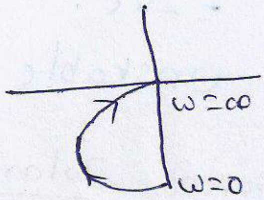
⑤ Type-1 | Order-1

$$G(s) = \frac{1}{s} = \frac{1}{j\omega} = \frac{1}{\omega} \angle -90^\circ$$



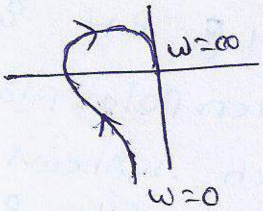
⑥ Type-1 | Order-2

$$G(s) = \frac{1}{s(1+sT)}$$



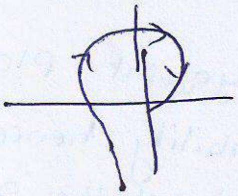
⑦ Type-1 | Order-4

$$G(s) = \frac{1}{s(1+T_1s)(1+T_2s)}$$



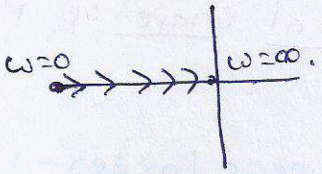
⑧ Type-1 | Order-4:

$$G(s) = \frac{1}{s(1+T_1s)(1+T_2s)(1+T_3s)}$$



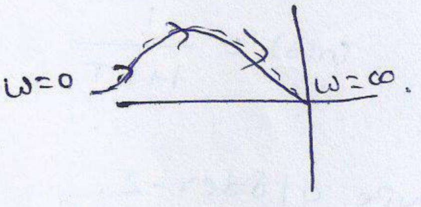
⑨ Type-2 | Order-2.

$$G(s) = \frac{1}{s^2} = \frac{1}{\omega^2} \angle -180^\circ$$



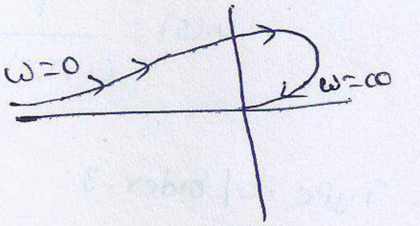
⑩ Type-2 | Order-3

$$G(s) = \frac{1}{s^2(1+Ts)}$$

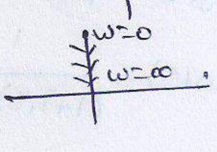


⑪ Type-2 | Order-~~4~~

$$G(s) = \frac{1}{s^2(1+T_1s)(1+T_2s)}$$

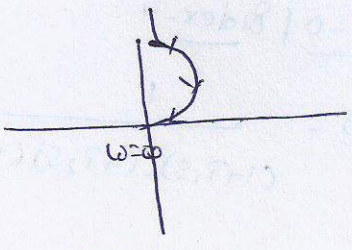


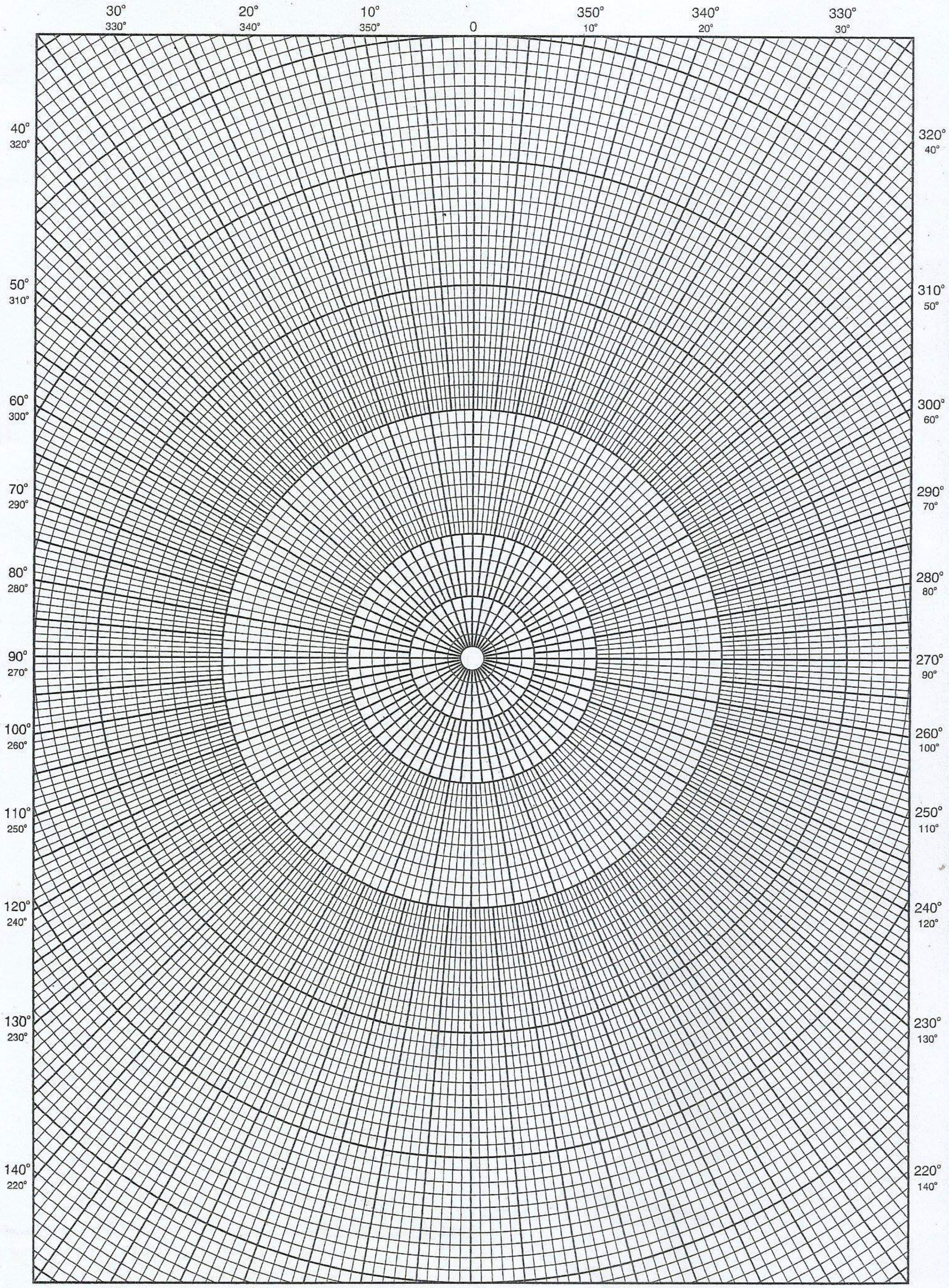
⑫ Type 3 | Order-3 $G(s) = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3} \angle -270$



⑬ Type-3 | Order-4

$$G(s) = \frac{1}{s^3(1+Ts)}$$

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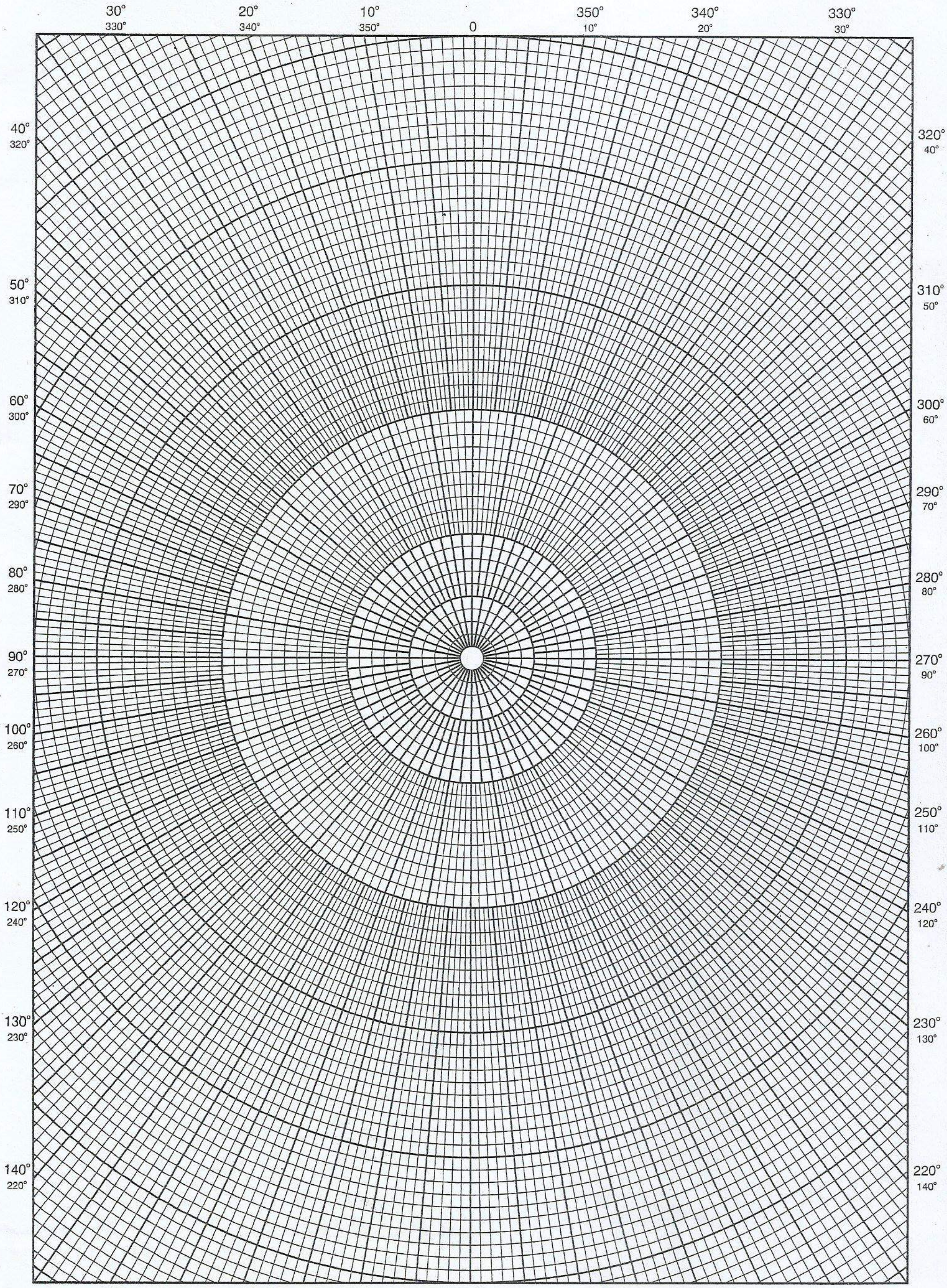
20° 340°
10° 350°
0
350° 10°
340° 20°
330° 30°
40° 320°
50° 310°
60° 300°
70° 290°
80° 280°
90° 270°
100° 260°
110° 250°
120° 240°
130° 230°
140° 220°
320° 40°
310° 50°
300° 60°
290° 70°
280° 80°
270° 90°
260° 100°
250° 110°
240° 120°
230° 130°
220° 140°
210° 150°
200° 160°
190° 170°
180° 180°
170° 190°
160° 200°
150° 210°
M.s.computers


30° 330°
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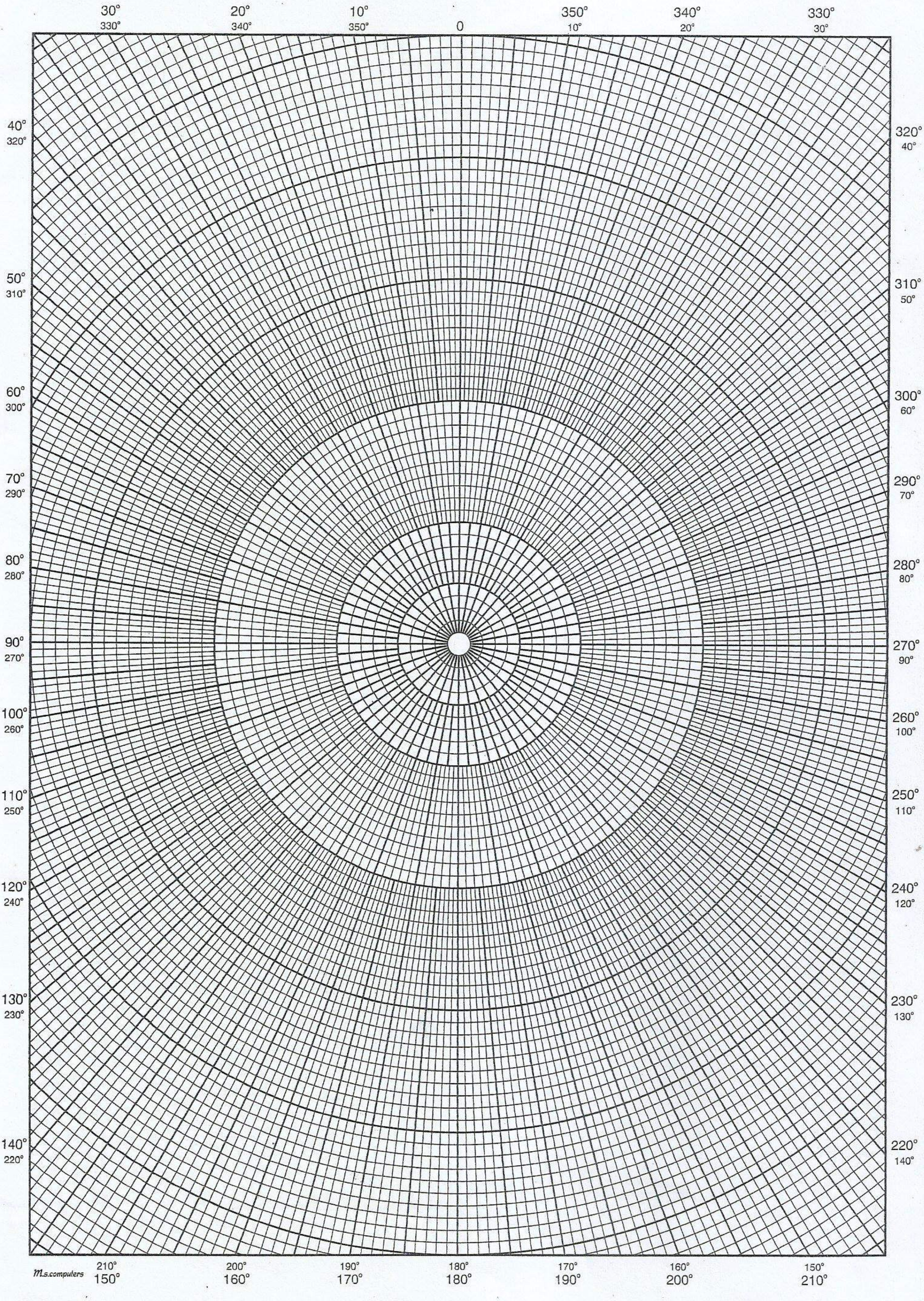
50° 310°
60° 300°
70° 290°
80° 280°
90° 270°
100° 260°
110° 250°
120° 240°
130° 230°
140° 220°
320° 40°
310° 50°
300° 60°
290° 70°
280° 80°
270° 90°
260° 100°
250° 110°
240° 120°
230° 130°
220° 140°
210° 150°
200° 160°
190° 170°
180° 180°
170° 190°
160° 200°
150° 210°
M.s.computers


30° 330°
20° 340°
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350° 10°
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330° 30°
40° 320°
50° 310°
60° 300°
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80° 280°
90° 270°
100° 260°
110° 250°
120° 240°
130° 230°
140° 220°
320° 40°
310° 50°
300° 60°
290° 70°
280° 80°
270° 90°
260° 100°
250° 110°
240° 120°
230° 130°
220° 140°
210° 150°
200° 160°
190° 170°
180° 180°
170° 190°
160° 200°
150° 210°
M.s.computers

Nyquist plot

→ It is basically a graphical method. By using Nyquist stability test we can predict the closed loop stability of a given system from OL data.

→ For unstable systems it indicates how to stabilise the system with the help of suitable compensating networks to obtain desired closed loop system specification.

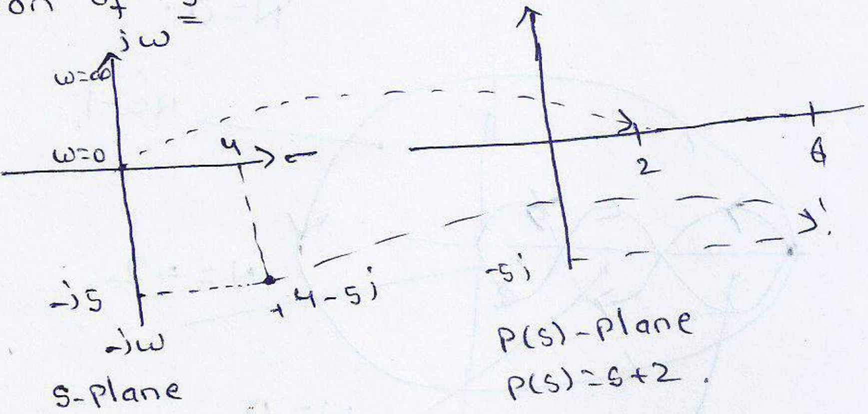
→ In addition to G.M & P.M from the Nyquist plot, the no. of right side poles can be calculated.

Theory of Nyquist plot:-

1. Principle of Mapping:-

"The mapping theorem states that every point in S-plane will get mapped onto corresponding point in P(S) plane where P(S) is any function of 's'"

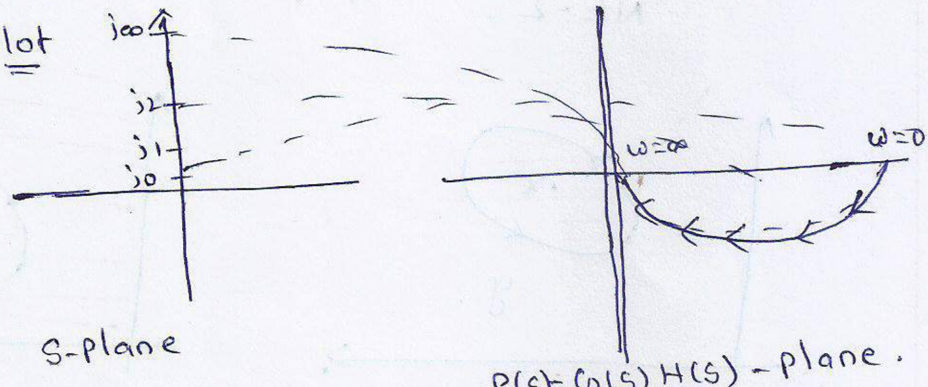
eg:- ①



$P(S) = S+2$.

eg:-

② Polar plot



$P(S) = G(S)\,H(S)$ - plane.

$$= \frac{1}{S+1} = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$$

2. Principle of Argument:-

The principle of argument may be stated as " if t S-plane closed contour encloses P-poles & Z-zeroes ($P > Z$) in R.H.S of S-plane then origin of P(S) plane is encircled P-Z in anticlockwise direction"

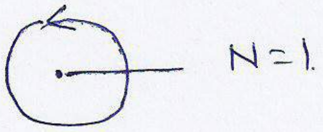
Concept of Encirclement & Enclosement :-

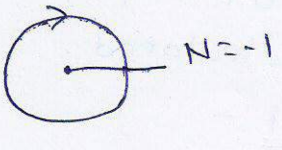
-> The Point is said to be enclosed by a contour if it lies to the right side of the direction of the contour.

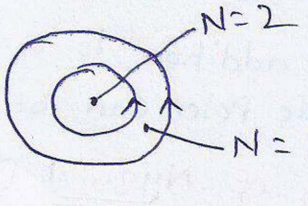
-> The Point is said to be encircled if it lies inside the closed path.

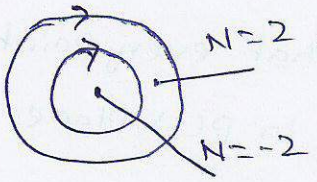
clock wise = -ve. Anticlockwise = +ve.

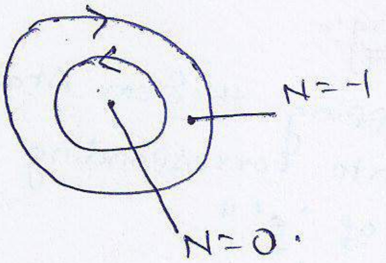
N = no. of encirclements.

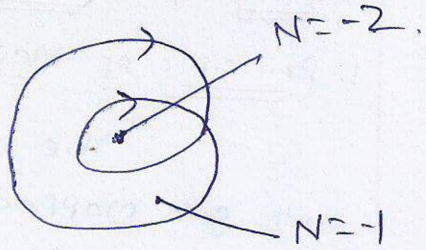


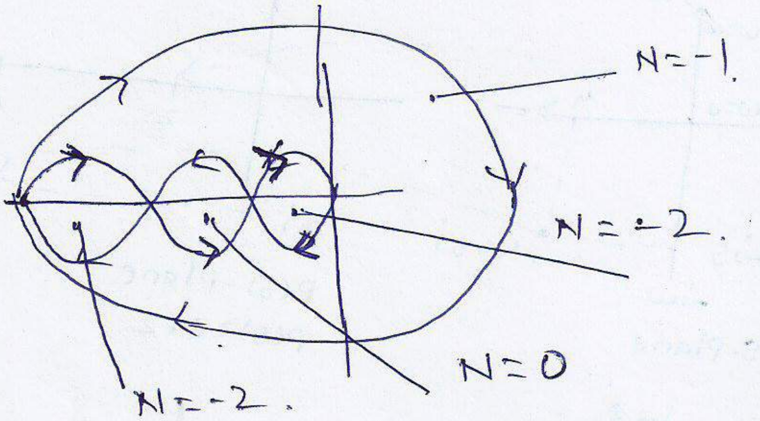


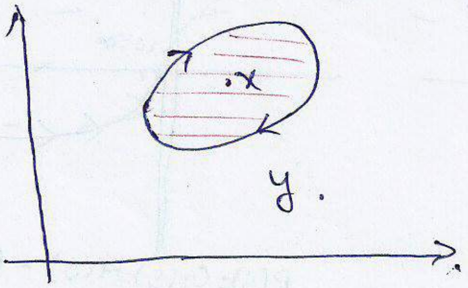


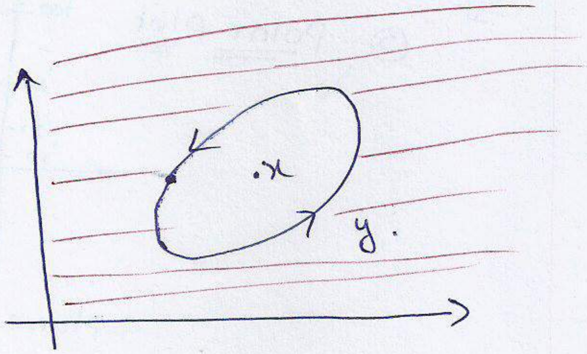












x – encircled & enclosed.

y -> not encircled (since o/s contour)

not enclosed ∵ left side

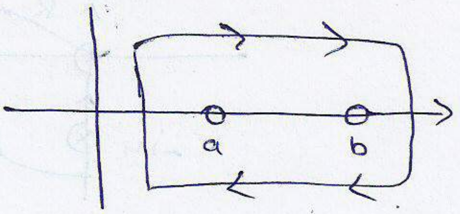
x – encircle.

x -> not enclosed

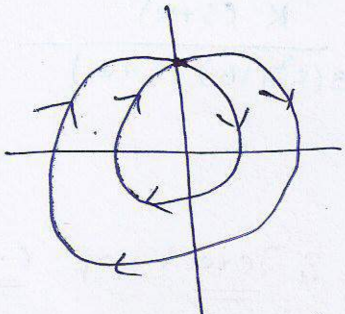
y – not encircle.

y -> enclosed.

Case 1:-

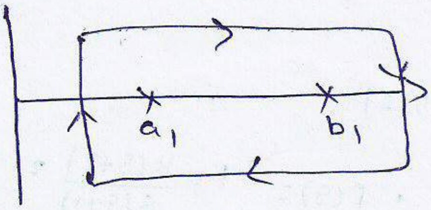


S_plane.

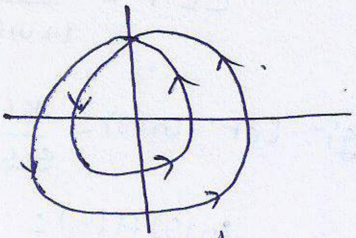


$P(s) = (S-a)(S-b)$

Case 2:-

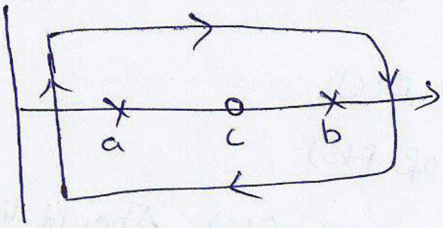


S_Plane

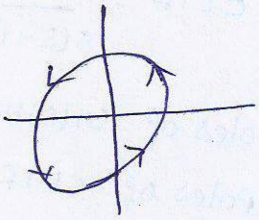


$$P(s) = \frac{1}{(S-a_1)(S-a_2)}$$

Case - 3:-



S_plane



$$P(s) = \frac{(S-c)}{(S-a)(S-b)}$$

No.of Encirclement $N = P - Z$.

③ Nyquist contour/Path:-

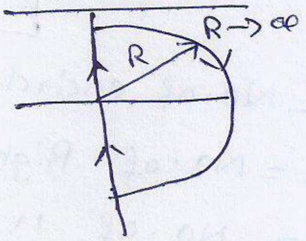
The Nyquist contour/Path encloses entire right side of S-plane and it should not pass through G(s)H(s) which are on the Imaginary axis/at the origin. Therefore small semi circles of radius $\epsilon\ [\epsilon \to 0]$ are drawn as shown in figure below.

G(s)H(s)

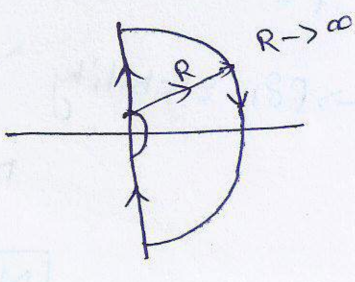
Nyquist contour

S-plane.

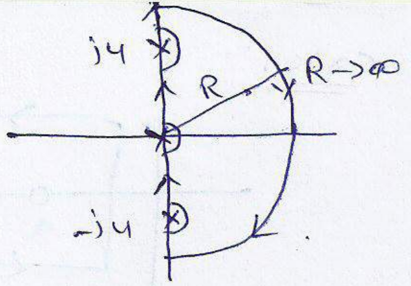
$$\frac{K}{S+1}$$



$$\frac{K}{S^2(S+2)}$$



$$\frac{K(s+4)}{s(s^2+16)(s+2)}$$



④ Poles & Zeroes of C.E :-

$$F(s) = 1 + G(s)H(s)$$

$$CLTF = \frac{G(s)}{1+G(s)H(s)}$$

eg:- Let $G(s) = \frac{K(s+2)}{s(s+1)}$ & $H(s) = 1$

$G(s)H(s) = \frac{K(s+2)}{s(s+1)}$ — ① ; $F(s) = 1 + \frac{K(s+2)}{s(s+1)} = \frac{s(s+1)+K(s+2)}{s(s+1)}$ → ②

$$CLTF = \frac{K(s+2)}{s(s+1)+K(s+2)} \quad -③$$

Poles of G(s)H(s) = Poles of F(s)

Poles of CLTF = Zeroes of F(s)

∴ for stability all the Zeroes of F(s) should lie in the left side of S-Plane.

⑤ Nyquist Stability criteria:-

It is mapping of Nyquist contour in G(s)H(s) plane. Thus plotted Nyquist plot will encircle (-1, j0) critical point as many no. of times as difference between the no. of right side Poles & Zeroes of F(s)

$$\boxed{N = P - Z}$$

N = No. of encirclements of (-1, j0) by the Nyquist plot.

Z = No. of Right side Zeroes of F(s) | CL poles.

P = No. of " " Poles of F(s) | G(s)H(s) poles | OL poles.

→ for stability 'Z' must be Zero

$$N = P - Z$$
$$= P - 0$$
$$\boxed{N = P}$$

→ Nyquist Stability criteria states that (-1, j0) . will be encircled in the ccw direction, as many no. of times as the no. of right side poles of G(s)H(s).

1.

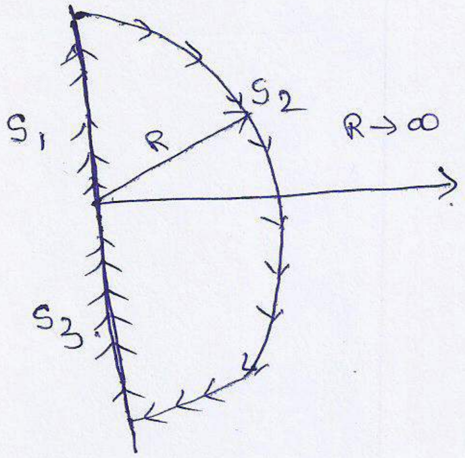
Sol:- $G(s)H(s) = \dfrac{K}{(1+T_1s)(1+T_2s)}$

OL Sinusoidal transfer function is $G(j\omega)H(j\omega) = \dfrac{K}{(1+j\omega T_1)(1+j\omega T_2)}$

$$G(j\omega)H(j\omega) = \frac{K(1-j\omega T_1)(1-j\omega T_2)}{(1+\omega^2 T_1)(1+\omega^2 T_2)}$$

S_1

1. $|G|\angle G$ at $\omega = 0$
2. $|G|\angle G$ at $\omega = \infty$
3. Intersection with real axis
4. " " Imaginary axis.



1. $K\angle 0°$
2. $0\angle -180°$

$$G(j\omega)H(j\omega) = \frac{K(1 - j\omega T_1 - j\omega T_2 + j^2\omega^2 T_1T_2)}{(1+\omega^2 T_1)(1+\omega^2 T_2)}$$

$$= \frac{K(1-\omega^2 T_1 T_2)}{(1+\omega^2 T_1)(1+\omega^2 T_2)} - \frac{jK\omega(T_1+T_2)}{(1+\omega^2 T_1)(1+\omega^2 T_2)}$$

As $\omega \to \infty$ $\quad \angle G(j\omega) = \tan^{-1}$

$= 180 - \tan^{-1}\omega T_1 + 180° - \tan^{-1}\omega T_2$

$= 180°$

Unit - 5

State Space Analysis of Continuous Systems

Syllabus:-

-> Concept of state, state variables and state model.

-> Derivation of state models from block diagrams

-> Diagonalization

-> Solving the Time Invariant state Equations

-> State Transition Matrix and it's properties

-> Concepts of Controllability & observability.

1. State space representation of Electrical system & Mechanical systems
2. State space representation from Differential Equations
   phase variable method.
3. Obtaining T.F from state model
4. Stability from state model
5. Solving state equations.
6. State transition matrix.
7. Controllability & observability.
8. State Diagrams.
9. Diagonalization.

Transfer function Approach	State Variable Approach
1. It is also called conventional (or) classical approach.	1. It is also called Modern Approach
2. It is based on the input/olp relation or Transfer fn.	2. It is based on description of the System equations interms of n first order differential eqns, which may be combined into first order vector-matrix Differential Eqns.
3. Applicable only to LTI systems.	3. Applicable to nonlinear, linear, time variants & time varying.
4. Generally Limited to Single ilp single olp system	4. Applicable to multiple ilp multiple olp systems.
5. In the Initial conditions are - neglected	5. Initial conditions are considered
6. classical design method are based on trail & error procedure. which yields only acceptable system	6. Design is not trial & error procedure. Design using this approach gives optimal systems
7. frequency domain approach	7. Time domain approach.
8. Only ilp, olp and Error signals are considered important. The ilp & olp variables must be measurable.	8. The state need not represent physical variables. They need not even be measurable (or) observable.
9. It requires Laplace transform for continous dat control System and Z-transform for discrete data control systems.	9. It formulates both continuous data control system and discrete data control systems in the same way.
10. The Internal variables can't be fed back.	10. The state variables can be fed-back.
11. The transfer function of a System is unique.	11. The state model of System is not unique

5.1 Concept of State, State Variables & State Model:-

State:- The state of the dynamic system is the smallest set of variables (called state variables) such that the knowledge of these variables at $t = t_0$ together with the knowledge of inputs for $t \geq t_0$, completely determine behaviour of the system for any time $t \geq t_0$.

State Variables:- In a dynamic system, the state of a system is characterized by a collection of minimum set of variables called state variables.

If atleast n variables $x_1, x_2 --- x_n$ are needed to completely describe the behaviour of a dynamic system, then those n variables are a set of state variables.

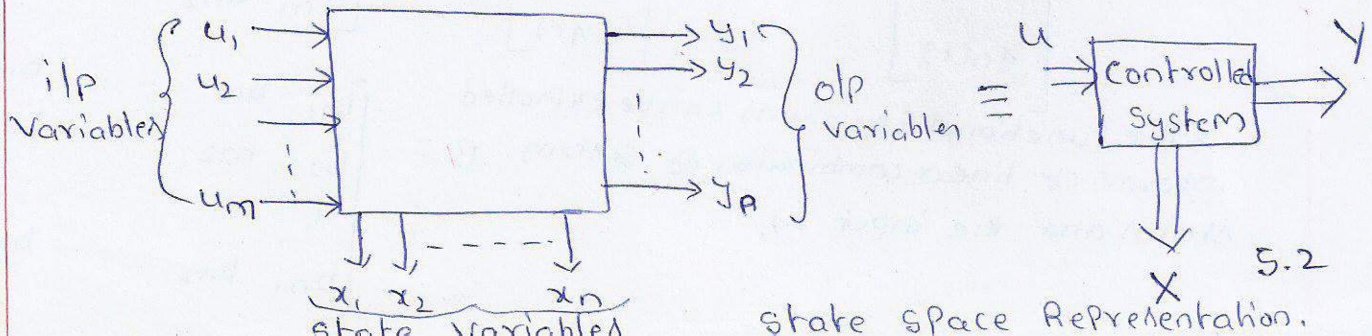
State vector:- If there are n variables $x_1, x_2, --- x_n$ to describe the state, then the vector X of n components is called state vector.

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

State Space:- The n-dimensional space whose co-ordinates axes consist of the x_1 axis, x_2 axis ---- x_n axis, where x_1, x_2 --- x_n are state variables is called the state space.

5.1.1 State Space Representation:-

State space representation of a given system consists of two equations (i) State Equation (ii) output Equation.



State Space Representation.

The state variable representation of a system can be - arranged In the form of n-first Order differential Equations.

$$\frac{dx_1}{dt} = \dot{x}_1 = f_1(x_1, x_2 ---- x_n; u_1, u_2, u_3, --- u_m)$$

$$\frac{dx_2}{dt} = \dot{x}_2 = f_2(x_1\ x_2 --- x_n; u_1, u_2 --- u_m)$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\frac{dx_n}{dt} = \dot{x}_n = f_n(x_1, x_2 --- x_n; u_1, u_2 --- u_m)$$

-> The dynamic system must involve elements that memorize the values of the i/p for $t \geq t_0$, Since integrators in continuous time control system serve as memory devices, the o/p of such integrators can be considered as variables the define Internal state of dynamic system. Thus the o/ps of Integrators serve as state variables.

-> State variable has memory it remembers it's past evolution (or) Immediate past history.

$$\dot{x}_1(t) = a_{11}x_1 + a_{12}x_2 + --- + a_{1n}x_n + b_{11}u_1 + b_{12}u_2 + --- + b_{1m}u_m$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + --- + a_{2n}x_n + b_{21}u_1 + --- + b_{2m}u_m$$

$$\vdots$$

$$\dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + --- a_{nn}x_n + b_{n1}u_1 + --- + b_{nm}u_m$$

~~X(t) =~~ $\dot{X}(t) = AX(t) + BU(t)$

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}, \quad U(t) = \begin{bmatrix} U_1(t) \\ u_2(t) \\ \vdots \\ U_m(t) \end{bmatrix} \quad A = \begin{bmatrix} a_{11} & a_{12} & --- & a_{1n} \\ a_{21} & a_{22} & --- & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & -- & a_{nn} \end{bmatrix}$$

-> The functional Equations can be expressed interms of linear combination of system states and the input as.

$$B = \begin{bmatrix} b_{11} & b_{12} & --- & b_{1m} \\ b_{21} & b_{22} & -- & b_{2m} \\ \vdots & & & \\ b_{n1} & b_{n2} & -- & b_{nm} \end{bmatrix}$$

→ o/p Variables at time 't' are linear combination of the - i/p and state variable at time 't', i.e.,

$$y_1(t) = c_{11}x_1(t) + c_{12}x_2(t) + \dots + c_{1n}x_n(t) + d_{11}u_1(t) + \dots \dots d_{1n}u_m(t)$$

$$\vdots$$

$$y_p(t) = c_{p1}x_1(t) + c_{p2}x_2(t) + \dots c_{pn}x_n(t) + d_{p1}u_1(t) \dots \dots d_{pm}u_m(t).$$

$$Y(t) = C\,X(t) + D\,U(t)$$

$X(t) \rightarrow n\times 1$ - state vector

$u(t) \rightarrow m\times 1$ - i/p vector

$Y(t) \rightarrow p\times 1$ - o/p vector

A : ~~m×m~~ n×n (evolution) System matrix

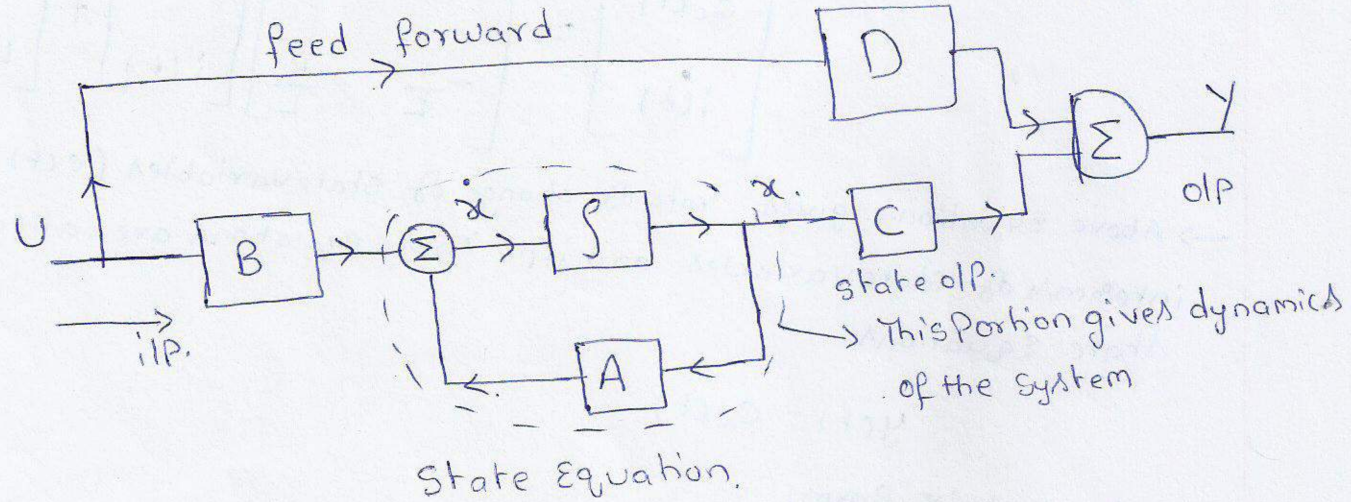
B : n×m i/p matrix. control

C : p×n observation output matrix.

D : p×m transmission matrix.

$$\dot{x}(t) = A X(t) + B U(t)$$: State Equations.

$$\dot{y}(t) = C X(t) + D U(t)$$: o/p Equations.

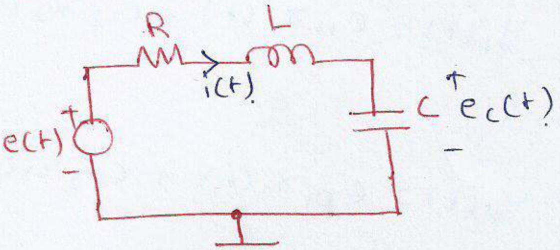


→ If feed forward Portion is zero, then D=0

Block diagram Representation of the state model

① Obtaining State model from differential Equations.
Electrical Systems:-

① Obtain state model for given electrical n/w



Sol:- i/p → e(t)

→ The o/p at any time 't' can be determined if initial voltage across capacitor $e_c(t_0)$ and initial current through ~~Capacitor~~ inductor $i(t_0)$ are known in addition to the values of the o/p input e(t) applied for $t > t_0$.

KVL

$$R\,i(t) + L\frac{di(t)}{dt} + e_c(t) = e(t)$$

$$C\frac{de_c(t)}{dt} = i(t)$$

$$\frac{de_c(t)}{dt} = \frac{1}{C}i(t)$$

$$\frac{di(t)}{dt} = \frac{1}{L}e(t) - \frac{R}{L}i(t) - \frac{1}{L}e_c(t)$$

In vector form

$$\begin{bmatrix} \dot{e}_c(t) \\ \dot{i}(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} e_c(t) \\ i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ L \end{bmatrix} e(t).$$

→ Above Equations gives rate of change of state variables (e(t) & i(t)) interms of state variables and i/p. These equations are called - state Equations.

$$y(t) = e_c(t)$$

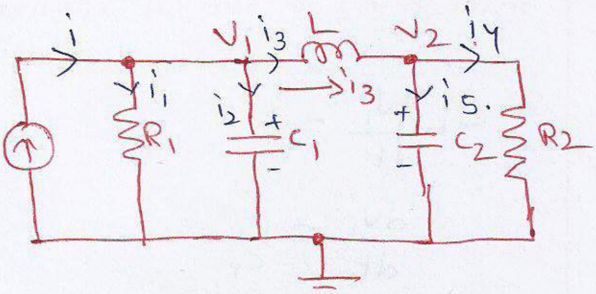
In matrix form,

$$y(t) = [1 \quad 0] \begin{bmatrix} e_c(t) \\ i(t) \end{bmatrix} \quad \rightarrow \text{o/p Equation}$$

→ State Equations and o/p Equations together are called dynamic Eq^{ns}.

→ The o/ps of the integrators are defined as the state variables.

② Obtain the state model of the n/w ~~from~~ assuming $R_1 = R_2 = 1\Omega$, $C_1 = C_2 = 1F$ & $L = 1H$.



Sol:- ~~A~~ KCL at node 1

$$i = \frac{V_1}{R_1} + \frac{C_1 dV_1}{dt} + i_3$$

$$\frac{dV_1}{dt} = -\frac{V_1}{R_1 C_1} - \frac{i_3}{C_1} + \frac{i}{C_1}$$

KCL at node 2.

$$i_3 = i_4 + i_5.$$

$$i_3 = C_2 \frac{dV_2}{dt} + \frac{V_2}{R_2}.$$

$$\frac{dV_2}{dt} = -\frac{V_2}{R_2 C_2} + \frac{i_3}{C_2}.$$

KVL for Loop consisting L.

$$-V_1 + L\frac{di_3}{dt} + V_2 = 0.$$

$$\frac{di_3}{dt} = \frac{V_1}{L} - \frac{V_2}{L}.$$

→ If the current through the Resistor R_2.

$$y_1 = i_5 = \frac{V_2}{R_2}. \qquad y_2 = i_5 R_2 = V_2.$$

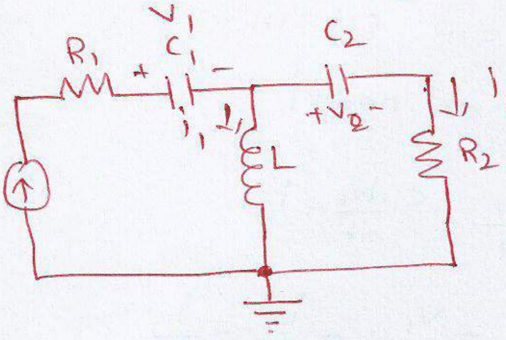
$$\begin{bmatrix} \frac{dV_1}{dt} \\ \frac{dV_2}{dt} \\ \frac{di_3}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 C_1} & 0 & \frac{1}{C_1} \\ 0 & -\frac{1}{R_2 C_2} & \frac{1}{C_2} \\ \frac{1}{L} & -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} \\ 0 \\ 0 \end{bmatrix} [i]$$

$$\begin{bmatrix} i_5 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{R_2} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Substituting above value.

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} i \qquad \begin{bmatrix} i_5 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ i_3 \end{bmatrix}$$

③ obtain the dynamic Equations of the n/w shown in fig. The current through R_2 is the o/p required.



$$C_1 \frac{dv_1}{dt} = i$$

$$\frac{dv_1}{dt} = \frac{i}{C_1}$$

current thru C_2.

$$C_2 \frac{dv_2}{dt} = i - i_1 \Rightarrow \frac{dv_2}{dt} = -\frac{1}{C_2} i_1 + \frac{1}{C_2} i$$

voltage across inductor

<u>KVL</u> Loop2.

$$L \frac{di_1}{dt} = v_2 + R_2 (i - i_1)$$

$$\frac{di_1}{dt} = \frac{1}{L} v_2 - \frac{R_2}{L} i_1 + \frac{R_2}{L} i$$

<u>o/p</u> $i_0 = i - i_1$

dynamic Equations in matrix form are

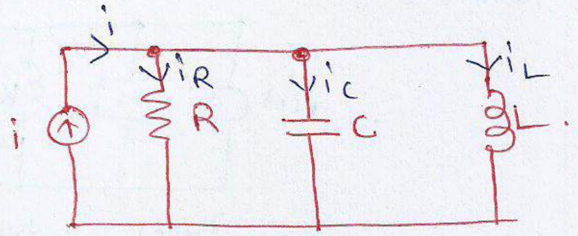
$$\begin{bmatrix} \frac{dv_1}{dt} \\ \frac{dv_2}{dt} \\ \frac{di_1}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{C_2}. \\ 0 & \frac{1}{L} & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_1 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} \\ \frac{1}{C_2} \\ \frac{R_2}{L} \end{bmatrix} [i]$$

$$i_0 = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_1 \end{bmatrix} + (1) i$$

The above matrix form gives state model.

④ write the statevariable formulation of the Parallel RLC ckt n/w shown in fig 10.7. The current through the inductor and voltage across Capacitor are the o/p variables.

Sol:-



KCL

$$i = i_R + i_C + i_L.$$

$$i = \frac{V}{R} + C\frac{dv}{dt} + i_L.$$

$$\frac{dv}{dt} = -\frac{1}{C}i_L - \frac{1}{RC}v + \frac{1}{C}i$$

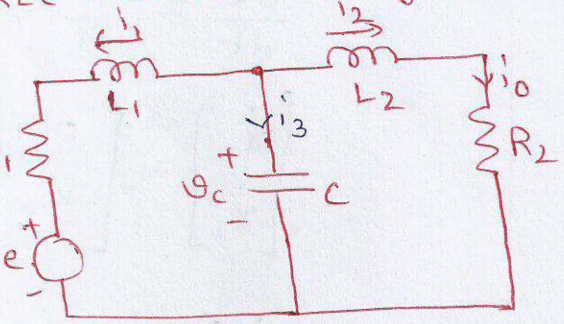
Voltage across the inductor

$$v = L\frac{di_L}{dt} \Rightarrow \frac{di_L}{dt} = \frac{V}{L}. \qquad i_0 = i_L, \; V_0 = V.$$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ V \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} [i].$$

$$\begin{bmatrix} i_0 \\ V_0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ V_0 \end{bmatrix}$$

⑤ obtain State space representation of RLC n/w shown in fig 10.8.



→ 3 Energy storage Element
= order 3
= 3 D.E's.

$i_1(t), i_2(t), \vartheta_C(t) \to$ Statevariables.

KCL at node. $i_1 + i_2 + C\frac{d\vartheta_c}{dt} = 0.$

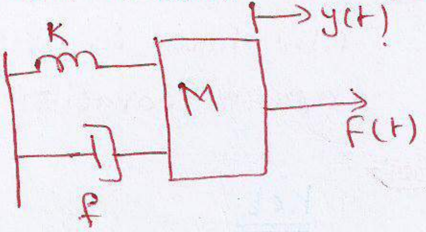
KVL ① $e = -R_1 i_1 - L_1\frac{di_1}{dt} + V_C = 0 \Rightarrow L_1\frac{di_1}{dt} + R_1 i_1 + e - \vartheta_c = 0.$

KVL ② $L_2\frac{di_2}{dt} + R_2 i_2 - \vartheta_c = 0$

$$\frac{di_1}{dt} = -\frac{R_1}{L_1}i_1 + \frac{1}{L_1}\vartheta_c - \frac{1}{L_1}e \; ; \; \frac{di_2}{dt} = -\frac{R_2}{L_2}i_2 + \frac{1}{L_2}\vartheta_c. \; ; \; \frac{d\vartheta_c}{dt} = -\frac{1}{C}i_1 - \frac{1}{C}i_2$$

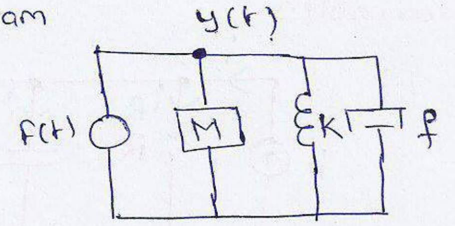
$$\begin{bmatrix} \dot{i}_1(t) \\ \dot{i}_2(t) \\ \dot{\vartheta}_B(t) \end{bmatrix} = \begin{bmatrix} -R_1/L_1 & 0 & \frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ -\frac{1}{C}. & -\frac{1}{C}. & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vartheta_c \end{bmatrix} + \begin{bmatrix} -1/L_1 \\ 0 \\ 0 \end{bmatrix} e \; ; \; \begin{bmatrix} i_0 \\ \vartheta_0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & R_2 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vartheta_c \end{bmatrix}$$

⑥ obtain a state model for the Mechanical system shown in figure



Sol:-

Nodal diagram

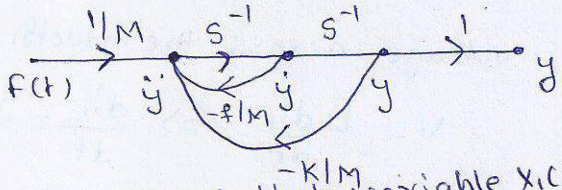


$$F(t) = M\frac{d^2y(t)}{dt^2} + f\frac{dy(t)}{dt} + Ky(t)$$

$$\ddot{y} = \frac{-f}{M} = \dot{y}$$

$$\frac{d^2y(t)}{dt2} = -\frac{f}{M}\frac{dy}{dt} - \frac{K}{M}y + \frac{1}{M}F(t).$$

By defining the o/ps of Integrators on the state diagrams as state variables x_1 and x_2.



$\dot{y} = \cancel{x_1}$, $\ddot{y} = \cancel{x_2}$.

→ choose y(t) as state variable $x_1(t)$ and succesive derivatives of it gives us remaining state variables.

$x_1(t) = y(t)$ $y = x_1$

$x_2(t) = \frac{dy(t)}{dt}$. $\dot{y} = x_2 = \frac{dx_1}{dt}$.

$\ddot{y} = \frac{dx_2}{dt}$.

$\cancel{x_3(t) = \frac{d^2y(t)}{dt}}$.

→ As order of the eqn is 3, only state variables are allowed.

$$y(t) = x_1(t)$$

$$\frac{dx_1}{dt} = x_2.$$

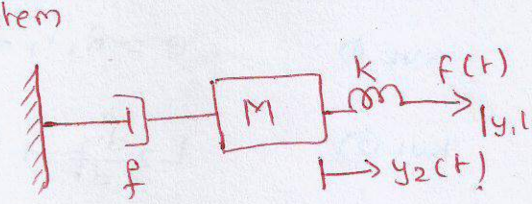
$$\frac{dx_2}{dt} = -\frac{f}{M}x_2 - \frac{K}{M}x_1 + \frac{1}{M}F(t)$$

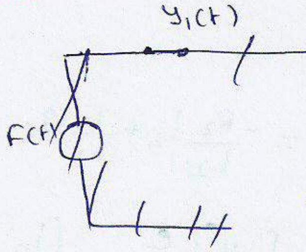
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

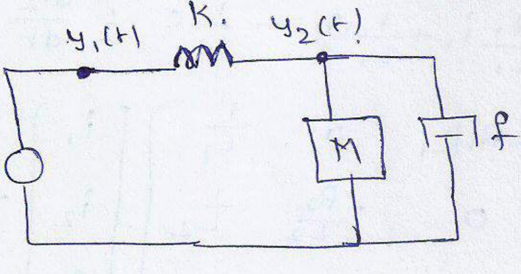
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K/M & -f/M \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \end{bmatrix}F(t).$$

$$\frac{Y(s)}{F(s)} = \frac{1/Ms^2}{1 - \left(-\frac{f}{Ms} - \frac{K}{Ms^2}\right)}$$

⑦ obtain the state model of mechanical system







$F(t) = K[y_1(t) - y_2(t)]$

$M\frac{d^2Y_2(t)}{dt^2} + K(y_2(t) - y_1(t)) + f\frac{dy_2(t)}{dt} = 0.$ $\quad y_1(t) = Y_2(t) + \frac{1}{K}F(t)$

- The o/p of Integrators are taken as statevariables

$x_1(t) = y_2(t)$ and $x_2(t) = \dot{y}_2(t)$

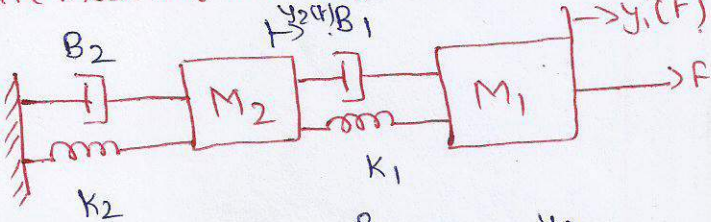
$$\frac{dx_1(t)}{dt} = x_2(t)$$

$$\frac{dx_2(t)}{dt} = -\frac{f}{M}x_2(t) + \frac{1}{M}F(t)$$

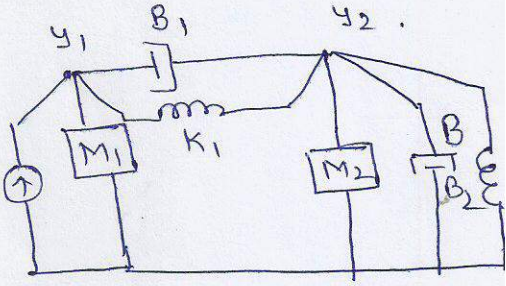
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -f/M \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} F(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \frac{1}{K}F(t)$$

⑧ Consider a mechanical system shown in figure. choosing suitable state variables, construct a state model of the system



$$f(t) = M_1\frac{d^2y_1(t)}{dt^2} + B_1\left(\frac{dy_1}{dt} - \frac{dy_2}{dt}\right) + K_1(y_1 - y_2)$$



$$0 = B_1\left(\frac{dy_2}{dt} - \frac{dy_1}{dt}\right) + K_1[y_2 - y_1]$$

$$M_2\frac{d^2y_2}{dt^2} + B\frac{dy_2}{dt} + K_2y_2.$$

$$\frac{d^2y_2}{dt^2} \xrightarrow{s^{-1}} \underset{x_4}{\frac{dy_2}{dt}} \xrightarrow{s^{-1}} \underset{x_3}{y_2}. \qquad \frac{d^2y_1}{dt^2} \xrightarrow{s^{-1}} \underset{x_2.}{\frac{dy_1}{dt}} \xrightarrow{s^{-1}} \underset{x_1}{y_1}$$

$x_1 = y_1$ $\quad x_3 = y_2$

$x_2 = \dot{y}_1$ $\quad x_4 = \dot{y}_2$

$F(t) = M_1\ddot{y}_1 + B_1(\dot{y}_1 - \dot{y}_2) + K_1(y_1 - y_2)$

$0 = M_2\ddot{y}_2 + B\dot{y}_2 + K_2y_2 + B_1(\dot{y}_2 - \dot{y}_1) + K_1(y_2 - y_1)$

$$\dot{x}_2 = \ddot{y}_1 = \frac{f(t)}{M_1} - \frac{B_1 x_2}{M_1} + \frac{B_1 x_4}{M_1} - \frac{K_1 x_1}{M_1} + \frac{K_1 x_3}{M_1}$$

$$\dot{x}_4 = \ddot{y}_2 = -\frac{(B_1+B_2)x_4}{M_2} - \frac{(K_1+K_2)x_3}{M_2.} + \frac{\beta B_1 x_2}{M_2} + \frac{K_1 x_1}{M_1}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K_1/M_1 & -B_1/M_1 & K_1/M_1 & B_1/M_1 \\ 0 & 0 & 0 & 1 \\ K_1/M_2 & B_1/M_2 & -\frac{(K_1+K_2)}{M_2} & -\frac{(B_1+B_2)}{M_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M_1 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} u(t) \\ f(t) \end{matrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

This is state model of the System.

③ obtain statemodel for system described by.

$$T(s) = \frac{Y(s)}{U(s)} = \frac{2s^3+7s^2+12s+8}{s^3+6s^2+11s+9}$$

$$T(s) = \frac{Y(S)}{U(S)} = \frac{Y_{+}(S)}{X_1(S)} \cdot \frac{X_1(S)}{U(S)}$$

The $\frac{X_1(S)}{U(S)} = \frac{1}{s^3+6s^2+11s+9}$

$$\frac{Y(S)}{X_1(S)} = 2s^3+7s^2+12s+8.$$

$$y = 2\dddot{x}_1 + 7\ddot{x}_2 + 12\dot{x}_1 + 8x$$

$$= 2\dot{x}_3 + 7\dot{x}_2 + 12\dot{x}_1 + 8$$

$$= 2(-9x_1 - 11x_2 - 6x_3 + u)$$

$$+ 7x_3 + 12x_2 + 8x_1$$

$$= -10x_1 - 10x_2 - 5x_3 + 2u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -9 & 11 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -10 & -10 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 24.$$

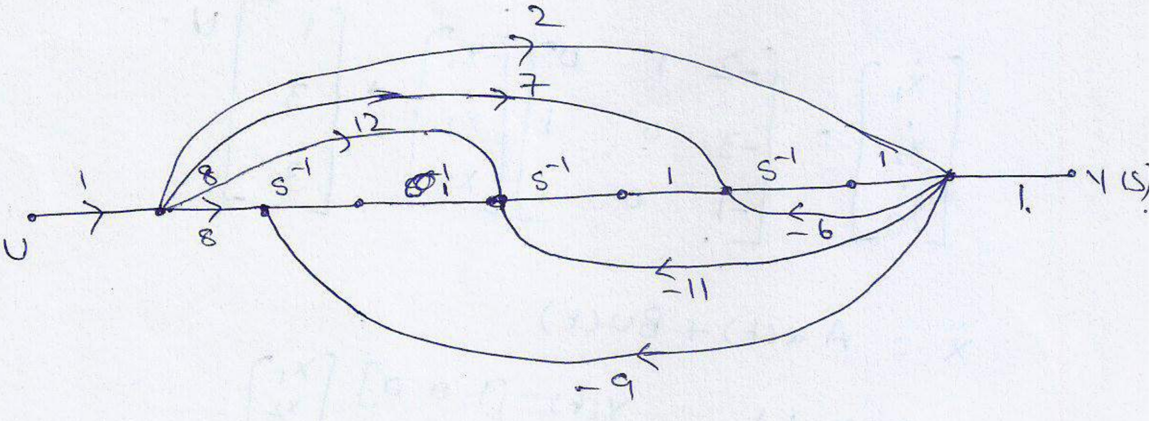
ⓒ obtain statemodel from T.F using SFG.

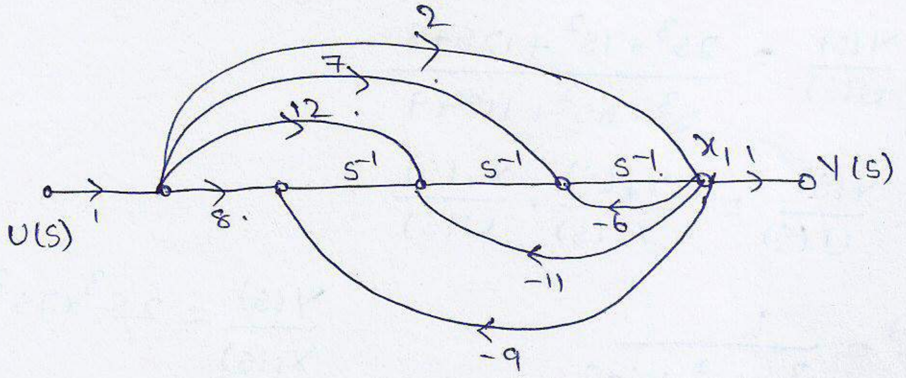
④ $T(s) = \frac{Y(S)}{U(S)} = \frac{2s^3+7s^2+12s+8}{s^3+6s^2+11s+9}$

Sol:- Above T.F canbe ~~w~~ written. $T.F = \frac{2+7s^{-1}+12s^{-2}+8s^{-3}}{1+6s^{-1}+11s^{-2}+9s^{-3}}$

$$T.F = \frac{Y(S)}{U(S)} = \frac{2+7s^{-1}+12s^{-2}+8s^{-3}}{1-(-6s^{-1}-11s^{-2}-9s^{-3})}$$

SFG have 3 feedback loops $-6s^{-1}, -11s^{-2}, -9s^{-3}$ and four forward paths $2, 7s^{-1}, 12s^{-2}$ & $8s^{-3}$.



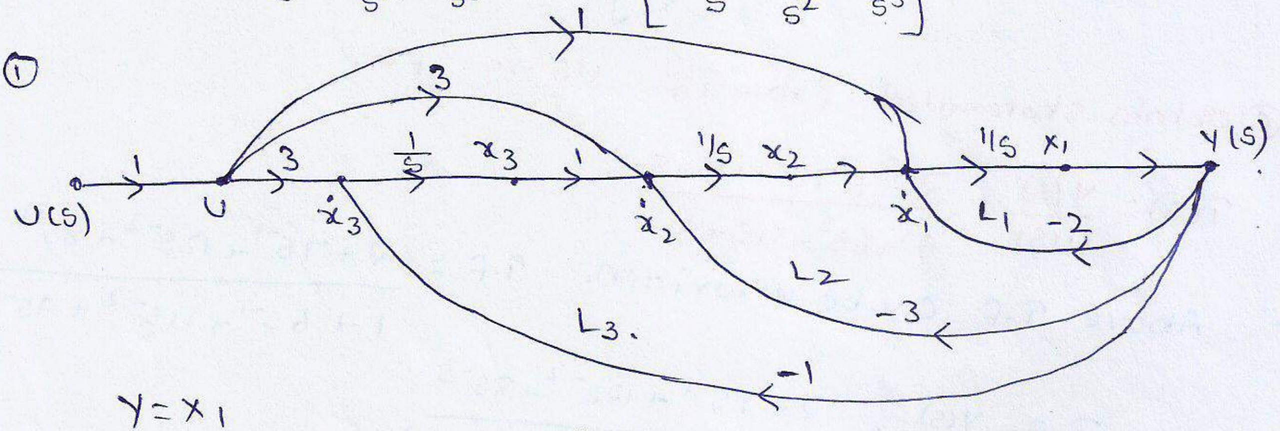


$y =$

(Pb) A feedback systemis characterized by a CLTF $T(s) = \frac{s^2+3s+3}{s^3+2s^2+3s+1}$. Draw a suitable SFG and obtain the statemodel.

Sol:- $$T(s) = \frac{\frac{1}{s}+\frac{3}{s^2}+\frac{3}{s^3}}{1+\frac{2}{s}+\frac{3}{s^2}+\frac{1}{s^3}} = \frac{\frac{1}{s}+\frac{3}{s^2}+\frac{3}{s^3}}{1-\left[-\frac{2}{s}-\frac{3}{s^2}-\frac{1}{s^3}\right]}$$

①



$Y = X_1$

$\dot{x}_1 = X_2 + U - 2Y \quad = X_2 - 2X_1 + U$

$\dot{x}_2 = 3U + x_3 - 3Y = -3X_1 + X_3 + 3U$

$\dot{x}_3 = 3U - Y = -X_1 + 3U$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -3 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix} U.$$

$\dot{x} = A x(t) + B U(t)$

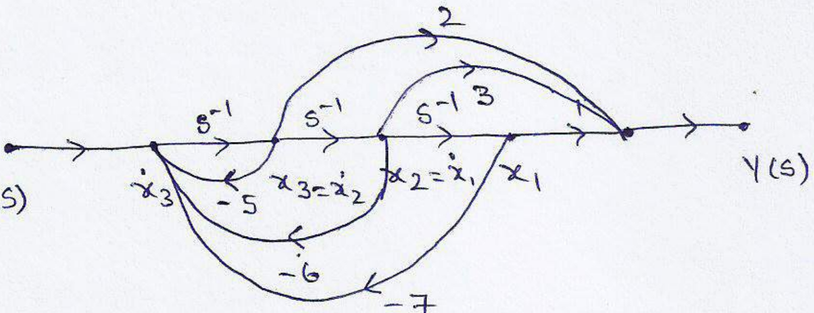
$Y(t) = X_1(t).$ $\quad Y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

(P6) obtain two companion forms of the system whose i/p - o/p transfe function is $\frac{Y(S)}{U(S)} = \frac{2S^2+3S+1}{S^3+5S^2+6S+7}$.

$$\frac{Y(S)}{U(S)} = \frac{2S^{-1}+3S^{-2}+S^{-3}}{1+5S^{-1}+6S^{-2}+7S^{-3}} \cdot \frac{X(S)}{X(S)}$$

$$Y(S) = 2S^{-1}X(S) + 3S^{-2}X(S) + S^{-3}X(S)$$

$$U(S) = X(S) + 5S^{-1}X(S) + 6S^{-2}X(S) + 7S^{-3}X(S)$$



State diagram (first companion form)

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

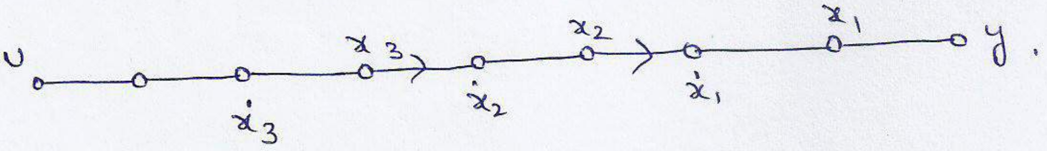
$\dot{x}_3 = u - 7x_1 - 6x_2 - 5x_3$

$y = x_1 + 3x_2 + 2x_3$.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -7 & -6 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u \qquad Y = \begin{bmatrix}1 & 3 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

second companion form

$$\frac{Y(S)}{U(S)} = \frac{\frac{2}{S}+\frac{3}{S^2}+\frac{1}{S^3}}{1+\frac{5}{S}+\frac{6}{S^2}+\frac{7}{S^3}}$$



Model 2:- obtaining state model from Transfer function.

a) If T.F of system has no zeroes, the state model of the system is obtained very easily as shown below.

$$T.F = \frac{b}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + --- + a_{n-1} s + a_n}$$

Corresponding D.E.

$$y^n + a_1 y^{n-1} + a_2 y^{n-2} + --- + a_{n-1} y^{(1)} + a_n y. = b.$$

Let

$x_1 = y$ $\qquad\qquad$ $\dot{x}_1 = x_2$

$x_2 = \dot{y}$ $\qquad\qquad$ $\dot{x}_2 = x_3$

$x_3 = \ddot{y}$ $\qquad\qquad$ $\vdots$

$\vdots$ $\qquad\qquad$ $\dot{x}_{n-1} = x_n$

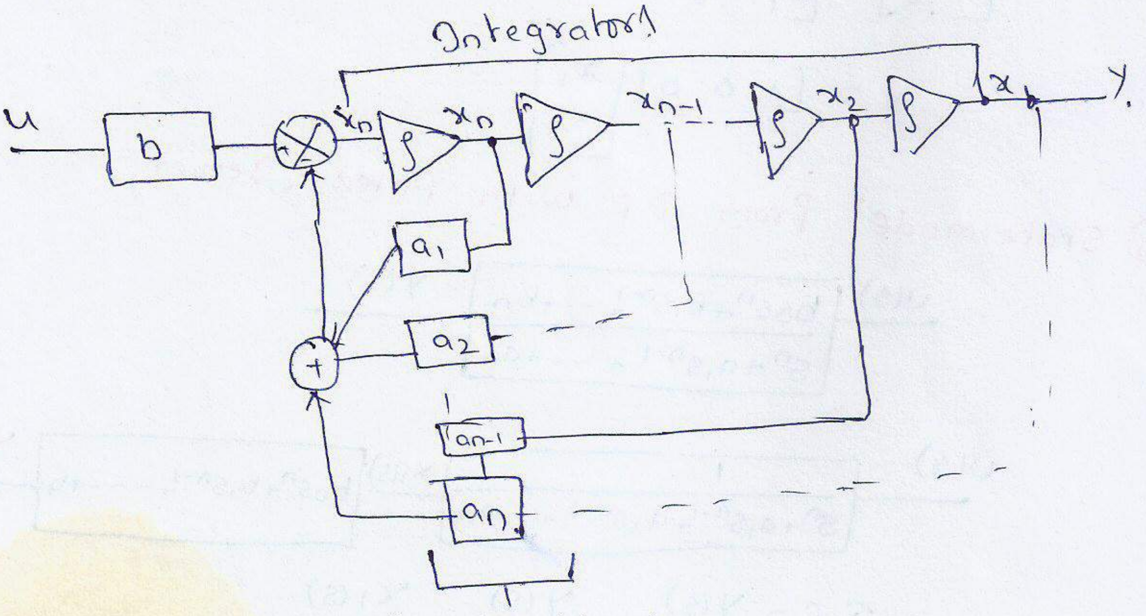
$x_n = y^{(n-1)}.$ $\qquad\qquad$ $\dot{x}_n = -a_n x_1 - a_{n-1} x_2 ---- a_1 x_n + bu$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & -- & 0 \\ 0 & 0 & 1 & 0 & -- & 0 \\ 0 & 0 & 0 & 1 & --- & 0 \\ -a_n & -a_{n-1} & & & --- & a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ b \end{bmatrix} U \qquad Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$\dot{X} = AX + BU. \qquad Y = CX(t)$$

→ Matrix A has special form. It has 1's in upper off-diagonal. It's las row is comprised of -ve of the coefficients of D.Es. all other elements are zero.

A is in Bush form or Companion form.



Block diagram Representation.

① obtain statemodel for the system described by.

$$T(S) = \frac{Y(S)}{U(S)} = \frac{1}{s^3+6s^2+10s+5.}$$

$$\dddot{y} + 6\ddot{y} + 10\dot{y} + 5y = u$$

$x_1 = y.$ i.e., $y = x_1$

$x_2 = \dot{y} = \dot{x}_1$ $\qquad \dot{x}_1 = x_2.$

$x_3 = \ddot{y} = \dot{x}_2.$ $\qquad \dot{x}_2 = x_3.$

$\dot{x}_3 = -5x_1 - 10x_2 - 6x_3 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -10 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u.$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

② $\frac{y(s)}{U(s)} = \frac{5}{s^3+6s+7}$ $\qquad \dddot{y} + 6\dot{y} + 7y. = 5u$

$x_1 = y$ $\qquad \dddot{y} = -6\dot{y} - 7y + 5u.$

$x_2 = \dot{y} = \dot{x}_1$ $\qquad \dot{x}_3 = -6x_2 - 7x_1 + 5u$

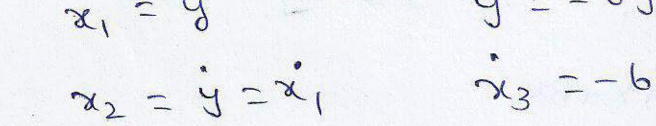
$x_3 = \ddot{y} = \dot{x}_2$ $\qquad \dot{x}_1 = x_2.$

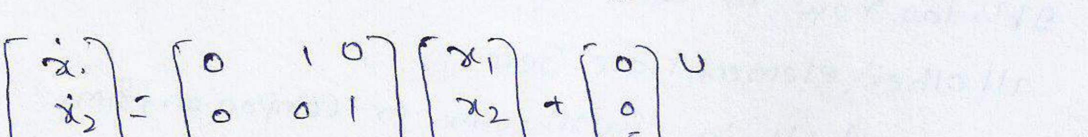
$\dot{x}_2 = x_3.$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

ⓑ state model from T.f with Poles & zeroes.





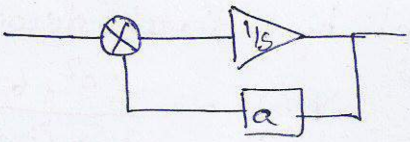
$$T.f = \frac{Y(S)}{U(S)} = \frac{Y(S)}{X_1(S)} \cdot \frac{X_1(S)}{U(S)}.$$

②

Direct Decomposition from Transfer function:-

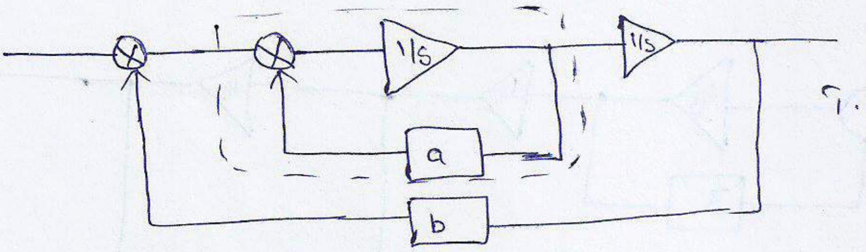
-> Denominator is of Transfer function is rearranged in specific form

Let $T(s) = \frac{1}{s+a}$



$= \frac{1}{s+a}$

-> If such a loop is added in the forward path of another such loop then



$$T.F = \frac{\frac{1}{s(s+a)}}{1+\frac{b}{s(s+a)}} = \frac{1}{s(s+a)+b} = \frac{1}{sx+b}$$

$x = s+a.$

-> If above loop is placed in forward path of another such loop.

we get. $T.F = \frac{1}{sy+c}$ where $= y = sx+b$

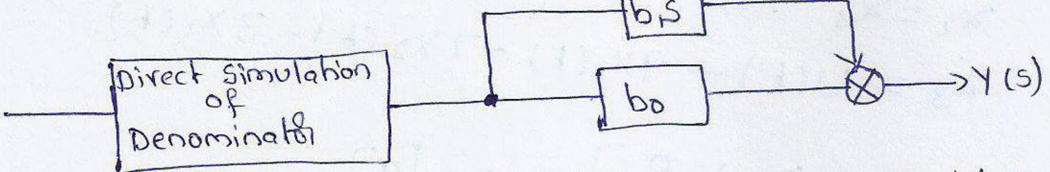
$s(sx+b)+c = . s(s(s+a)+b)+c.$

$s^2+as+b \Rightarrow \{s(s+a)+b\}$

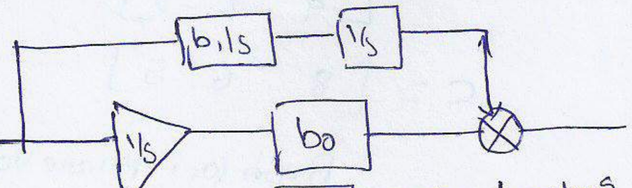
$s^3+as^2+bs+c \Rightarrow \{(s(s+a)+b)s+c\}.$

$s^4+as^3+bs^2+cs+d \Rightarrow \{([(s+a)s+b]s+c)s+d]$ so on.

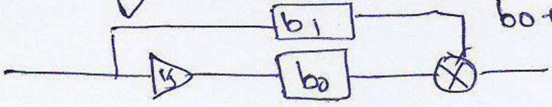
-> If numerator is b_1s+b_0

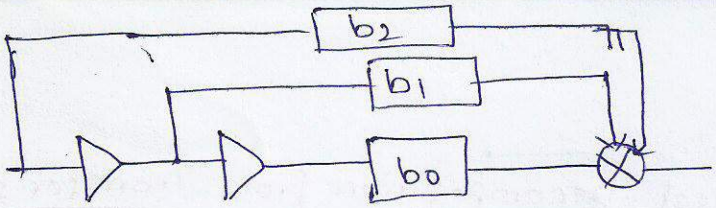


If we shift take off point before Integrator block then



b_0+b_1s

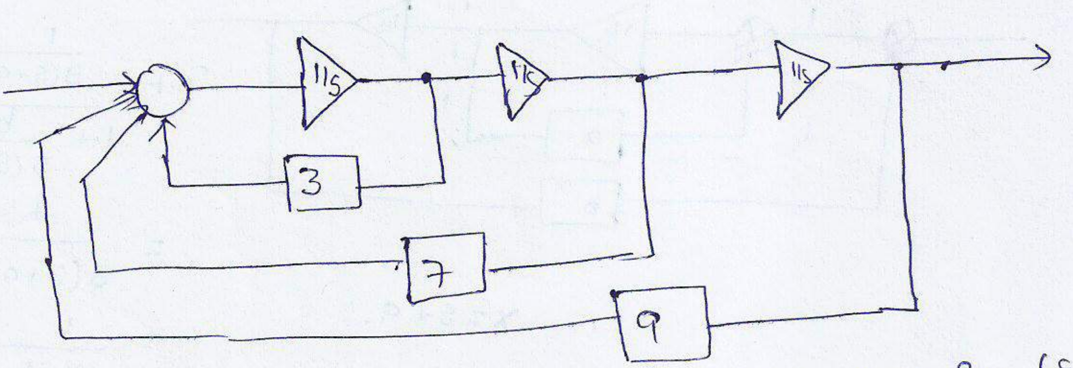




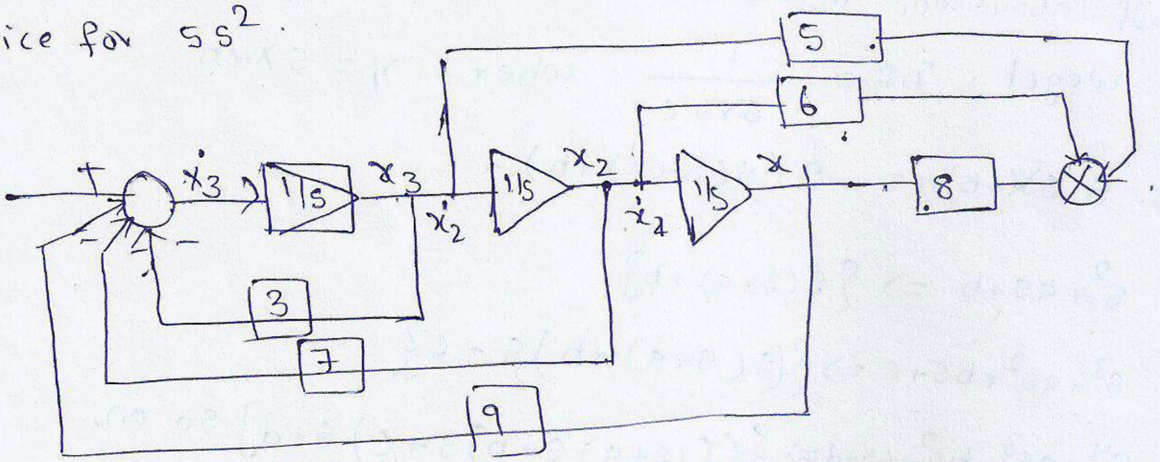
$b_0 + b_1 s + b_2 s^2$.

(P6) obtain state model by direct decomposition method of a system whose T.F is. $\frac{Y(s)}{U(s)} = \frac{5s^2+6s+8}{s^3+3s^2+7s+9}$.

Sol:- Decompose denominator $s^3+3s^2+7s+9 = [s(s+3)+7]s+9$



To simulate, numerator, shift take off point once for 6s, shift twice for $5s^2$.



$\dot{x}_1 = x_2$.

$\dot{x}_2 = x_3$

$\dot{x}_3 = U(t) - 9x_1(t) - 7x_2(t) - 3x_1(t)$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -9 & -7 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = [8 \quad 6 \quad 5] \quad D = [0].$$

Bush (or) phase variable form.

State space representation using Canonical variables!-

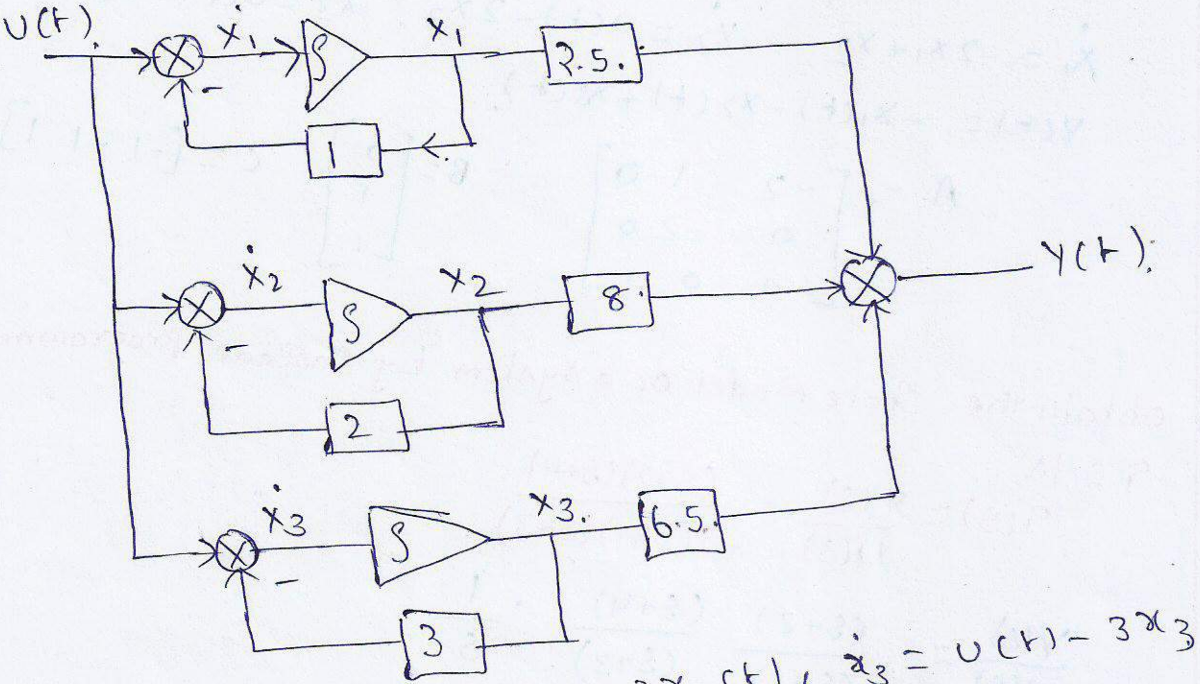
(Parallel programming)

(Pb) Obtain state model in foster's form of a system whose T.F is.
↳ non repeated roots.

$$T.F = \frac{s^2+4}{(s+1)(s+2)(s+3)}$$

Sol:- Find partial fraction expansion of it.

$$\frac{s^2+4}{(s+1)(s+2)(s+3)} = \frac{2.5}{s+1} - \frac{8}{s+2} + \frac{6.5}{s+3}$$



$\dot{x}_1 = -x_1 + u(t)$, $\dot{x}_2 = u(t) - 2x_2(t)$, $\dot{x}_3 = u(t) - 3x_3$

$y(t) = 2.5\, x_1(t) - 8x_2(t) + 6.5\, x_3(t)$

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 2.5 & -8 & 6.5 \end{bmatrix} \quad D = 0.$$

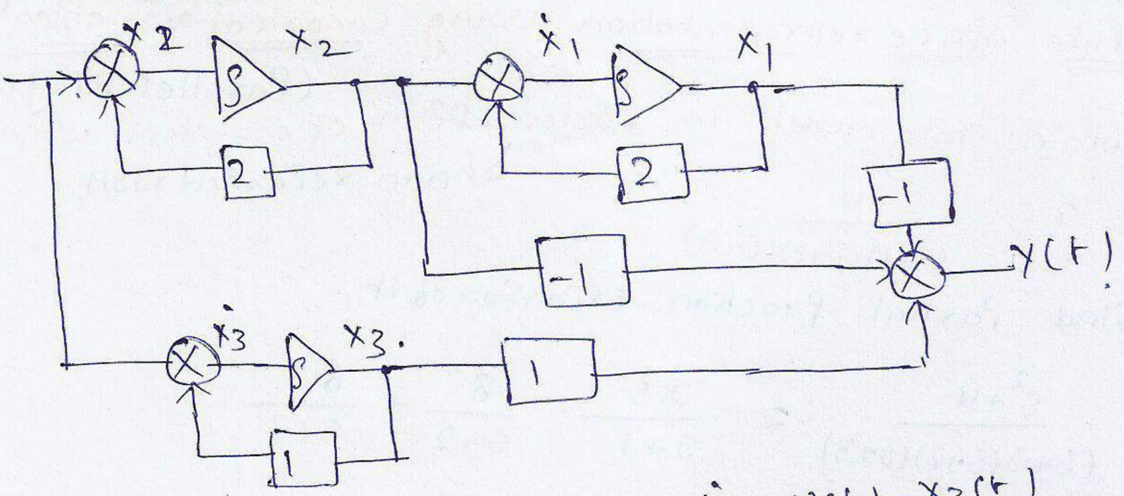
(Pb) Obtain state model from Jordan's canonical form of a system

$$T.F = \frac{1}{(s+2)^2(s+1)}$$

$$T.F = \frac{A}{(s+2)^2} + \frac{B}{s+2} + \frac{C}{s+1} \qquad A = -1,\ B = -1,\ C = 1$$

$$T.F = \frac{-1}{(s+2)^2} - \frac{1}{(s+2)} + \frac{1}{(s+1)}$$

→ Simulate (repeated) first term by series integrators. while other non repeated terms by parallel integrators.



$\dot{x}_1 = -2x_1 + x_2$. $\dot{x}_2 = u(t) - 2x_2$. $\dot{x}_3 = u(t) - x_3(t)$

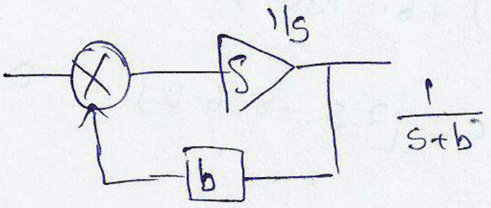
$y(t) = -x_1(t) - x_2(t) + x_3(t)$.

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} -1 & -1 & 1 \end{bmatrix}, D = 0$$

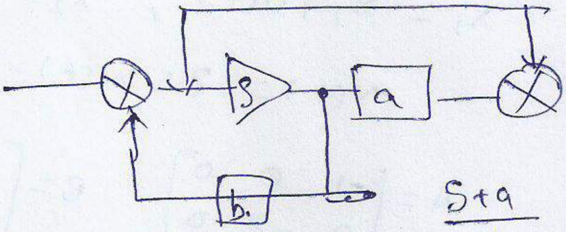
(Pb) obtain the state model of a system by cascade programming whose T.F is

$$T(S) = \frac{Y(S)}{U(S)} = \frac{(S+2)(S+4)}{S(S+1)(S+3)}$$

$$\frac{Y(S)}{U(S)} = \frac{(S+2)}{(S+1)} \frac{(S+4)}{(S+3)} \cdot \frac{1}{S}$$



$\frac{1}{S+b}$



$\frac{S+a}{S+b}$.

Model 4 :- To obtain Transfer function from state model

State Eqns., $\dot{x}(t) = Ax(t) + Bu(t)$

$y(t) = Ax(t) + Bu(t)$

Applying Laplace transform

$$\cancel{\dot{x}}\ [SX(S) - X(0)] = AX(S) + BU(S)$$

$$Y(S) = \overset{C}{\cancel{A}}X(S) + \overset{D}{\cancel{B}}U(S)$$

for T.F, $X(0) = 0$

$$SX(S) - AX(S) = BU(S)$$

$$X(S)[SI - A] = BU(S)$$

$$X(S) = (SI - A)^{-1} . B . U(S)$$

$$Y(S) = [C . (SI - A)^{-1} . B + D]\,U(S)$$

$$\boxed{T.F = \frac{Y(S)}{U(S)} = C . [SI - A]^{-1} . B + D . \quad \text{Transfer Matrix .}}$$

Pb ①

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 \\ 1 \end{bmatrix} x$$

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} S+1 & 0 \\ 0 & S+2 \end{bmatrix}$$

$$(SI - A)^{-1} = \frac{adj(A)}{|A|} \qquad \begin{bmatrix} d & -b \\ \cancel{a} & \cancel{c} \\ -c & a \end{bmatrix}$$

$$|SI - A| \ \cancel{|A|} = (S+1)(S+2) \qquad adj A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$adj(\cancel{SI}-A) = \begin{bmatrix} S+2 & 0 \\ 0 & S+1 \end{bmatrix}$$

$$(SI - A)^{-1} = \begin{bmatrix} \frac{1}{(S+1)} & 0 \\ 0 & \frac{1}{S+2} \end{bmatrix}$$

$$(SI-A)^{-1}.B = \begin{bmatrix} \frac{1}{S+1} & 0 \\ 0 & \frac{1}{S+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{S+1} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{S+1} \\ 0 \end{bmatrix}$$

$$C.[SI-A]^{-1}.B = \underset{1\times 2}{\begin{bmatrix} 1 & 1 \end{bmatrix}} . \underset{2\times 1.}{\begin{bmatrix} \frac{1}{S+1} \\ 0 \end{bmatrix}} = \frac{1}{S+1}.$$

$$\frac{Y(S)}{U(S)} = \frac{1}{S+1}$$

② obtain T.f of system described by the statemodel.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U_1(t) \\ U_2(t) \end{bmatrix}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$T.f = C.[SI-A]^{-1}.B+D.$$

$$[SI-A] = \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} S & -1 & 0 \\ 0 & S & -1 \\ +1 & 2 & S+3 \end{bmatrix}$$

$$|SI-A| = S(S(S+3)+2)+1(+1)$$

$$= S(S^2+3S+2)+1 \quad = S^3+3S^2+2S+1$$

$$adj[SI-A) = \begin{bmatrix} \overset{+}{S(S+3)+2.} & \overset{-}{-1} & \overset{+}{-S.} \\ - & + & - \\ + & & + \end{bmatrix} = \begin{bmatrix} (S+2)(S+1) & -(S+3) & 1 \\ -1 & S(S+3) & S \\ -S & -(2S+1) & S^2 \end{bmatrix}$$

$$T.f = C[SI-A]^{-1}.B+D$$

$$= \frac{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (S+2)(S+1) & -(S+3) & 1 \\ -1 & S(S+3) & S \\ -S & -(2S+1) & S^2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}{S^3+2S^2+2S+1.}$$

$$T.F = \frac{\begin{bmatrix} (s+3) & 1 \\ -(2s+1) & s^2 \end{bmatrix}}{s^3+3s^2+2s+1}$$

③ obtain transfer function of system described by .

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u ; \quad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

x

$sX(s) - X(0) = AX(s) + BU(s)$

$[sI - A]\, X(s)$

$$T.F = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2+3s+2} .$$

$$= \frac{\begin{bmatrix} s+3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{s^2+3s+2 .} = \frac{1}{s^2+3s+2 .}$$

Models:- stability.

$$\frac{Y(s)}{U(s)} = C[sI-A]^{-1}B + D$$

$$= C \cdot \frac{Adj[sI-A]}{|sI-A|} \cdot B + D .$$

$$= \frac{C \cdot Adj[sI-A] \cdot B + D \cdot |sI-A|}{|sI-A|}$$

Zeroes $\Rightarrow C \cdot Adj[sI-A] \cdot B + |sI-A| \cdot D = 0 .$

Poles $= 1 + G(s)H(s) = 0 \quad |sI-A| = 0 .$

Eigen values of System matrix A $= |sI-A| = 0$ CL - Poles .

→ Stability of a system can be determined by determinging the location of eigenvalues.

Pb Determine the stability of the system whose A matrix is.

a) $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ 1,1 unstable. c) $= \begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix}$ 0, -1 marginally stable.

d) $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ j2, -j2, MS.

e) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix}$ -1, -1, -2, S

f) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$ -1, -2, -3 S

g) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}$ 1,1,1, unstable.

Sol
a)

$$|\lambda I - A| =$$

$$|SI - A| = \begin{bmatrix} S & 2 \\ -1 & S+3 \end{bmatrix} = S(S+3)+2 = S = -1, -2.$$

Stable.

Model 3 :- Diagonalization :-

→ The state model is generally not convenient for investigation of system properties and evaluation of time response. The canonical state model wherein A is in diagonal form is the most suitable for this purpose. The techniques for transforming a general state model into canonical one is referred to as Diagonalization techniques.

→ consider an nth order Multi i/p - multi o/p state model

$$\dot{x}(t) = Ax(t) + BU(t)$$

$$y(t) = Cx(t) + DU(t)$$

we have to transform these dynamic equations into another set of eq^ns of the same dimensions. Let us define new state vector $z(t)$

$$x(t) = Pz(t)$$

where P is an non singular matrix $(n \times n)$.

$$P\dot{z}(t) = APz(t) + BU(t)$$

$$\dot{z}(t) = P^{-1}APz(t) + \overset{P^{-1}}{B}U(t)$$

$$y(t) = \bar{C}Pz(t) + DU(t)$$

$$\bar{A} = P^{-1}AP \qquad \bar{B} = P^{-1}B \qquad \bar{C} = CP \qquad \bar{D} = D.$$

The transformation described above is called similarity transformation, since in the transformed system, such properties as the characteristic equation, eigenvectors, eigen values and the transfer f^n are all preserved by the transformation.

Eigen values:-

The roots of characteristic equation $|\lambda I - A| = 0$ i.e., $\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots + a_n = 0$ are often referred to as eigen values of the matrix A

→ Roots of characteristic equation $|SI - A| = 0$

$s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n = 0$, in terms of s are the same as poles of CLTF.

Eigen values of A ≡ Poles of CLTF.

Eigen vector:-

Any non zero vector X_i such that $AX_i = \lambda_i X_i$ is said to be eigen vector associated with eigen value λ_i

Let $\lambda = \lambda_i$ satisfies the eqn,

$$(\lambda_i I - A)X = 0$$

(Pb) Consider a state model with matrix A is $\begin{bmatrix} 0 & 2 & 0 \\ 4 & 0 & 1 \\ -48 & -34 & -9 \end{bmatrix}$

Determine characteristic eqn, Eigenvalues, Eigen vectors, Modal matrix. Also prove that $M^{-1}AM$ results a diagonal matrix.

Sol:- a) The characteristic eqn $|\lambda I - A| = 0$

$$\lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 2 & 0 \\ 4 & 0 & 1 \\ -48 & -34 & -9 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \lambda & -2 & 0 \\ -4 & \lambda & -1 \\ 48 & 34 & \lambda+9 \end{vmatrix} = 0$$

$$\lambda^3 + 9\lambda^2 + 26\lambda + 24 = 0 \quad - \text{C.E.}$$

b) Eigen values = roots of C.E.

$$\lambda_1 = -2, \ \lambda_2 = -3, \ \lambda_3 = -4.$$

c) To obtain Eigenvectors, obtain $[\lambda_i I - A]$ for each eigen value by substituting value of λ in equation (1)

$$\lambda_1 = -2 \quad [\lambda_i I - A] = \begin{bmatrix} -2 & -2 & 0 \\ -4 & -2 & -1 \\ 48 & 34 & 7 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \end{bmatrix} \quad C_{11}, C_{12}, C_{13} \rightarrow \text{cofactors of row 1.}$$

$$M_1 = \begin{bmatrix} 20 \\ -20 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

$$\lambda_2 = -3 \quad [\lambda_i I - A] = \begin{bmatrix} -3 & -2 & 0 \\ -4 & -3 & -1 \\ 48 & 34 & 6 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \end{bmatrix} = \begin{bmatrix} 16 \\ -24 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

~~Eigen values:-~~

~~If there exists a vector x such that A transforms it to vector λx then x is called the solution of equation,~~

$$\cancel{AX = \lambda X}$$

$$\cancel{\lambda X - AX = 0 \Rightarrow [\lambda I - A]x = 0.}$$

$\lambda_3 = -4$, $[\lambda_i I - A] = \begin{bmatrix} -4 & -2 & 0 \\ -4 & -4 & -1 \\ 48 & 34 & 5 \end{bmatrix}$; $M_3 = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix} = \begin{bmatrix} 14 \\ -28 \\ 56 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

M_1, M_2, M_3 are called eigen vector corresponding to eigen values λ_1, λ_2, & λ_3.

d) Modal matrix.

$$M = [M_1 \vdots M_2 \vdots M_3] = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -3 & -2 \\ -2 & 1 & 4 \end{bmatrix}$$

Let's Prove $M^{-1}AM$ is diagonal matrix.

$M^{-1} = \frac{Adj[M]}{det[M]}$. $M^{-1} = \begin{bmatrix} 10 & 7 & 1 \\ -8 & -6 & -1 \\ 7 & 5 & 1 \end{bmatrix}$

$AM = \begin{bmatrix} 2 & -6 & -4 \\ 2 & 9 & 8 \\ 4 & -3 & -16 \end{bmatrix}$; $M^{-1}AM = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \Lambda$

$M^{-1}AM$ is diagonal matrix.

⑮ Reduce the given state model in to its canonical form by diagonalising matrix 'A'.

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U(t)$$

$$Y(t) = [1 \ 0 \ 0] X(t)$$

Sol:- Eigen values, eigen vectors & modelmatrix ~~of A~~.

$|\lambda I - A| = 0$ $\lambda = -1, -2, -3$.

$\lambda_1 = -1$, $[\lambda_1 I - A] = \begin{bmatrix} -1 & -1 & 1 \\ 6 & 10 & -6 \\ 6 & 11 & -6 \end{bmatrix}$; $M_1 = \begin{bmatrix} c_{11} \\ c_{12} \\ c_{13} \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

for $\lambda_2 = -2$

$$[\lambda_2 I - A] = \begin{bmatrix} -2 & -1 & 1 \\ 6 & 9 & -6 \\ 6 & 11 & -7 \end{bmatrix} ; \quad M_2 = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{13} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

for $\lambda_3 = -3$

$$[\lambda_3 I - A] = \begin{bmatrix} -3 & -1 & 1 \\ 6 & 8 & -6 \\ 6 & 11 & -8 \end{bmatrix} , \quad M_3 = \begin{bmatrix} 1 \\ 6 \\ 9 \end{bmatrix}$$

$$M = [M_1 \vdots M_2 \vdots M_3] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 6 \\ 1 & 4 & 9 \end{bmatrix}$$

$$M^{-1}AM = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \Lambda$$

$$\tilde{B} = M^{-1}B \quad ; \quad M^{-1} = \begin{bmatrix} 0.4285 & 0.3571 & -0.2857 \\ -0.4285 & -0.5714 & +0.4285 \\ 0.1428 & 0.2142 & -0.1428 \end{bmatrix}$$

$$\tilde{B} = M^{-1}B = \begin{bmatrix} -0.2857 \\ 0.4285 \\ -0.1428 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}.$$

$$\tilde{C} = CM = [1 \ \ 1 \ \ 1]$$

(Pb) Diagonalize the system matrix given below.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -5 & -4 \end{bmatrix}$$

Sol:- $|\lambda I - A| = \lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0 \quad (\lambda+1)(\lambda+2)(\lambda+3) = 0$

$$\lambda = -1, -2, -3.$$

$$\lambda = -1, \quad [\lambda_1 I - A] = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 6 & 11 & 5 \end{bmatrix} , \quad M_1 = \begin{bmatrix} 6 \\ -6 \\ 6 \end{bmatrix}$$

$$\lambda = -2, \quad [\lambda_2 I - A] = \begin{bmatrix} -2 & -1 & 0 \\ 0 & -2 & -1 \\ 6 & 11 & 4 \end{bmatrix} , \quad M_2 = \begin{bmatrix} 2 \\ -6 \\ 18 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$$

$$\lambda = -3, \quad [\lambda_3 I - A] = \begin{bmatrix} -3 & -1 & 0 \\ 0 & -3 & -1 \\ 6 & 11 & 3 \end{bmatrix} , \quad M_3 = \begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$$

$$M = [M_1 \vdots M_2 \vdots M_3] = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}.$$

Since matrix is in companion form and has distinct eigen values, the Modal matrix M can be written directly in Vander Monde form as

$$M = \begin{bmatrix} 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & -3 \\ 1 & 4 & 9 \end{bmatrix}$$

diagonal matrix is given by $M^{-1}AM = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

ⓑ To diagonalise matrix A first determine eigenvalues, eigenvectors,

$$|\lambda I - A| = \lambda^3 + 4\lambda^2 + 5\lambda + 2 = 0 = (\lambda+1)(\lambda+1)(\lambda+2) = 0$$

$$\lambda = -1, -1, -2.$$

$\rightarrow$ $[\lambda_1 I - A] = \begin{bmatrix} \lambda_1 & -1 & 0 \\ 0 & \lambda_1 & -1 \\ 2 & 5 & \lambda_1 + 4 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 1 & -1 \\ 2 & 5 & -3 \end{bmatrix}.$

$$M_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$\rightarrow$ $[\lambda_2 I - A]$: Since eigen values are repeated, the second eigen vector (generalized eigen vector) can be obtained by differential the cofactors corresponding to first row of $[\lambda_1 I - A] = 0$

$$M_2 = \begin{bmatrix} C_{21} \\ C_{22} \\ C_{23} \end{bmatrix} = \begin{bmatrix} \frac{d}{d\lambda_1}[\lambda_1(\lambda_1+4)+5] \\ \frac{d}{d\lambda_2}(-2) \\ \frac{d}{d\lambda_3}(-2\lambda_1) \end{bmatrix} = \begin{bmatrix} 2\lambda_1 + 4 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$$

$\rightarrow$ $[\lambda_3 I - A] = \begin{bmatrix} -2 & -1 & 0 \\ 0 & -2 & -1 \\ 2 & 5 & 2 \end{bmatrix}$ $\quad M_3 = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$

$$M = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & -2 \\ 1 & -1 & 4 \end{bmatrix}$$

$$M^{-1}AM = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

→ The Modal matrix obtained in not in modified VanderMonde form.

$$M^{-1} = \begin{bmatrix} -2 & -5 & -2 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix} \qquad M^{-1}AM = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

→ Since system matrix A is in Companion form, the model matrix could have been written directly in modified Vander monde form as

$$M = \begin{bmatrix} 1 & \frac{d}{d\lambda_1}(1) & 1 \\ \lambda_1 & \frac{d}{d\lambda_1}(\lambda_1) & \lambda 2 \\ \lambda_1^2 & \frac{d}{d\lambda_1}\lambda_1^2 & \lambda_2^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ \lambda_1 & 1 & \lambda_2 \\ \lambda_1^2 & 2\lambda_1 & \lambda_2^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -2 \\ 1 & -2 & 4 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} 0 & -2 & -1 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Diagonal matrix $M^{-1}AM = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

This shows that even though modal matrix M not unique., the resultant diagonal matrix ,is unique.

Model Controllability & observability:-

→ A system is said tobe completely state controllable at time to, if it is possible by means of an unconstrained control vector u(t) to transfer the system from an initial state $x(t_o)$ to any other desired state of a in a finite interval of time.

Kalman's test for controllability:-

$$\dot{X}(t) = AX(t) + BU(t)$$

$A_{n\times n}$ $B_{m\times 1}$

→ The composite matrix is given by $Q_c = [B : AB : A^2B : \dots A^{n-1}B]$

n → no. of State variables.

B, AB, A^2B --- are columns.

Q_c - matrix for controllability.

→ The system is completely controllable iff the rank of composite matrix Q_c is 'n'.

→ To find the rank of matrix means to search for highest order determinant which is non singular i.e., whose value is non zero. This order of determinant is the rank of the given matrix.

Thus rif r×r matrix has a non zero value in a given matrix. then the rank of matrix is r and determinant having order r+1 and or more than that has a zero value.

① Evaluate controllability of the system.

$$\dot{X} = AX + BU \quad A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

n = 2.

$$Q_c = [B \; AB] = \{$$

$$AB = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = \det = 0$$

Hence rank of $Q_c \Rightarrow r = 1$. $Q_c \neq n$

This system is not state controllable.

② Find the controllability of the system

$$\dot{x} = \begin{bmatrix}1 & 1\\ 2 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

$n=2$.

$Q_c = \cancel{A}[B \,.\, AB]$

$$AB = \begin{bmatrix}1 & 1\\ 2 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}0+1\\ -1\end{bmatrix} = \begin{bmatrix}1\\ -1\end{bmatrix}$$

$$Q_c = \begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$$ det $\neq 0$.

$r=2$.

Rank $Q_c = 2 = n$. completely controllable.

③ $$A = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -2 & -3\end{bmatrix} \quad B = \begin{bmatrix}0 & 1\\ 0 & 0\\ 1 & 0\end{bmatrix}$$

$Q_c = [B \;\; AB \;\; A^2B]$

$$AB = \begin{bmatrix}0 & 0\\ 1 & 0\\ -3 & 0\end{bmatrix} \quad A^2B = \begin{bmatrix}1 & 0\\ -3 & 0\\ 7 & 0\end{bmatrix} \quad Q_c = \begin{bmatrix}0 & 1 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & -3 & 0\\ 1 & 0 & -3 & 0 & 7 & 0\end{bmatrix}$$

consider $$\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 3\end{bmatrix}$$ det $=1$ = non zero.

Rank $Q_c = 3 = n$ completely controllable.

④ $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -11 & -6\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u$$

$n=3$ $$AB = \begin{bmatrix}0\\ 1\\ -6\end{bmatrix} \quad A^2B = \begin{bmatrix}1\\ -6\\ 25\end{bmatrix} \quad Q_c = \begin{bmatrix}0 & 0 & 1\\ 0 & 1 & -6\\ 1 & -6 & -25\end{bmatrix}$$

det $= -1$ non zero

Rank $Q_c = 3 = n$ completely controllable.

Observability:-

The observability is related to the problem of determining the system state by measuring the o/p for finite length of time.

" A system is said to be completely observable, if every state $X(t_0)$ can be completely identified by measurements of the o/ps $Y(t)$ over a finite interval time. If the system is not completely observable means that few of its state variables are not practically measurable and are shidded from the observation.

Kalman's test for observability:-

The composite matrix $Q_0 = [C^T : A^T C^T : \cancel{A^T} - - - - - - (A^T)^{n-1} C^T]$

C^T : Transpose of C.

A^T : " " A

→ If rank of $Q_0 = n$ then system is completely observable.

① Evaluate the observability of the system.

Sol:-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad Y(t) = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

order of System $n = 2$,

$$A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \quad C = [1 \quad 0] \quad A^T = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix} \quad C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Q_0 = [C^T \quad A^T C^T] \qquad A^T C^T = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

determinant = 1 = non zero. $Q_0 = 2 = n$

Hence the system is completely Obsesvable.

② $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $C = [3 \quad 4 \quad 1]$

$Q_0 = [C^T : A^T C^T : (A^T)^2 C^T]$. det = 0

$$\cancel{A}\ Q_0 = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 1 & -2 \\ 1 & 1 & -2 \end{bmatrix}$$

Hence a non zero det existing in Q_0 is having order less than 3

Rank of $Q \neq 3 \neq n$

not completely obsesvable.

③ $\dot{x} = \begin{bmatrix} -0.2 & 0.4 \\ 0.1 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$

$y(t) = [1 \ 0] \, x(t)$. find the complete observability of system

$A = \begin{bmatrix} -0.2 & 0.4 \\ 0.1 & -0.1 \end{bmatrix} \quad c = [1 \ 0]$

$A^T c^T = \begin{bmatrix} -0.2 \\ 0.4 \end{bmatrix} \quad Q_0 = \begin{bmatrix} 1 & -0.2 \\ 0 & 0.4 \end{bmatrix}$

det $= 0.4 \neq 0$ Rank of $Q_0 = 2 = n$

completely observable.

Model Solving Time Invariant State Equations:-

$$\dot{X}(t) = A X(t) + B U(t).$$

→ If A is a constant matrix and I/P control forces are zero then the equation takes the form

$$\dot{X}(t) = A X(t) \quad \text{— homogoneous Eq}^n.$$

→ If A is a constant matrix & matrix U(t) is non zero vector. then the equations i.e., i/p control forces are applied to the system the form

$$\dot{X}(t) = A X(t) + B U(t) \rightarrow \text{Non homogeneous Eq}^n.$$

→ To solve the homogeneous (unforced) state equation

$$\dot{x}(t) = A x(t), \quad x(0) = x_0.$$

consider a scalar case,

$$\frac{dx}{dt} = a x(t), \quad x(0) = x_0.$$

Solution for above Eqn

$$x(t) = e^{at} \cdot x_0$$

$$= \left(1 + at + \frac{a^2 t^2}{2!} + \frac{a^3 t^3}{3!} + \cdots \right) x_0.$$

By analogy with the scalar case, the vector state Eqn has Solution

$$X(t) = \left[I + At + \frac{A^2 t^2}{2!} + \frac{A^3 t^3}{3!} + \cdots \right) X_0.$$

e^{At} → Matrix Exponential.

$$e^{At} = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + \cdots$$

$$\therefore\ X(t) = e^{At} X_0.$$

The solution ~~abov~~ shows that initial state X_0 at $t=0$, is driven to a state $X(t)$ at time 't'.

→ Since transition in state is carried out by the matrix exponential e^{At}, e^{At} is known as State Transition Matrix. and is denoted by $\phi(t)$ i.e., $\phi(t) = e^{At}$.

→ STM depends only on the system Matrix A, it is also called STM of A.

Solution of Non homogeneous State Equation:- $(U(t) \neq 0)$

$$\dot{X}(t) = AX(t) + BU(t)\ ;\ X(0) = x_0.$$

$$\dot{X}(t) - AX(t) = BU(t)$$

Multiplying e^{-At} on both sides.

$$e^{-At}\left[\dot{X}(t) - AX(t)\right] = \frac{d}{dt}\left[e^{-At}X(t)\right] = e^{-At}.BU(t)$$

Integrating this w.r.t to t between the limits 0 & t, give.

$$\int_0^t \frac{d}{dt}\left[e^{-At}X(t)\right] = e^{-At}X(t)\Big]_0^t = \int_0^t e^{-A\tau}BU(\tau)\,d\tau.$$

$$e^{-At}X(t) - X(0) = \int_0^t e^{-A\tau}BU(\tau)\,d\tau$$

Interms of STM, X(t)

$$X(t) = e^{At}X(0) + e^{At}\int_0^t e^{-A\tau}BU(\tau)\,d\tau.$$

$$X(t) = e^{At}\left[X(0) + \int_0^t e^{-A\tau}.BU(\tau)\,d\tau\right]$$

$$\boxed{X(t) = \phi(t)X(0) + \int_0^t \phi(t-\tau)\,BU(\tau)\,d\tau}$$

→ If initial time is t_0, then

$$X(t) = \phi(t-t_0)X(t_0) + \int_0^t \phi(t-\tau)BU(\tau)\,d\tau$$

Significance of STM:-

—>It represent the free response of the system.

—>It is dependent only on system matrix. (A)

—>It describes the change of state from the initial time t=0, to any time t, when the i/ps are zero.

Properties of STM:-

1. $\phi(0) = I$, Proof : $\phi(0) = e^{A \times 0} = I$.

$$e^{At} = I + At + \frac{A^2t^2}{2!} + ---$$

2. $\phi^{-1}(t) = \phi(t)$

Proof: $\phi^{-1}(t) = \frac{1}{\phi(t)} = \frac{1}{e^{At}} = e^{-At} = \phi(-t)$

3. $\phi(t_2-t_1)\,\phi(t_1-t_0) = \phi(t_2-t_0)$ for any t_2, t_1, t_0

Proof:- $\phi(t_2-t_1)\,\phi(t_1-t_0) = e^{A(t_2-t_1)} \cdot e^{A(t_1-t_0)} = e^{A(t_2-t_0)}$

$= \phi(t_2-t_0)$

4. $[\phi(t)]^K = \phi[Kt]$

Proof $[\phi(t)]^K = \phi(t) \cdot \phi(t) - - - K\text{time} = e^{At} \cdot e^{At} - - - = e^{AKt}$.

$= \phi(Kt)$

5. $\phi(t_1+t_2) = \phi(t_1)\,\phi(t_2) = \phi(t_2)\,\phi(t_1)$

$\phi(t_1+t_2) = \phi(t_1)\, e^{A(t_1+t_2)} = e^{At_1} \cdot e^{At_2}$.

$= \phi(t_1)\,\phi(t_2) = \phi(t_2) \cdot \phi(t_1)$

Computation of STM (State Transition Matrix.)

(1) By using Infinite Series Method.

Pb (1) Compute the STM by Infinite Series Method.

(a) $A = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Sol (a)

$$\phi(t) = I + At + \frac{A^2t^2}{2!} + \frac{A^3t^3}{3!} + - - - -$$

$$A^2 = \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix} \quad A^3 = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$$

$$\therefore \phi(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} t + \begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix} \frac{t^2}{2!} + - - -$$

$$\phi(t) = \begin{bmatrix} 1 - \frac{t^2}{2} + \frac{t^3}{3} + - - - & t - t^2 + t^3/3 + - - - \\ -t + t^2 + \frac{t^3}{2} + - - - & 1 - 2t + \frac{3t^2}{2} - \frac{2t^3}{3} - - - \end{bmatrix}$$

$$= \begin{bmatrix} e^{-t} + t & te^{-t} \\ -te^{-t} & e^{-t} - te^{-t} \end{bmatrix}$$

$$e^{-t} = 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} + - - -$$

(b) $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$$\phi(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} t + \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \frac{t^2}{2!} + \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \frac{t^3}{3!} + - - - -$$

$$= \begin{bmatrix} 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + - - - & t + t^2 + \frac{t^3}{2} + - - - \\ 0 & 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + - - - \end{bmatrix}$$

$$= \begin{bmatrix} e^t & te^t \\ 0 & e^t \end{bmatrix}$$

(2) Computation of STM by Laplace Transformation.

a) Free Response $[U(t)=0]$

$$\dot{X}(t) = AX(t)$$

Applying Laplace Transformation

$$sX(s) - X(0) = AX(s)$$

$$X(s)[sI-A] = X(0)$$

$$X(s) = [sI-A]^{-1}X(0) \quad -(1)$$

$$\phi(s) = [sI-A]^{-1} = \text{Resolvent matrix.}$$

Taking ILT, for (1)

$$X(t) = L^{-1}[sI-A]^{-1}X(0)$$

$$\boxed{\phi(t) = e^{At} = L^{-1}[sI-A]^{-1} = L^{-1}[\phi(s)]}$$

b) Forced Response:-

~~As~~ $\dot{X}(t) = AX(t) + BU(t)$

$$sX(s) - X(0) = AX(s) + BU(s)$$

$$[sI-A]X(s) = X(0) + BU(s)$$

$$X(s) = [sI-A]^{-1}X(0) + B\cdot[sI-A]^{-1}U(s)$$

$$= \phi(s)X(0) + B\cdot\phi(s)U(s)$$

$$= \phi(s)[X(0) + BU(s)]$$

I.L.T

$$X(t) = L^{-1}[\phi(s)[X(0) + BU(s)]]$$

$$= \phi(t)[X(0)] + L^{-1}[\phi(s)BU(s)]$$

Applying convolution theorem,

$$X(t) = \phi(t)X(0) + \int_0^t \phi(t-\tau)BU(\tau)d\tau.$$

Zero i/p Response ZIR

Zero state. Response ZSR.

① obtain STM of state model whose system matrix A is given by.

a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ b) - $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ c) $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

Sol:- a) $[SI - A] = \begin{bmatrix} S-1 & -1 \\ 0 & S-1 \end{bmatrix}$

$$\phi(S) = [SI-A]^{-1} = \frac{adj[SI-A]}{det[SI-A]} = \frac{\begin{bmatrix} S-1 & 1 \\ 0 & S-1 \end{bmatrix}}{(S-1)^2}.$$

$$STM = \phi(t) = L^{-1}[\phi(S)] = L^{-1}[(SI-A)^{-1}]$$

$$= L^{-1}\begin{bmatrix} \frac{1}{S-1} & \frac{1}{(S-1)^2} \\ 0 & \frac{1}{S-1} \end{bmatrix} = \begin{bmatrix} e^t & te^t \\ 0 & e^t \end{bmatrix}$$

b) $[SI-A] = \begin{bmatrix} S & -1 \\ 1 & S+2 \end{bmatrix}$

$$\phi(S) = [SI-A]^{-1} = \frac{\begin{bmatrix} (S+2) & 1 \\ -1 & S \end{bmatrix}}{(S+1)^2}.$$

$$S.T.M = \phi(t) = L^{-1}[\phi(S)] = L^{-1}\begin{bmatrix} \frac{S+2}{(S+1)^2} & \frac{1}{(S+1)^2} \\ \frac{-1}{(S+1)^2} & \frac{S}{(S+1)^2} \end{bmatrix} = \begin{bmatrix} (1+t)e^{-t} & te^{-t} \\ -te^{-t} & (1-t)e^{-t} \end{bmatrix}$$

c) $[SI-A] = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$

$$\phi(S) = [SI-A]^{-1} = \frac{\begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}}{S^2+3S+2}.$$

$$S.T.M = \phi(t) = L^{-1}[\phi(S)] = L^{-1}\begin{bmatrix} \frac{S+3}{(S+1)(S+2)} & \frac{1}{(S+1)(S+2)} \\ \frac{-2}{(S+1)(S+2)} & \frac{S}{(S+1)(S+2)} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

③ Computation of the STM Using Cayley Hamilton theorem:-

→ for large system this Method is more convenient.

Procedure:-

① Find Eigen values of System matrix A.

② If all the eigen values are distinct, solve n simultaneous eqns given by the equation

$$f(\lambda_i) = \alpha_0 + \alpha_1 \lambda_i + \alpha_2 \lambda_i^2 + \cdots + \alpha_{n-1} \lambda_i^{n-1}$$

for coefficients $\alpha_0, \alpha_1, \cdots \alpha_{n-1}$

→ If any of the eigen values are repeated, then obtain one Independent Equation by substituting that eigenvalue in the above Equation.

③ Substitute the coefficients α_i obtained in step 2 in Eqn

$$f(A) = \alpha_0 I + \alpha_1 A + \alpha_2 A^2 + \cdots + \alpha_{n-1} A^2$$

to obtain matrix Polynomial.

Ⓟⓑ obtain the STM for the statemodel whose A matrix is given below using Cayley Hamilton theorem.

a) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ b) $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$ c) $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

a) ① $|\lambda I - A| = \begin{vmatrix} \lambda-1 & -1 \\ 0 & \lambda-1 \end{vmatrix} = \lambda^2 - 2\lambda + 1 = (\lambda-1)(\lambda-1) = 0.$

$\lambda_1 = 1, \lambda_2 = 1.$

② we know that

$$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda. \quad -(1).$$

if $\lambda = 1$ $\quad e^t = \alpha_0 + \alpha_1$

D(1) above Eqn w.r.t λ.

$$t \cdot e^{\lambda t}\Big|_{\lambda=1} = t e^t = \alpha_1$$

$$\alpha_0 = -\alpha_1 + e^t = -te^t + e^t$$

$\phi(t) = e^{At} = \alpha_0 I + \alpha_1 A.$

$$= \alpha_0 \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \alpha_1 \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} = \begin{bmatrix}\alpha_0+\alpha_1 & \alpha_1\\ 0 & \alpha_0+\alpha_1\end{bmatrix}$$

$$= \begin{bmatrix}e^t & te^t\\ 0 & e^t\end{bmatrix}$$

ⓑ $A = \begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}$

$|\lambda I - A| = \lambda^2 + 2\lambda + 1 = 0 \Rightarrow \lambda_1 = \lambda_2 = -1$

$f(\lambda) = \alpha_0 + \alpha_1 \lambda$

$e^{\lambda t} = \alpha_0 + \alpha_1 \lambda. \quad — (1)$

Sub $\lambda = -1$ $\quad e^{-t} = \alpha_0 - \alpha_1. \quad \Rightarrow \alpha_0 = e^{-t}(1+t).$

B.(1) w.r.t λ

$e^{\lambda t}.t = \alpha_1. \quad \Rightarrow \alpha_1 = te^{-t}.$

$f(A) = e^{At} = \alpha_0 I + \alpha_1 A$

$$= \begin{bmatrix}\alpha_0 & 0\\ 0 & \alpha_0\end{bmatrix} + \cancel{\alpha_0}\begin{bmatrix}0 & \alpha_1\\ -\alpha_1 & -2\alpha_1\end{bmatrix}$$

$$= \begin{bmatrix}(1+t)e^{-t} & te^{-t}\\ -te^{-t} & (1-t)e^{-t}\end{bmatrix}$$

ⓒ $|\lambda I - A| = 0, \quad \lambda_1 = -1, \lambda_2 = -2.$

$f(\lambda) = \alpha_0 + \alpha_1 \lambda = e^{\lambda_1 t}.$

$\alpha_0 - \alpha_1 = e^{-t}.$

$\alpha_0 - 2\alpha_1 = e^{-2t}.$

Solving $\alpha_0 = 2e^{-t} - e^{-2t}$; $\alpha_1 = e^{-t} - e^{-2t}.$

$e\, f(A) = e^{At} = \alpha_0 I + \alpha_1 A.$

$$= \begin{bmatrix}2e^{-t} - e^{-2t} & e^{-t} - e^{-2t}\\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t}\end{bmatrix}$$