Vidya Jyothi Institute of Technology (Autonomous)

(Accredited by NAAC, Approved By A.I.C.T.E., New Delhi, Permanently Affiliated to JNTU, Hyderabad)

(Aziz Nagar, C.B.Post, Hyderabad -500075)

COURSE HANDOUT
APPLIED PHYSICS

Course Overview:

The Applied Physics course provides the knowledge to understand the concepts of Physics which plays an important role in analysis and design thinking process of complex engineering solutions.The course helps the students to understand the behavior of electronic devices with an insight of material science knowledge required for an engineer.

Course Outcomes:

1. Identify various optical phenomena of light
2. Discuss the basic principles of quantum mechanics
3. Classify solids based on the band theory
4. Elucidate the characteristics of semiconductors and semiconductor devices
5. Explain the working principle of optical fibers and lasers

Course Syllabus:

UNIT – I: Wave Optics:

Principle of Superposition, coherence. Interference - Interference in thin films by reflection, Newton's Rings. Diffraction – Fraunhofer and Fresnel Diffraction, Fraunhofer diffraction due to single slit, Plane Diffraction Grating, resolving power of Grating(qualitative treatment). Polarization – Polarization of light waves, Plane of vibration, plane of polarization, Double refraction, Nicol's Prism, Applications of Polarization.

UNIT-II: Free electron theory and Introduction to Quantum Mechanics:

Classical free electron Theory, Electrical Conductivity and Ohm's Law – Drawbacks.

Introduction to quantum physics: Black body radiation and Planck's Law(qualitative treatment), wave-particle duality, de-Broglie hypothesis of matter waves, Heisenberg uncertainty principle, time independent Schrodinger equation, Born interpretation of wave function, particle in an infinite potential well (one dimension).

UNIT-III: Band theory of solids and semiconductors:

Kronig-Penny model(qualitative treatment), E-k diagram, Energy bands in solids, classification of materials into metals, semiconductors, and insulators, Effective mass, Density of States(qualitative treatment), Fermi distribution function, Fermi level and its importance.

Intrinsic semiconductors, carrier concentration in intrinsic semiconductors, energy band diagram and position of Fermi level in intrinsic semiconductors, equation for electrical conductivity of semiconductors, extrinsic semiconductors (qualitative treatment).

UNIT-IV: Semiconductor Devices:

Direct and indirect band-gap semiconductors, Formation of p-n junction, energy diagram of PN junction, I-V characteristics of PN junction diode, Photo diode, solar cell-efficiency, light emitting diode and their characteristics, semiconductor laser: device structure and characteristics, Hall effect and its applications.

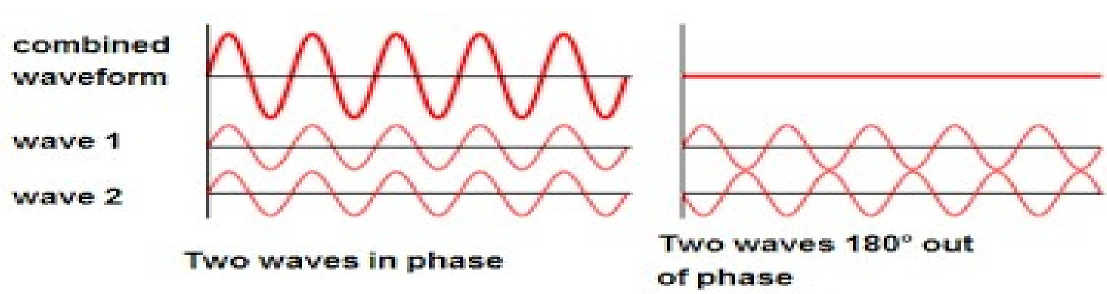
UNIT-V: Fiber Optics and Lasers:

Introduction, total internal reflection, acceptance angle and numerical aperture, step and graded index fibers, applications of optical fibers.

Introduction to interaction of radiation with matter: stimulated absorption, spontaneous emission and stimulated emission, characteristics of a laser, population inversion, important components of a laser: active medium, pumping source, optical resonator. Construction and working of Ruby laser, He-Ne laser, applications of lasers.

UNIT–I Wave Optics

Superposition principle: when two or more waves travel simultaneously in a medium, the resultant displacement at any point is due to the algebraic sum of the displacement due to individual waves. This phenomenon is called superposition principle.



$Y=Y_1+Y_2$ $Y=Y_1-Y_2$

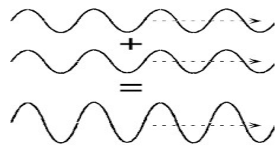
Coherent source: The two light sources with same frequency, same wavelength, same time period and constant phase difference between them are said to be coherent source.

Interference: When light waves are superimposed, the modification in the distribution of intensity in the region of superposition is called Interference.

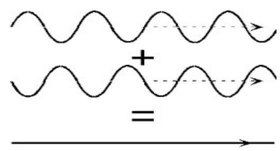
Example: Colours observed on soap bubbles, on oil film formed on water when viewed under sunlight, compact discs.

Conditions for Interference:

1) 1. Constructive interference:If two waves interfere in phase, gives constructive interference and Resultant Intensity is maximum.



2. Destructive interference: If two waves interfere out of phase, gives destructive interference and Resultant Intensity is minimum.

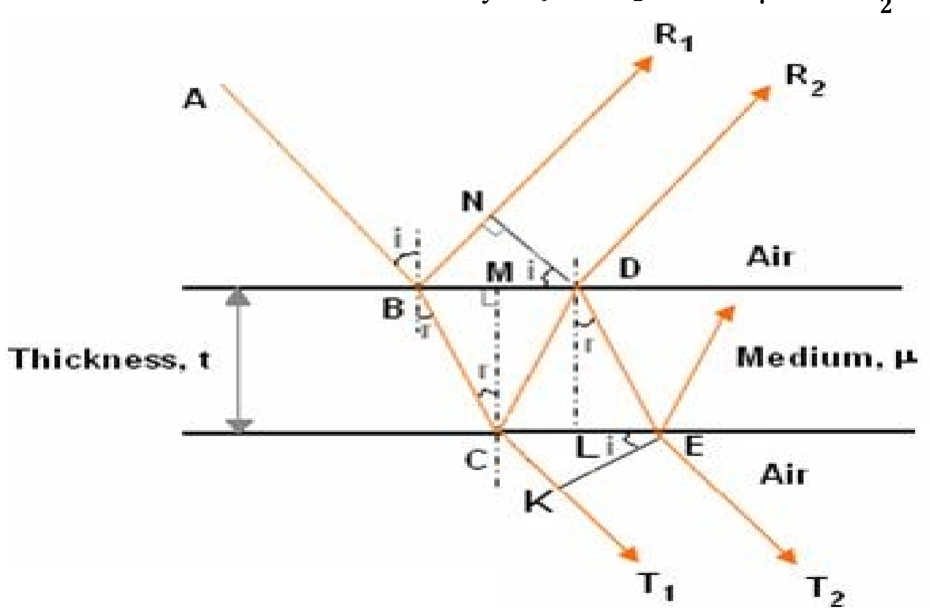


Interference in thin film (by reflected rays)

Thin film: A film is a transparent material with uniform thickness. A film is said to be thin film when thickness in the range of 1μm to 10 μm.

Example: Thin film may be glass, Mica, air enclosed between two transparent sheets, soap bubble.

Path Difference between the two reflected rays R_1 and $R_2$is $\Delta = \mathbf{2\ \mu t\ cosr - \frac{\lambda}{2}}$



Condition for Bright band:

The thin film appears bright, if path difference $2\ \mu t\ \cos r - \frac{\lambda}{2} = n\ \lambda$

Or $\quad 2\ \mu t\ \cos r = (2n+1)\frac{\lambda}{2} \quad$ Where, n = 0,1, 2,…..

Condition for Dark band:

The thin film appears dark, if path difference $2\ \mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

Or $\quad 2\ \mu t \cos r = n\lambda \quad$ where n = 0,1, 2, …

If thickness of the film is extremely small when compared to λ, then $2\mu t\cos r$ can be neglected. Then in equation 1, net path difference is $\frac{\lambda}{2}$, Hence destructive interference occurs and film will appear always dark.

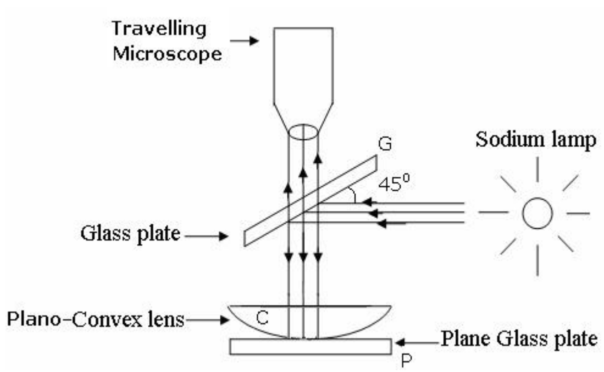
Applications

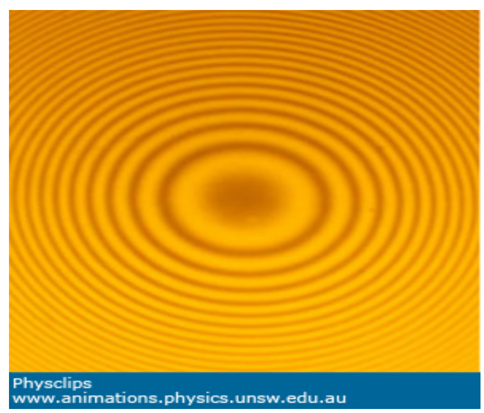
- Anti-reflection coating on glasses and camera lenses
- Optical filters.

Newton's Rings experiment:

Introduction: Interference phenomenon of light waves can be observed by, when a plano convex lens is placed on top of a glass plate, the lens illuminated by a monochromatic light source and circular rings can be observed. These circular rings are called Newton's Rings.

Experimental arrangement:





Path difference = $2t+\frac{\lambda}{2}$

Condition for bright rings (constructive interference):

When, path difference = nλ, constructive interference occurs.

$2t+\frac{\lambda}{2} = n\lambda$ Or $\quad 2t = (2n-1)\frac{\lambda}{2}$

Where, n = 0,1,2,3, …

Condition for dark rings (destructive interference):

When, path difference = $(2n+1)\frac{\lambda}{2}$, destructive interference occurs.

$2t+\frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$ Or $\quad 2t = n\lambda$

Where, n = 0,1,2,3, …..

$\mathbf{r} = \sqrt{\frac{(2n-1)R\lambda}{2}}$ …… is the **radius of bright rings**.

Where, n = 1,2,3,…..

$\mathbf{r_n} = \sqrt{nR\lambda}$ **……** is the **radius of dark rings.**

Wavelength of given source of light can be calculated by the formula,

$$\lambda = \frac{D^2(n+m) - D^2 n}{4mR}$$

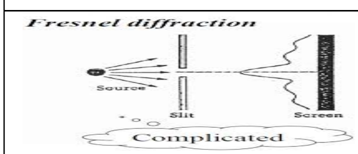
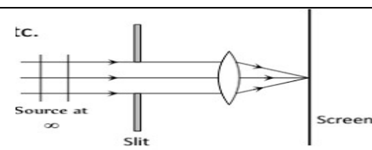
DIFFRACTION

Definition: The bending of light waves around the edges of an obstacle is called diffraction.

Condition for Diffraction: Diffraction phenomenon can be observed when the dimensions of obstacle or aperture is comparable to wavelength.

There are two classes of diffractions.

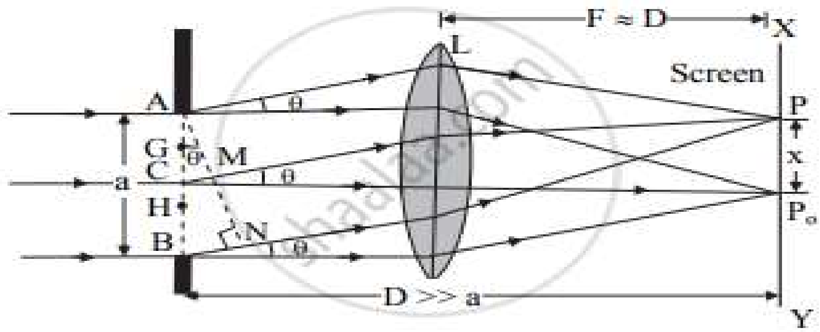
1. Fresnel diffraction
2. Fraunhofer diffraction

Fresnel diffraction	Fraunhofer diffraction
	
Either point source or screen or both kept at finite distance from obstacle	Source and screen are effectively at infinite distance from obstacle.
The wave front under goes diffraction either spherical or cylindrical.	The wave front under goes diffraction is a plane wave front.
Experiment is simple but analysis is difficult.	Experiment is simple but analysis also simple.
No lens are used	Lens are used

Fraunhofer diffraction due to single slit:

Introduction: In Fraunhofer diffraction incident wave front must be plane wave front, hence two lenses are used one is collimating and another is converging.

Experimental arrangement:



Path difference between the rays emitted from points A and B is Δ = BN=**a** sinθ

Where, a = slit width andθ = angle of diffraction.

Condition for Minima:

If path difference $\Delta = \lambda, 2\,\lambda, 3\,\lambda, \ldots\ldots,$

$$a\sin\theta_n = n\,\lambda$$

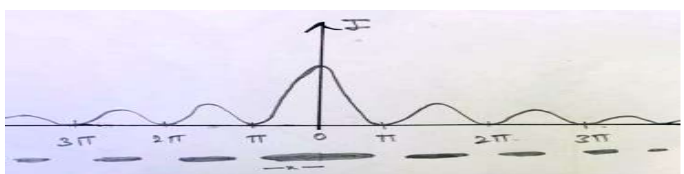
Where, n= 1,2,3,…….and θ_n= corresponding directions of n^{th} minima

Condition for Maxima:

If path difference $\Delta = \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \ldots\ldots (2n+1)\frac{\lambda}{2}$

$$a\sin\theta_n = (2n+1)\frac{\lambda}{2}$$

Where, n= 1,2,3,…….andθ_n= corresponding directions of n^{th} maxima

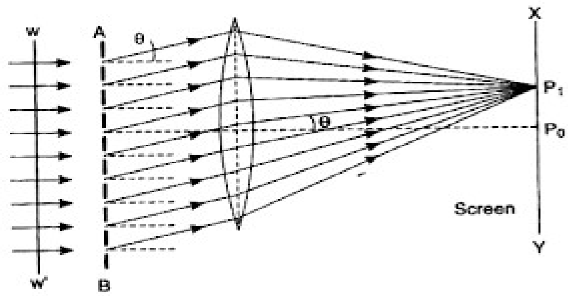


Calculate the slit width:

$\mathbf{a} = \frac{F\,\lambda}{x}$**W**here, a is slit width

Fraunhofer diffraction due to N- slit:

(Diffraction Grating)



Condition for maxima

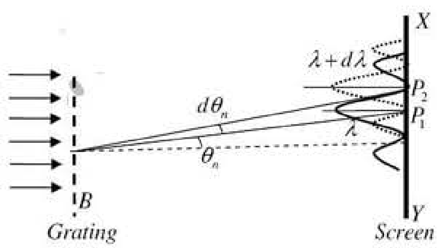
$\sin\theta_m = mN\lambda$

Condition of maximum number of orders possible with grating.

$\mathbf{m} = < \frac{1}{\lambda N}$ Since, m is not a decimal it is an integer, this gives the maximum number of possible orders.

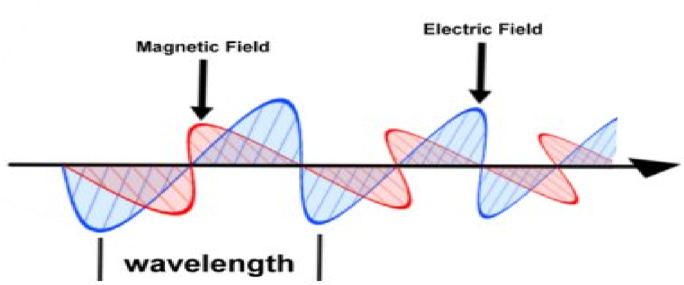
Resolving power of Grating:

The ability of grating to form separate spectral lines of two very close wave lengths is called resolving power of plane diffraction grating.



POLARIZATION

Introduction: The phenomenon of polarization, establishes the transverse nature of light. Light is electromagnetic in nature. It consists of oscillating electric and magnetic fields perpendicular to each other and also to the direction of propagation of the wave.

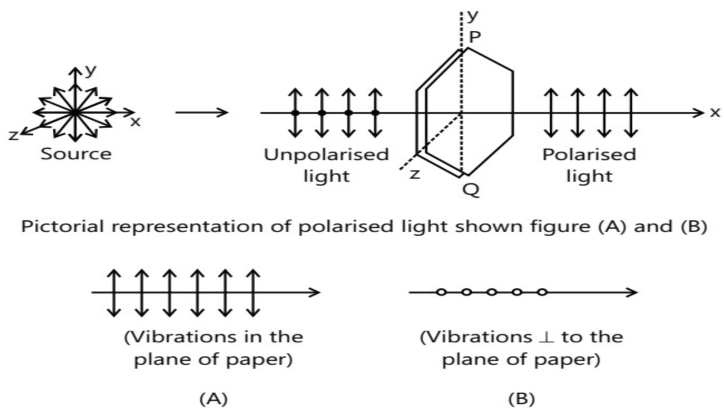


Electromagnetic wave traveling by 3Dlens.com

Example: Waves produced on stretched string are transverse.

- Polarization is a property of certain type of wave that describes the orientation of oscillations.

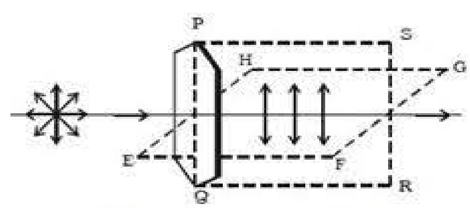
- Ordinary light (bulb, sunlight,candle) or unpolarized light can be represented as



Plane polarized light: The vibrations confined to a single plane(either in the direction along the plane of paper or in the direction perpendicular to the plane of paper) is called plane polarized light.

Plane of Vibration: The plane in which the vibrations occurs is called plane of vibrations (plane PQRS).

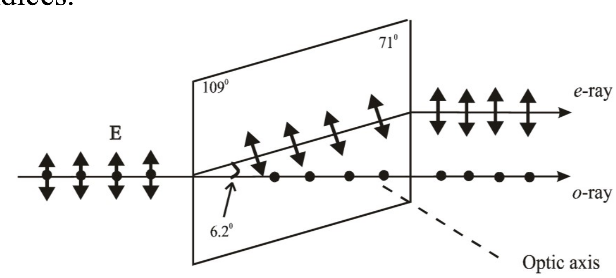
Plane of polarization: The perpendicular to the plane of vibrations is called plane of polarization (plane EFGH).



PQRS = Plane of vibration
EFGH = Plane of polarization

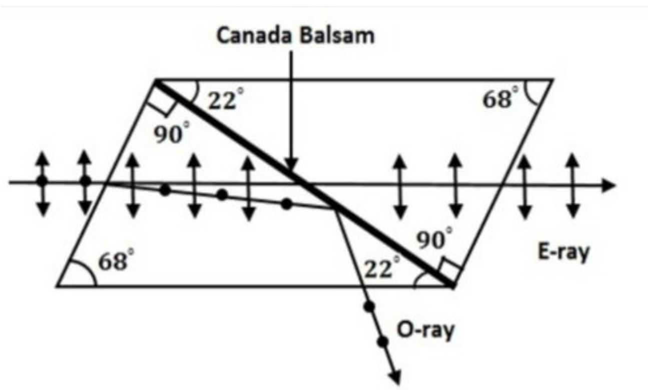
Double Refraction (Birefringence):

When unpolarized light passes through certain anisotropic crystals such as calcite or quartz, velocity of propagation of two components of unpolarized light vary. This means that the material exhibits two different refractive indices.



Nicol's Prism:

Device used to produce plane polarized light calcite crystal is modified such that it eliminates one of the two refracted rays by total internal reflection (TIR).



Applications of Polarization:

- In polarized glasses Eg: Sun glasses
- In 3D movies.
- Polarized sheet is used as polarizer and analyzer.
- Optical activity of Quartz crystal can be measured with the help of polarized light.
- Polarized light is used in the study of structure of nucleic acid.

Formulae

Condition for Bright band in Thin Films is $2\ \mu t \cos r = (2n+1)\frac{\lambda}{2}$

Condition for Dark band in Thin Films is $2\ \mu t \cos r = n\ \lambda$

Wavelength of given source of light can be calculated by the formula in Newton Rings is

$$\lambda = \frac{D^2(n+m) - D^2 n}{4mR}$$

Sample multiple choice questions:

1. Light propagates rectilinearly, due to [ ]
   a) Wave nature
   b) Wavelength
   c) Velocity
   d) Frequency

Answer:

2. The phenomenon of bending of light around the corners of small obstacles is [ ]
   a) Interference
   b) Diffraction
   c) Polarization

d) Reflection

Answer: b

3. Polarization of light proves [ ]

a) quantum nature of light

b) transverse wave nature of light

c) longitudinal wave nature of light

d)corpuscular nature of light

Answer: b

4.Superposition of two waves in the same phase to produce maximum intensity is [ ]

a) constructive interference

b) destructive interference

c) coherence

d) none of these

Answer: a

5.The phase difference between two points on the same wavefront is [ ]

a) two

b) one

c) zero

d)none of these

Answer: c

Sample problems:

1. Two interfering beams have amplitude ratios of 2:1, calculate the intensity ratio of maxima to minima in the interference fringes.

Solution: $\frac{A1}{A2} = \frac{2}{1}$

We know that, $I = A^2$

Amplitude for bright fringes $A = A_1+A_2$

$=2+1 = 3$

Therefore $I_1= 9$

and Amplitude for dark fringes $A = A_1-A_2$

$=2-1 = 1$

Therefore $I_2= 1$

$\frac{Imaxi}{Iminima} = \frac{I_1}{I_2} = \frac{9}{I}$

2. In case of Newton's rings experiment wavelength of light source is 5400 Å and the radius of the 8^{th} dark ring is 3.6×10^{-3} m. Find the radius of curvature of lens.

Solution: Given, $\lambda = 5400$ Å $= 5400 \times 10^{-10}$ m

$r_8 = 3.6 \times 10^{-3}$ m

$n = 8$

$r_n= \sqrt{(nR\lambda)}$

$R = r_n^2 / n\lambda$

$$R = \frac{(3.6 \times 10^{-3})2}{(8 \times 5400 \times 10^{-10})} = 3m$$

3.The first diffraction minima due to a single slit is at θ = 30^0 for a light of wavelength 5000 Å, find the width of slit.

Solution: Given, θ = 30^0, n = 1, λ = 5000 Å = 5000 x 10^{-10} m

$$\text{Slit width (a)} = \frac{n\lambda}{\sin\theta}$$

$$a = \frac{1 \text{x} 5000 \text{x} 10^{-1}}{\sin 30^0} = 1 \text{x} 10^{-6} \text{ m}$$

UNIT-II: Free electron theory and Introduction to Quantum Mechanics.

Introduction to electron theory

The electron theory aims to explain the structure and bulk properties of solids through their electronic structure. The electron theory is applicable to all solids i.e., both metals and non-metals. It explains the electrical, thermal and magnetic properties of solids etc. The theory has been developed in three mainstages.

The classical free electron theory

The classical free electron theory was introduced by P. Drude in 1900 and developed by Lorentz in 1909. According to this theory, the metals containing the free electrons and obey the laws of classicalmechanics. This theory was developedto explain some of the thermal and electrical properties of metals.

The classical free electron theory is based on the following postulates or assumptions

1. The valence electrons of atoms are free to move about the whole volume of the metal, like the molecules of a perfect gas in acontainer.
2. The free electrons move in random direction and collide with both positive ions fixed to the lattice and the collisions are elastic in nature i.e., there is no loss of energy.
3. The electron velocities in a metal obey classical Maxwell-Boltzmann distribution of velocities and the RMS velocity of these electrons is given by $\bar{C} = \sqrt{\frac{3KT}{M}}$.
4. Mutual repulsion between electrons is ignored and hence potential energy is taken as zero. Therefore, the total energy of the electron is equal to its kinetic energy.
5. As the potential experienced by the free electrons inside the metal is zero and at the walls, infinite, the electrons are confined inside the metal specimen.

Merits of Classical Free Electron theory:

- It verifies Ohm's law.
- It explains the electrical and thermal conductivities of metals.
- It could explain that the ratio of thermal conductivity to electrical conductivity of a metal is directly proportional to absolute temperature (Weidmann-Franz law).
- It explains the optical properties of metals.

Drawbacks:

- The phenomena such as photoelectric effect, Compton effect and the spectral distribution of blackbody ration could not be explained.
- The theoretically predicted value of specific heat of a metal does not agree with the experimentally obtained value.
- Electrical conductivity of semiconductors could not be explained.
- Ferromagnetism could not be explained.

Introduction to quantum physics:

Quantum mechanics explains the behavior of matter and its interactions with energy on the scale of atoms and subatomic particles.

Black body radiation and Planck's law

When a blackbody is heated, it radiates electromagnetic waves of all possible wavelengths. The distribution of total radiated energy among different wavelengths is called spectral distribution of energy.

The energy distribution among various wavelengths at different temperatures of the black body is as shown in the figure.

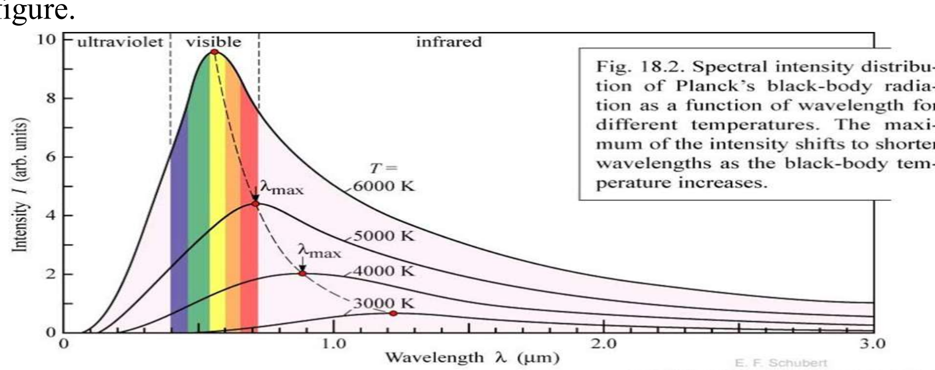


Fig. 18.2. Spectral intensity distribution of Planck's black-body radiation as a function of wavelength for different temperatures. The maximum of the intensity shifts to shorter wavelengths as the black-body temperature increases.

Max Planck derived the following relation for spectral distribution of black body radiation

.

$$E_\lambda d\lambda = \frac{8\Pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda KT}}-1}d\lambda$$

This relation was found to be valid for all wavelengths of black body radiation spectrum.

Wave particle duality and de-Broglie hypothesis:

Light or radiation has dual nature i.e., particle nature (Ex: Compton Effect and photo electric effect) and wave nature (Example: interference and diffraction). This idea of dual nature of radiation was extended to material particle by the scientist de-Broglie in 1924 and put forward the hypothesis.

de-Broglie hypothesis:

Any moving particle has got a wave associated with it. Such waves are named as matter waves or de-Broglie matter waves.

Expression for the wavelength of matter wave:

de-Broglie derived the expression for the wavelength of matter wave

$$\boldsymbol{\lambda = \frac{h}{mv}}$$

Heisenberg Uncertainty Principle

Heisenberg stated that the simultaneous determination of exact position and momentum of a moving particle is impossible.

If Δx is error in the measuremen to position of the particle along X-axis, Δp is Error in the measurement of momentum

Then **(Δx).(Δp)≥Ђ/2**

Where ђ=h/2π and 'h'is Plank's constant

The above relation represents the uncertainty involved in measurement of both the position and momentum of the particle.

The uncertainty relation between energy and time is given below.

If the time during which a system occupies a certain state is not greater than Δt, then the energy of the state can not be known within ΔE,

i e **(Δ E) (Δ t) ≥ ђ / 2**

Schrodinger's Time Independent Wave Equation:

Schrödinger, in 1926, developed wave equation for the matter wave associated with moving particle of mass "m", "e" and "V" are the total energy and potential energies of the particle. $\nabla^2\Psi + \frac{\{2m(E-V)\}}{ђ^2}\Psi = 0$

This is the Schrödinger Time Independent Wave Equation.

Physical Significance of Wave Function:

Max Born in 1926 gave a satisfactory interpretation of the wave function Ψ associated with a moving particle. He postulated that the square of the magnitude of the wave function $|\Psi|^2$ (or $\Psi\Psi*$, $\Psi*$ is complex conjugate of Ψ), evaluate data particular point represents the probability of finding the particle at the point.

$|\Psi|^2$is called the probability density and Ψ is the probability amplitude. Thus, the probability of the particle within an element volume dv is $|\Psi|^2$dv. Since the particle is certainly somewhere, the integral at$|\Psi|^2$ dv over all space must be unity i.e.

$$\iiint |\Psi|^2 \, dv = 1$$

In one dimension, say along x axis

$$\int |\Psi|^2 \, dx = 1$$

A wave function that obeys the above equations is said to be normalized.

Application of Schrodinger's wave equation: Particlein one Dimensional Potential Box:

Consider a particle of mass 'm' moving along the x-axis between x=0 and x=a inside a one-dimensional box of infinite height and width

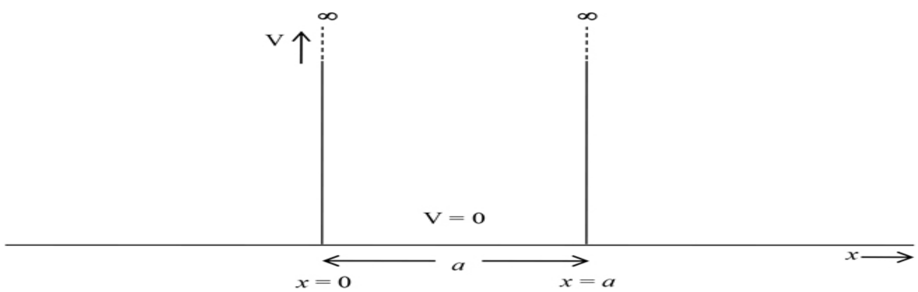


Fig. Particle in a potential well of infinite height.

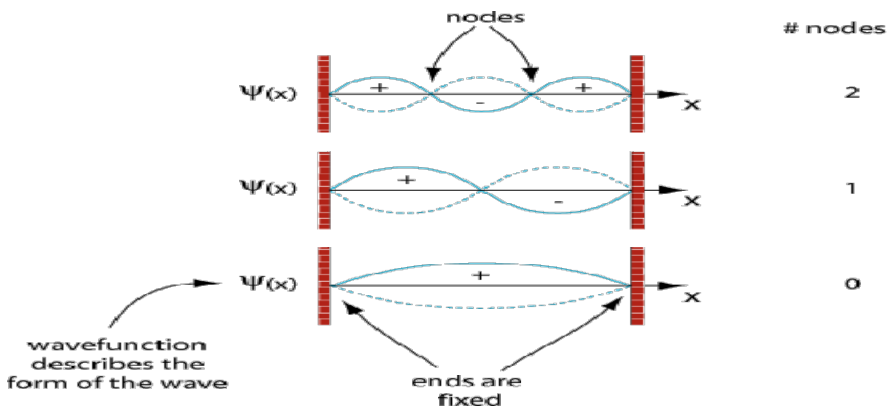
The energy of the particle is quantized and is given by

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2} = \frac{n^2 h^2}{8ma^2} \text{-------}(1)$$

and the wave function is given by

$$\Psi(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}\right) x \text{-------}(2)$$

Eqn. (1) represents an energy level for each value of n and the corresponding wave function is given in eqn. (2). Therefore, the particle in the box can have discrete values of energies which arequantized. Thenormalizedwavefunctions$\Psi 1, \Psi 2, \Psi 3$ given by eqn (2) are plotted.



Important Formulae:

1. Spectral distribution of black body radiation,

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda KT}} - 1} d\lambda$$

2. Wavelength of matter wave associated with a moving particle is,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

3. For an electron of mass "m" accelerated by a voltage "V", the wavelength associated with matter wave is

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{12.26}{\sqrt{V}}\square$$

4. If "Δx" is error in the measurement of position of the particle along X-axis, "Δp_x" is the error in the measurement of momentum, then

$$(\Delta x)(\Delta p_x) \geq h/4\pi$$

5. The energy of the particle moving in a one-dimensional potential box is quantized and is given by

$$E_n = \frac{n^2 h^2}{8ma^2}$$

Constants:

1. $\pi = 3.142$
2. $h = 6.626\times10^{-34}$ J·Hz^{-1}
3. Mass of the electron(m)= 9.1 x 10^{-31} Kg.
4. Charge of an electron(e)= 1.602 x 10^{-19} coulombs.
5. 1 eV (electron volt) = 1.6 x 10^{-19} Joules.

Sample multiple choice questions:

1. Which of the following was explained by classical free electron theory? [ ]
   A) Photoelectric effect
   B) Compton Effect
   C) Spectral distribution of black body radiation
   D) Ohms law

   Answer: d
2. According to classical free electron theory, the electrons moving inside a metal behaves as [ ]
   A) Photons
   B) Gas molecules
   C) Waves
   D) Electromagnetic radiation
3. The velocity of matter wave is
   A) equal to the velocity of particle
   B) Equal to the velocity of light
   C) Less than the velocity of light
   D) More than the velocity of light

3. The wavefunction is said to be normalized when the quantity $\iiint |\Psi|^2$ dv is equal to [ ]
   A) 0
   B) 1
   C) Infinity
   D) Less than 1

Sample problems:

1. Calculate the wavelength associated with a moving electron of kinetic energy equal to 15eV.

   Solution: $\lambda = \frac{h}{m\mathrm{v}} = \frac{h}{\sqrt{2mE}} = \frac{6.626x10^{-34}}{\sqrt{2x9.1x10^{-31}x15x1.6x10^{-19}}} = 0.316\ x\ 10^{-9}\ meters$

2. Calculate the de-Broglie wavelength associated with an electron accelerated by 100 volts.

   Solution: $\lambda = \frac{12.26}{\sqrt{V}} = \frac{12.26}{\sqrt{100}} = 1.226\ \text{Å}$

3. Calculate the minimum uncertainty in momentum of an electron confined in a 1 nm box.

   Solution: According to uncertainty principle, $(\Delta x)(\Delta p_x) \geq h/4\pi$

   $\Delta x = 1\text{nm} = 1 \text{ x } 10^{-9}$ meter

   $(\Delta p_x) \geq (h/4\pi)/(\Delta x) =$

   $(\Delta px) \geq \frac{\frac{h}{4\pi}}{(\Delta x)} = 1.536 \text{ x } 10^{-25}$ Kg.m/sec

4. Find the lowest energy of a neutron confined to a nucleus of size 10^{-14} meter. Given mass of neutron is 1.6×10^{-27} Kg.

Solution: $E_n = \frac{n^2h^2}{8ma^2} = \frac{1^2(6.626 \, x \, 10^{-34})^2}{8 \, x \, (1.6 \, x \, 10^{-27}) \, x \, (10^{-14})^2} = 2.1 \, x \, 10^6 \, eV$

UNIT-III: Band Theoryof Solids& Semiconductors.

Kronig – Penny Model:

According to Kronig-penny theory the electrons move in a periodic potential produced by the positive ion cores. The potential energy of the electron varies periodically with the periodicity of positive ion core and potential energy of electron is zero at the positive ion sites (near the nucleus of the positive ion) and maximum between the two ions. The two positive ions are separated by the interatomic distance ‘a’ as shown in fig.

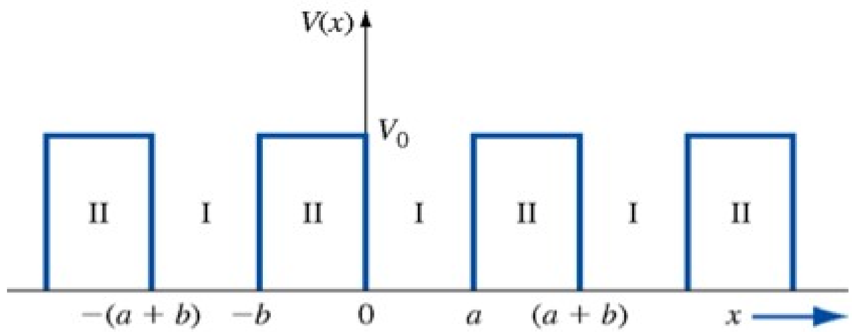


Fig. One dimensional Periodic potential in a crystal

Region-I $0<x<a$

The 1-dimensional Schrodinger Time Independent wave equation in this region is,

$\frac{d^2\Psi}{dx^2} + \alpha^2 \Psi = 0$ ----------------------(1) $\quad [\frac{2m}{\hbar^2} E = \alpha^2]$

Region-II $-b<x<0$

The 1-dimensional Schrodinger Time Independent wave equation in this region is,

$\frac{d^2\Psi}{dx^2} - \beta^2 \Psi = 0$ --------------------- (2) $\quad [\frac{2m}{\hbar^2} (V_0 - E) = \beta^2]$

These equations can be solved with the help of block theorem. The final solution of eq (1& 2) is given in the form of the following condition.

$$\boxed{\frac{P}{\alpha a}\,\mathrm{Sin}\alpha a + \mathrm{Cos}\alpha a = \mathrm{Cos}\,Ka} \text{ ----------- (3)}$$

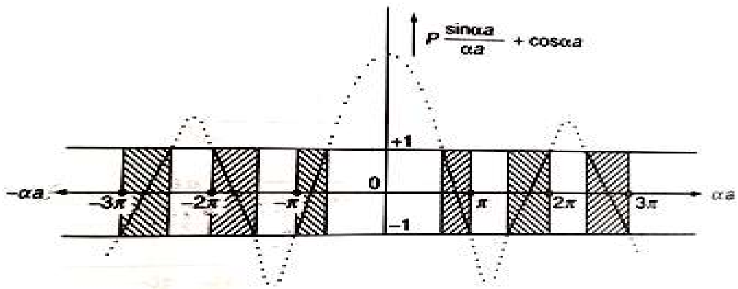
It means that the solutions for the wave functions exist only if equation (3) is satisfied

Where, P = Scattering power of the potential barrier

$$P = \frac{maV_0\omega}{\hbar^2} \quad ; \; V_0\omega = \text{Barrier strength.}$$

Kronig – Penny Graph:

A plot of the function "(Psinαa/αa + cosαa)" Versus "αa" is shown in the following figure. Coska imposes a limitation on the values of the left side function.i.e., from +1to -1.



Conclusions:

(i). The energy spectrum has a number of allowed energy and forbidden energy bonds

(ii). As the value of ' ' increases the width of the allowed ENERGY band increases and width of the forbidden band decreases

(iii). As scattering power (P) increases, the strength of potential barrier increases and the width of the allowed energy band decreases.

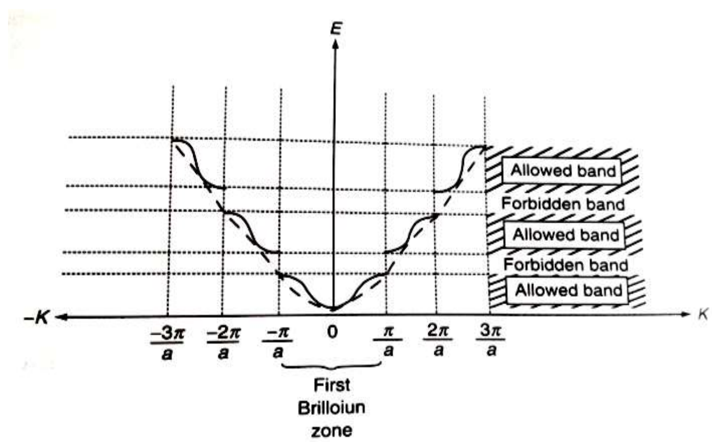
(iv) If P→ ∞, the allowed energy band becomes infinitely narrow and the energy spectrumisa line spectrum

(v) When P→ 0 then electrons move freely over the lattice without energy potential. Hence all the energies are allowed to the electrons and produce the continuous spectrum

Brilliouin Zones (or) E-K Diagram:

Brillouin zones are the boundaries that are marked by the values of waves vector K, in which the electrons can have allowed energy values. These represent the allowed values of K of the electrons in 1D, 2D & 3D.

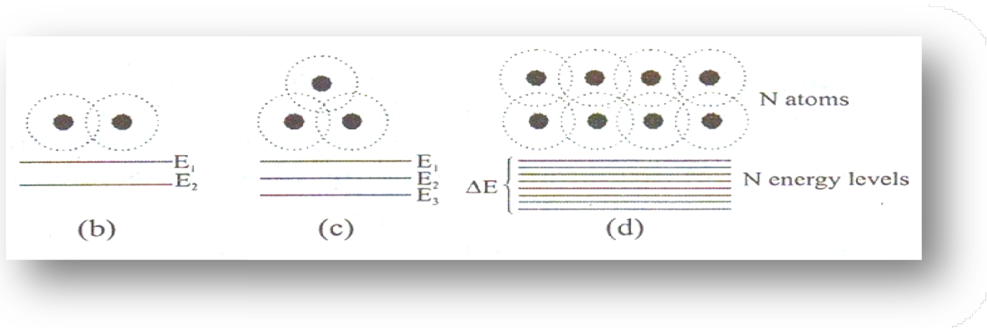
Graph: A graph is drawn between the total energy (E) and the wave vector k, for various values of k. i.e, $K = \frac{n\pi}{a}$; n = ±1, ±2, ±3,…..



Origin of Energy band formation in Solids:

Band theory of solids explains the formation of energy bands and determines whether a solid is a conductor, insulator or a semiconductor.

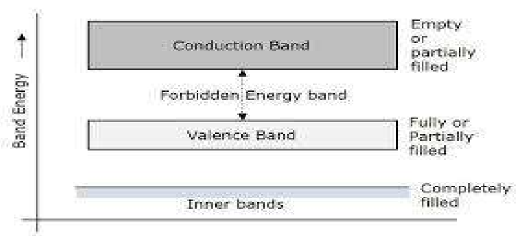
Energy Bands



If 'N' no. of atoms of equal energy levels are brought closer to form a solid, then forms a closely spaced continuous energy levels, called asenergy bands.

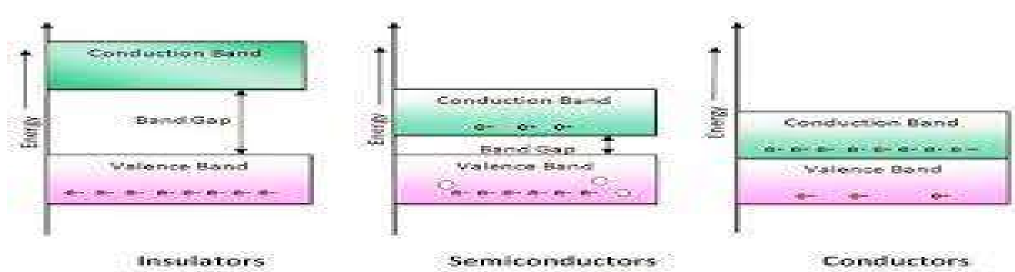
Energy Band Diagram

Energy band can also be defined as, the range of energies possessed by an electron in a solid. It consists of (i) Inner filled bands. (ii) Valence bands(iii) Conduction bands(iv) Forbidden band (or) energy gap.



Classification of Solids:

Based on band theory, arrangement of electrons and forbidden energy gap, the solid materials are classified into 3 types. They are Conductors, insulators and Semi conductors



Effective mass of the electron:

When an electron in a period potential is accelerated by an electric field (or) magnetic field, then the mass of the electron is called effective mass (m*).

$$m^* = \left[\frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} \right]$$

Density of states:

The number of energy states per unit volume is known as **density of energy states**.

$$\text{Density of energy states } g(E)\, dE = \frac{4\pi}{h^3}(2m)^{3/2}(E)^{1/2}\, dE$$

Fermi Dirac Distribution:

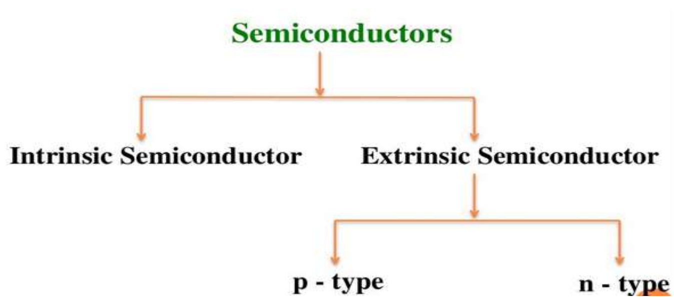
$$F(E_i) = \frac{1}{\exp\left(\frac{E_i - E_f}{kT}\right) + 1}$$

Where $F(E_i)$ is called Fermi function which is defined as the probability of electron occupation in the given energy state (E_i) at thermal equilibrium. E_f is Fermi energy, E_i is energy of i^{th} state and k is Boltzmann constant.

Semiconductors:

The materials which have electrical conductivity lies between conductors and insulators are called semiconductors. In these materials electrons and holes both are responsible for electrical conduction.

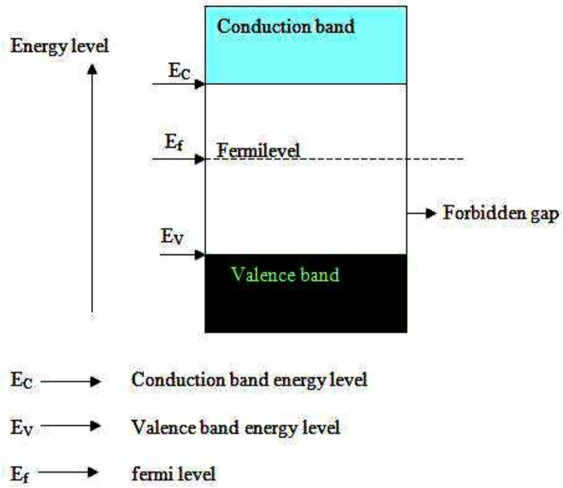
Semiconductors mainly classified as follows



Intrinsic semiconductors:

A pure semiconductor which is not doped is termed as intrinsic semiconductor.

In an intrinsic semiconductor, for every electron freed from the bond, there will be one hole created. It means that, the no. of conduction electrons is equal to the no. of holes at any given temperature.



In an intrinsic semiconductor, electrons and holes are equal in numbers.

Thus $n = p = n_i$

Carrier concentration in intrinsic semiconductors:

Density of Electrons:

$$n = 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2} \exp\left(\frac{E_f - E_c}{KT}\right)$$

Density of holes**:**

$$P = 2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2} exp\left[\frac{E_V - E_F}{KT}\right]$$

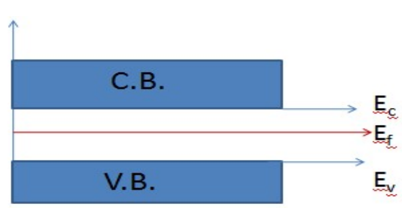
In intrinsic semiconductors, $n = p = n_i$ is called intrinsic carrier concentration

$$ni = \sqrt{n_i^2} = 2\left(\frac{2\pi KT}{h^2}\right)^{3/2}\left(m_c^* m_h^*\right)^{3/4} exp\left(\frac{-Eg}{2KT}\right)$$

Fermi level in intrinsic semiconductors:

The Fermi energy level for intrinsic semiconductor is lies at the middle of the energy gap.

$$\boldsymbol{E_f} = \left[\frac{\boldsymbol{E_c} + \boldsymbol{E_v}}{\mathbf{2}}\right]$$



Electrical conductivity in intrinsic semiconductors:

In case of semiconductors, electrical conductivity (σ) is directly proportional to the temperature

(T)

$$\ln \sigma_i = \ln A - Eg/2KT$$

Extrinsic Semiconductors:

Intrinsic Semiconductors are rarely used in semiconductor devices as their conductivity is not sufficiently high. The electrical conductivity is extremely sensitive to certain types of impurity. It is the ability to modify electrical characteristics of the material by adding chosen impurities that make extrinsic semiconductors important and interesting.

Extrinsic or doped semiconductors are classified into two main types according to the type of charge carries that predominate. They are the n-type and the p-type.

N-Type Semiconductors:

Doping with a pentavalent impurity like phosphorous, arsenic or antimony the semiconductor becomes rich in conduction electrons. It is called n-type the bond structure of an n-type semiconductor is shown in Fig.

P-Type Semiconductors:

p-type semiconductors have holes as majority charge carries. They are produced by doping an intrinsic semiconductor with trivalent impurities (e.g. boron, aluminum, gallium, or indium).

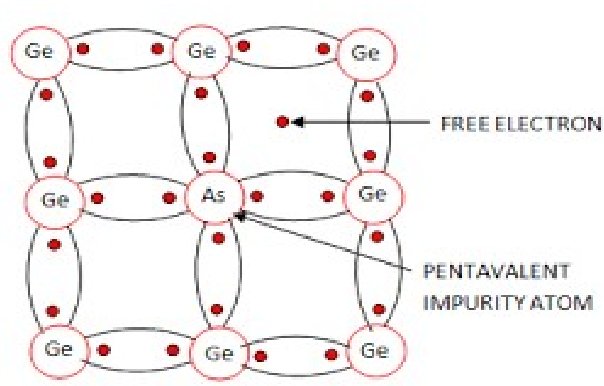


Fig. n-type semiconductor

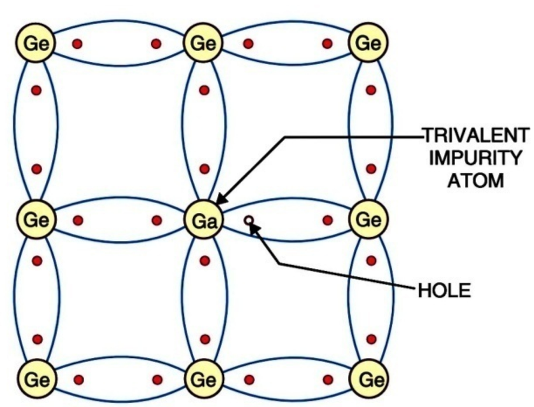


Fig. p-type semiconductor

Sample multiple choice questions:

1. In Kronig-Penney model, as the scattering power of the potential barrier, P → ∞ then theallowed energy bands:

   A)Reduce to single energy levels

   B) Reduce to smaller bands

   C) Increase to bigger bands

D) None

Answer: A

2. The process of adding impurities to a pure semiconductor is called

A) Mixing

B) Doping

C) Diffusing

D) None of the above

Answer: B

3. When a pentavalent impurity is added to a pure semiconductor, it becomes ……..

A) an insulator

B) an intrinsic semiconductor

C) p-type semiconductor

D) *n*-type semiconductor

Answer: D

4.A semiconductor has ……. temperature coefficient of resistance.

A) positive

B) zero

C) negative

D) none of the above

Answer: C

5.In an intrinsic semiconductor, the number of free electrons …….

A) equals the number of holes

B) is greater than the number of holes

C) is less than the number of holes

D) none of the above

Answer: A

Sample Problems:

1. The Hall coefficient of a semiconductor is $-6.85 \times 10^{-5} m^3/c$ then calculate the density of charge carriers

Sol: $\mathbf{R_H = 1/ne}$

$n = 1/R_H e$

$= 1/-6.85 \times 10^{-5} \times 1.6 \times 10^{-19} = -6.85 \times 10^{-5}\, m^3$

2. The intrinsic carrier concentration in Ge is $2.37 \times 10^{19}/m^3$. if the electron and hole mobilities are 0.38 and $0.18 m^2V^{-1}s^{-1}$ respectively, calculate the conductivity.

Sol: $\boldsymbol{\sigma = n_i e(\mu_e + \mu_h)}$

$= 2.37 \times 10^{19} \times 1.6 \times 10^{-19}(0.38 + 0.18) = 2.1235\Omega^{-1}m^{-1}$

<u>**Unit-IV: Semiconductor Devices**</u>

Direct and indirect band gap semiconductors:

In a **direct band gap semiconductor**, minimum-energy state in the conduction band (CB -minima) and the maximum-energy state in the valence band (VB-maxima) occur at the same value of momentum (k) in the Brillouin zone.

If the k-vectors (Propagation constant or wave vector) are the different for minimum-energy state in the conduction band (CB -minima) and the maximum-energy state in the valence band (VB-maxima) then, it is called a "**Indirect band gap semiconductor**". Best examples of indirect band gap semiconductors are Si and Ge.

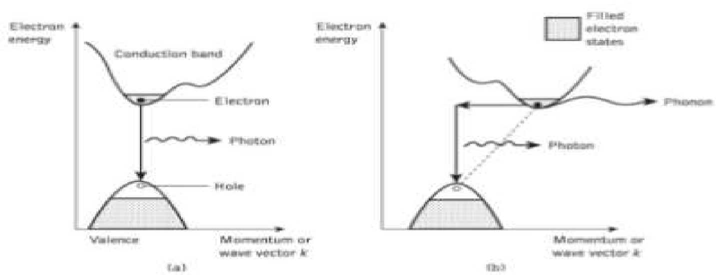
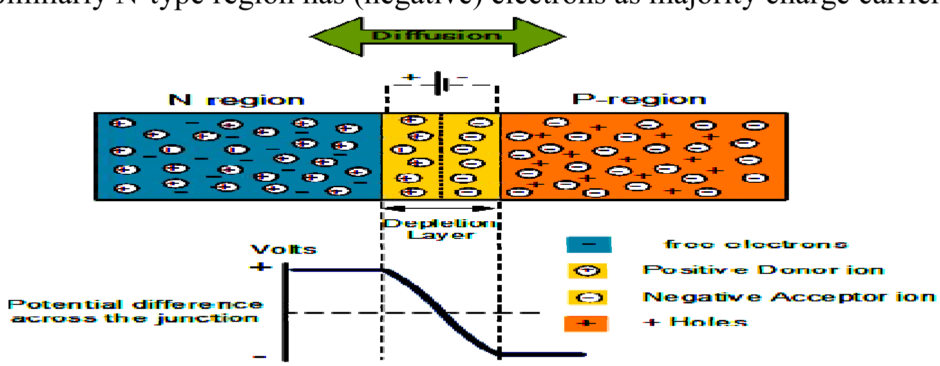


Fig. Energy-momentum (E-k) diagrams of (a) direct band gap semiconductor, (b) indirect band gap semiconductor

Formation of P-N Junction diode:

Junction diode is formed by placing of P type crystal in contact with n-type crystal and subjecting to high pressure so that it becomes a single piece. The surface of contact of P and N-type crystals is called junction. A P-N Junction is shown in fig. The P type region has (positive) holes as majority carriers. Similarly N-type region has (negative) electrons as majority charge carriers.



The region on either side of the junction which becomes depleted (free) of the mobile charge carries is called the depletion region. The thickness of this region is of the order of 10^{-6}m. The potential difference across the depletion region is called the potential barriers.

Energy diagram of PN diode:

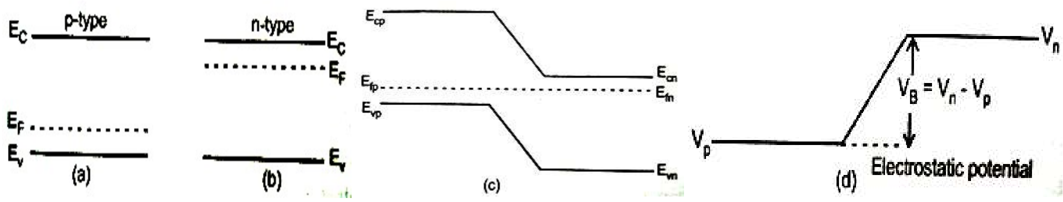


Fig: (a) and (b) Energy level diagram of p-type semiconductors respectively (c)Energy level diagram of PN junction and (d) formation of potential barrier across the junction

V-I Characteristics of a junction diode:

Graphs drawn between bias voltage and circuit current of a junction diode are called characteristics of the diode.

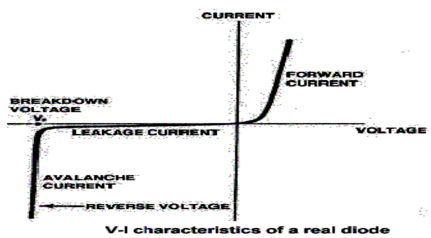
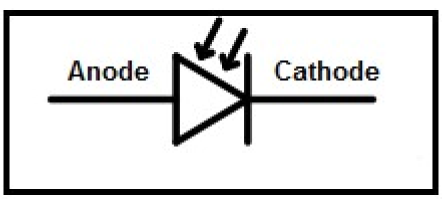


Fig: V-I Characteristics of a PN junction

Photo diode:

A photodiode is a semiconductor p-n junction device that converts light into an electrical current. The current is generated when photons are absorbed in the photodiode.



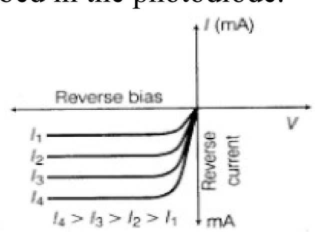


Fig: Symbol of Photo diode **Fig**: V-I characteristics of photodiode:

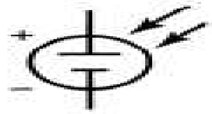
The function of the photo diode junction is the opposite an LED function. In an LED, photons are released in response to the current flow through the junction. In a photo diode, the photons are absorbed resulting in the generation of the carriers that manifest as current through the junction.

Applications:

Photodiodes are used in safety electronics, fire and smoke detectors, medical instruments that analyze samples like blood gas monitors, pulse oximeters, consumer electronics devices such as compact disc players, remote control devices, exposure meters in camera, photo sensors, solar cell panels.

Solar Cell:

A special p–n junction diode which converts sun light into electrical energy is known as ***solar cell or photo voltaic device.***



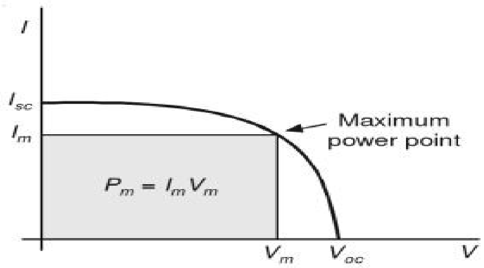


Fig: Symbol of solar cell **Fig**: I – V Characteristics

Three steps are involved in working of a solar cell, when light falls on it.

1. Generation of charge carriers (electron-hole pair)
2. Separation of charge carriers
3. Collection of charge carriers

Solar Cell Efficiency:

The efficiency is the most commonly used parameter to represent the performance of a solar cell. Efficiency is defined as the ratio of energy output from the solar cell to input energy from the sun. The efficiency of solar cell depends upon climate and latitude

Applications:

Solar cells are used in solar panels, power calculators, watches, irrigation systems, satellites and space vehicles as most important long duration power supply.

Light Emitting Diode (LED):

LED is a p–n junction device which emits light when forward biased, by a phenomenon called electro luminescence in the in the UV, Visible or IR regions of the electromagnetic spectrum. The quanta of light energy released are approximately proportional to the band gap of the semiconductors.

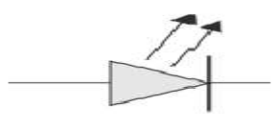


Fig: Symbol of LED **Fig**: I – V Characteristics

The wavelength of emitted photon is

$$\lambda = \frac{hc}{E_g}$$

Where, $h = plancks\ constant\ (6.625 \times 10^{-34}\ JS)$

$$\lambda = \frac{hc}{E_g} = \frac{6.625\times10^{-34}\times3\times10^{8}\times10^{10}}{1.602\times10^{-19}E_g} = \frac{12400}{E_g}\ A^{\circ}$$

Where, E_g is the energy gap in electron volt.

Therefore, color of the emitted light depends on the type of material used.

Advantages of LEDs in electronic Display:

- Output is bright and the intensity can be controlled easily by varying current.
- They can be operated over a wide range of temperature 0 to 700^{o}C
- Very fast response time in the order of ns and hence very useful as source for optical Communication.
- Available in different colors.
- Very small in size and hence can be closely packed for high density display.
- As long life (about 105 hours) and high degree reliability.
- Very rugged and hence suitable for any environment.

Semiconductor laser:

The semiconductor laser is also called diode laser. Diode lasers are always operated in forward bias. If p and n type materials are prepared from the same material then the p-n junction, is called as Homo junction semiconductor laser source. If p and n type materials are prepared from different materials then they are called as Hetero junction semiconductor laser source.

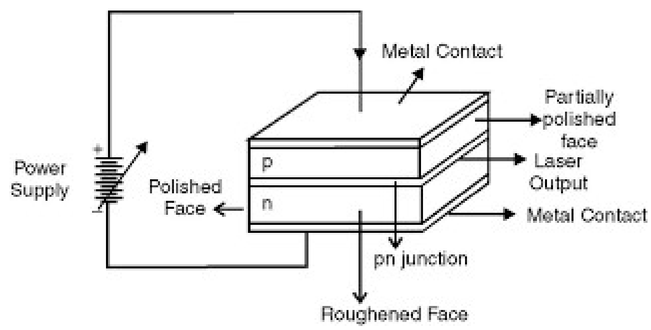


Fig: Semiconductor Laser

The connections in this p-n junction circuit are called forward bias. The electrons are minority charge carriers in p-side and holes are the minority charge carriers in n-side. The continuous injection of charge carriers creates the population inversion of minority carriers is n and p side respectively. The excess minority charge carries diffuse away from the region recombining with majority carriers of the n and p type material, resulting in the release of photons. Further, the emitted photons increase the recombination of injected electrons from the n-region and holes in p-region by inducing more recombinations thus the stimulated emission takes place more effectively at the the junction. In the case of GaAs homo junction which has an energy gap of 1.44eV gives a laser beam of wavelength around 8600A^{0}

Hall Effect:

When a magnetic field is applied perpendicular to a current carrying conductor or semiconductor, a voltage (electric field) is developed across the specimen in a direction perpendicular to both the current and magnetic field. This phenomenon is called **Hall Effect** and generated voltage is called as the ‘Hall voltage’.

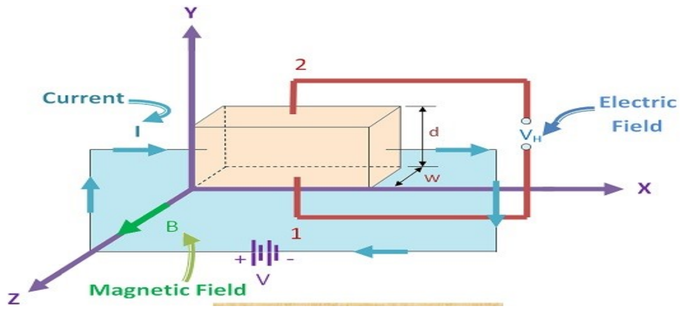


Fig: Hall Effect

Hall Coefficient:

The Hall field E_H, for a given material depends on the current density J, and the applied field B

i.e., $E_H \propto JB$

$E_H = R_H JB$

Where R_H is called the Hall Coefficient

Applications:

The Hall coefficient is useful for the determination of the concentration, mobility of the charge carriers and the type of semiconductors. This effect is also useful for the determination of the magnetic field in most of the magnetometers.

Sample multiple choice questions:

1. A photodiode is normally _________bias. [ ]

A) Forward bias
B) Reverse bias
C) Forward bias than Reverse bias
D) No biasing required

Answer: B

2. Which of the following material can be used to produce infrared LED? [ ]

A) Si
B) CdS
C) GaAs
D) PbS

Answer: C

3. For n-type semiconductor the Hall coefficient is related to carrier concentration n by [ ]

A) R_H= -ne
B) R_H= n/e
C) R_H= e/m
D) R_H= -1/ne

Answer: D

4. A solar cell is a ___________ [ ]

A) P-type semiconductor

B) N-type semiconductor

C) Intrinsic semiconductor

D) P-N Junction

Answer: D

5. Stimulated emission by recombination of injected carriers is encouraged in ______ [ ]

A) Semiconductor diode laser

B) He-Ne laser

C) Ruby laser

D) Co_2laser

Answer: A

Sample Problems:

1. If RH of a specimen is $3.66x10^{-4}m^3/c$. Its resistivity is $8.93x\ 10^{-3}$ Ωm. Find out the carrier concentration 'n'

Sol: $R_H = -1/ne$

$n = -1/R_H\ e = 1/3.66x10^{-4}x1.6x\ 10^{-19}$

$n = 1.708x\ 10^{22}/m^3$

2. Calculate the wavelength of radiation emitted by a LED made up of GaAs semiconductor with band gap 1.43eV.

Sol: $\lambda = hc/Eg$

$= 6.625x10^{-34}x3x10^8/1.43x1.6x10^{-19}$

$= 0.868\mu m$

UNIT- V Fiber Optics

Introduction

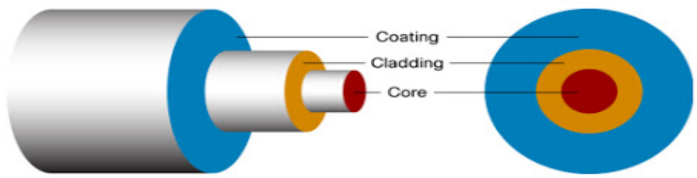
An opticalfiber is a flexible, transparent cable made by drawing glass (silica) or plastic to a cylindrical wire of diameter slightly thicker than that of a human hair.

Fibers are used instead of metal wires because signals travel along them with less loss

Fibers are immune to electromagnetic interference, a problem from which metal wires suffer.

Components of an optical fiber

A typical optical fiber comprises three main co-axial sections: Core, Cladding and outer Jacket/protective buffer coating.



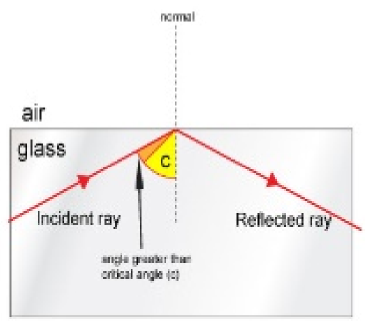
Core: The innermost cylindrical region which carries light. It is the denser medium and is made up glass/ plastic.

Cladding: The middle layer, which serves to confine the light to core. It is the rarer medium as its refractive index is slightly less than that of core.

Outer jacket/ Protective buffer coating: The outermost layer which protects the fiber from physical damage and environmental effects.

Principle of optical fiber: Optical fibers work on the principle of total internal reflection.

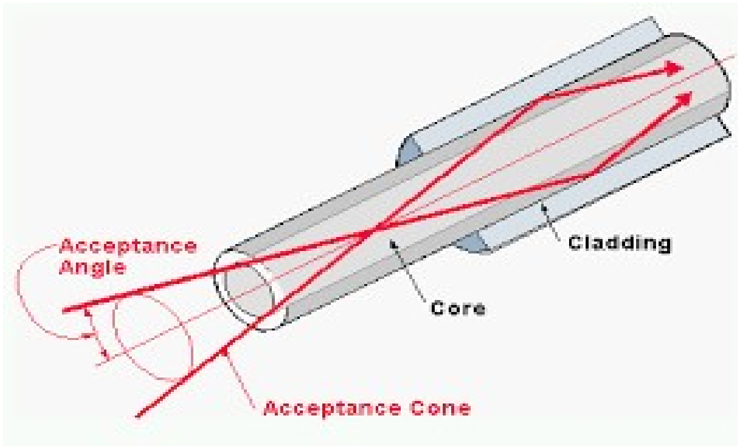
For total internal reflection



- The light must travel from a denser medium to a rarer medium.
- Angle of incidence should be greater than the critical angle.

Acceptance angle and Numerical aperture

The maximum possible launching angle with the axis of the fiber up to which a light ray accepted into the core of the fiber is called ***acceptance angle***. By rotating the acceptance angle about the core axis, a cone will be appeared and is called **acceptance cone**. The light rays that enter the fiber beyond acceptance cone refracts into cladding.



The sine of the acceptance angle is called **numericalaperture**. So it is a measure of light collecting capacity of given optical fiber.

Numerical aperture = $\sin\theta_{max} = \sqrt{n_1^2 - n_2^2} = n_1\sqrt{2\Delta}$

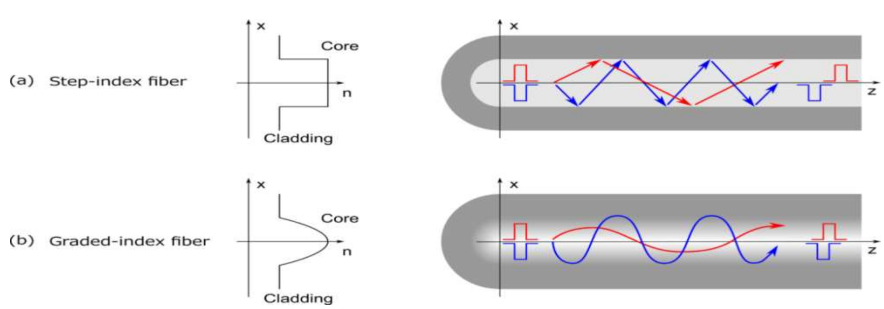
Where θ_{max} – Acceptance angle

n_1, n_2 are refractive indices of core and cladding respectively,

Δ is Fractional refractive index change, $\boxed{\boldsymbol{\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}}}$.

Step index fiber: Step index fiber is a fiber in which the core is with uniform refractive index and there is a sharp decrease in the index of refraction at the cladding. Index profiles are in the shape of step.

Graded index fiber: Graded index fiber is a type of fiber where the refractive index of the core is maximum at the center and decreases towards core-cladding interface. Index profiles is in the shape of a parabolic curve.



Applications of Optical Fiber

Optical fibers find applications in various fields. Some of them are

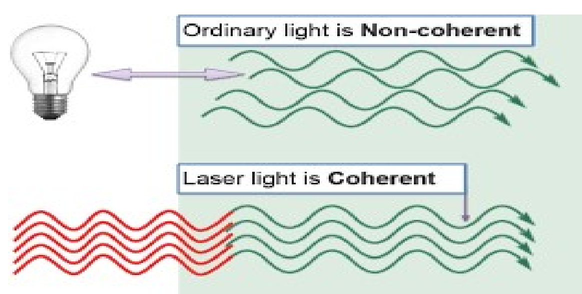
- Medical: Used as light guides, imaging tools.
- Defence /Government: Used as hydrophones for seismic waves and SONAR, as wiring in aircraft, submarines and other vehicles and also for field networking.
- Telecommunication: For transmission of information from transmitter to receiver.
- Networking: Used to connect users and servers in a variety of network settings and help to increase the speed and accuracy of data transmission.
- Industrial/Commercial**:** Used as sensory devices to make temperature, pressure and other measurements and as wiring in automobiles and in industrial settings.
- Broadcast/CATV**:** Broadcast/cable companies are using fiber optic cables for wiring CATV, HDTV, internet and other applications.

LASERS

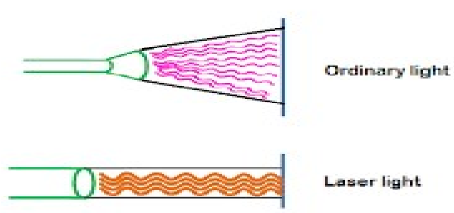
LASER acronym Light Amplification by stimulated Emission of radiation

Characteristics of Lasers:

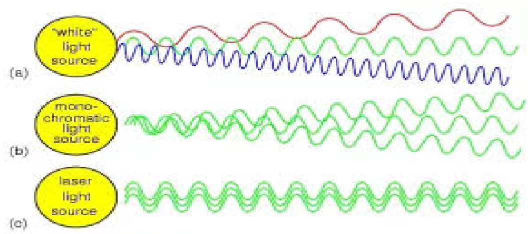
1. **Coherence:**Laser has high degree of coherence. As compared to ordinary light.



2. **Directionality:**ordinary light is highly divergent where as laser light is highly directional .



3. **Monochromaticity:**Laser beam is highly monochromatic (Single wavelength) than other sources of light.



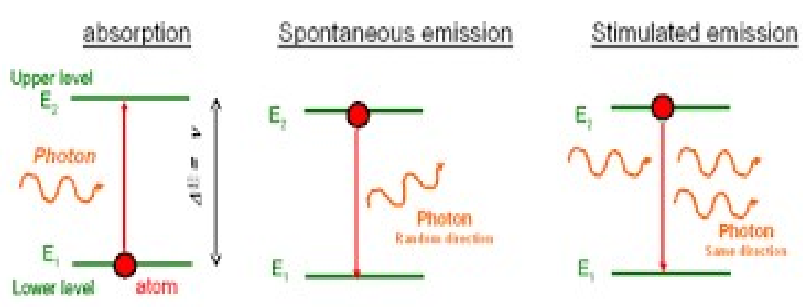
4. **High Intensity:**The intensity of laser light is thousand times more intense than an ordinary light.

Stimulated absorption, Spontaneous and stimulated emission:

1. Stimulated absorption: An atom in lower level of energy E1 goes to higher energy level E2 , when it absorbs a photon whose energy is equal to (E2-E1), this is known as stimulated absorbs.

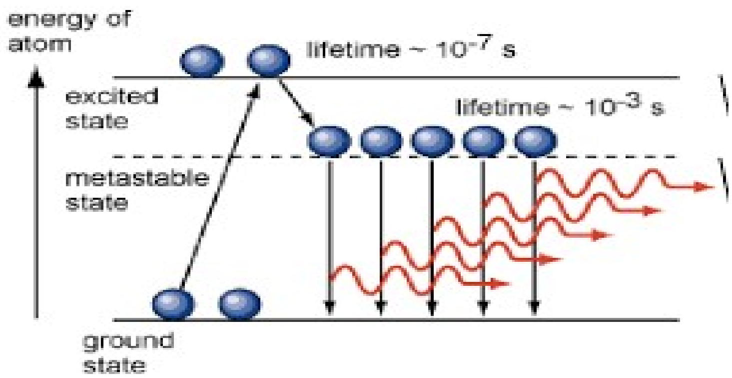
2.Sponteneous Emission: when the atom absorbs a photon energy it returns to ground state by emitting photon of energy $E=E_2-E_1=h\nu$, The emission occurs without any help from surrounding radiation this is known as spontaneous emission.

3. Stimulated emission: The atom in the excited state can also return to the ground state by trigging or inducement of photon of energy which is equal to energy of incident photon ie. $E=E_2-E_1=h\nu$, is known as stimulated emission. Thus, results into 2 photons of coherent and directional.



Population inversion and Meta stable state:

Population inversion: The process by which the population of higher energy state is made more than that of specified lower energy state is called as population inversion. ie. $N_2>N_1$.

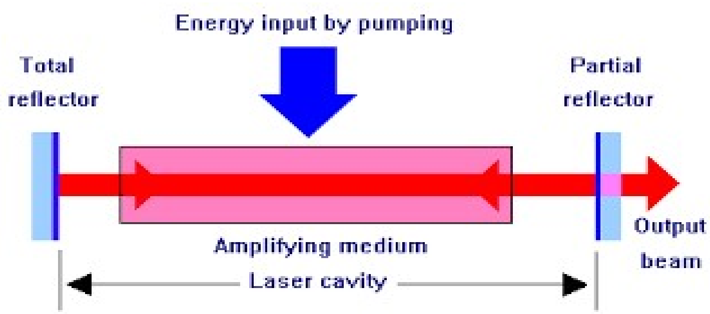


Note: Life time of higher energy state is 10^{-8} s and Life time of metastable state is 10^{-3} s

Meta stable state: It is the energy state in which atoms can stay longer time hence population inversion can achieve called meta stable state.

Main components of Laser: There are 3 main components of Laser.

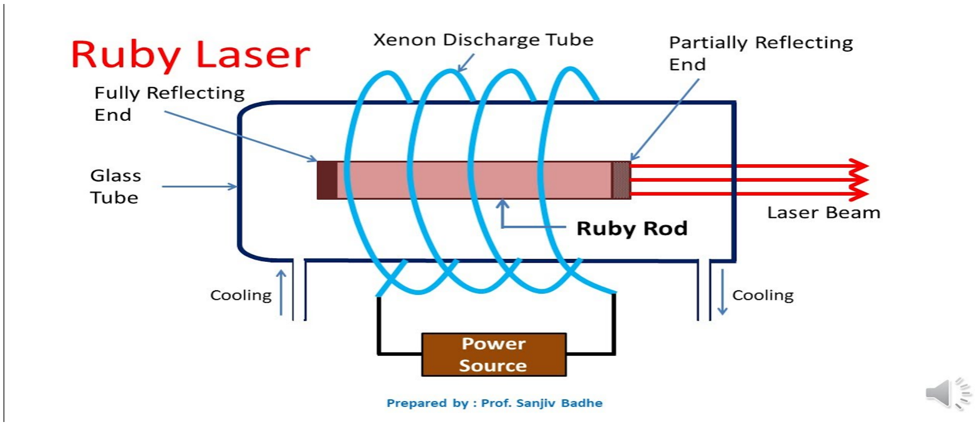
1. Active medium
2. Energy Source
3. Optical resonator



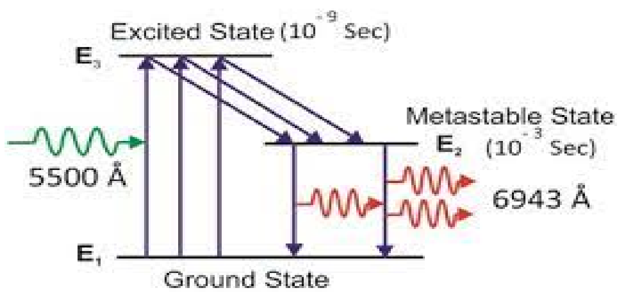
RUBY LASER

Introduction

- Ruby laser is a 3-level solid state laser discovered by T.H.Maiman in 1960. Ruby rod is a crystal used as active medium and the laser ou put is 694.3nm.
- **Construction:**

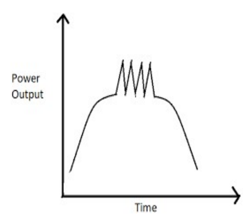


Working:



Energy Level Diagram of Ruby LASER

Output beam characteristics: Output consists spikes about 1μs, so output is in pulsed form.



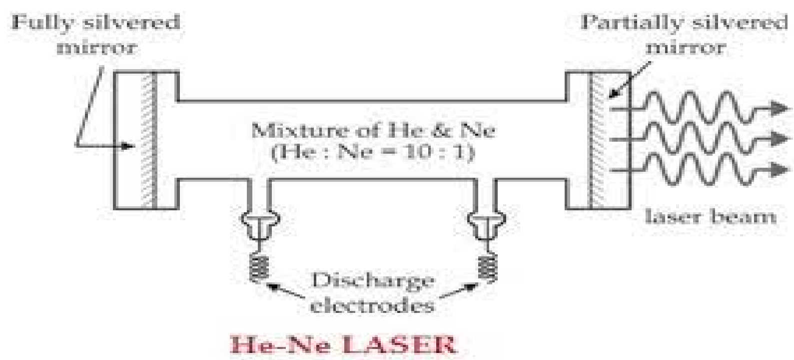
Applications:

- It is used in holography,
- It is used in LIDAR.
- It is used in remote sensing.
- It is used in Ophthalmology.
- It is used in drilling small areas.
- In military, used as target designators and range finders.

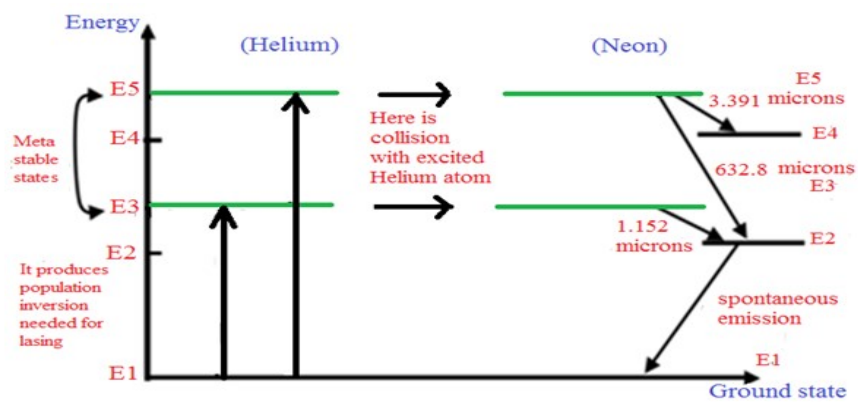
He-Ne LASER:

Introduction: He-Ne is the gas laser discovered by Ali Javan in December 1960, in Bell laboratory. This is designed to get continuous output beam. Here He-Ne gas is the active medium, Ne are the active centres achieves the population inversion and stimulated emission takes place of wavelength 632.8 nm. Or 6328 Å

Construction :



Working:



Applications :

- Due to high power it is used in open air communication.
- It is used to produce holograms.
- Widely used in in laboratories for all interferometric experiments.
- Widely used in metrology in surveying.
- He –Ne laser scanner used to read bar decoder.

Applications of LASER

1. Laser in industry:

Welding:

- Dissimilar metal can be welded.
- Micro welding can be done with great ease.

Cutting:

- Any desired shape cuts easily, complicated cuts made easy with laser.
- With high power CO_2 laser glass, quartz and diamonds can be cut easily.

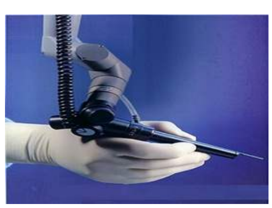
Drilling:

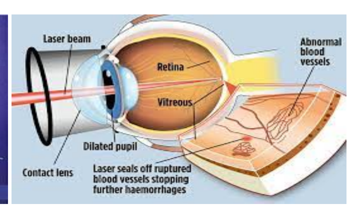
- Lasers are used to drill holes in difficult to drill material such as ceramic, etc.
- Hole of micron order can be easily drilled.

2. Laser in electronic industry:

- Scribbling: drawing fine lines in brittle ceramic and semiconductor wafers scribbling done with laser.
- Soldering: thin sheets 25 micron can soldered without any damage of sheets.
- Trimming: film register trimming made easy with laser.

3.Laser in medicine:





- Ophthalmologist using for eye treatment.

- Lasers scalps are used for bloodless surgeries.
- Lasers are used angioplasty for removal artery block in heart.
- In dermatology laser are used to remove freckles, acne, birth marks and tattoos.

4. Lasers in scientific fields and military:

- Laser in metrology survey to measure distance like earth to moon .
- Lasers act like weapon, target finder and ranging.
- Lasers used to find the enemy targets.

5. Lasers in communication:

- It is widely used in open air communication (satellite), because it is free from dust, fog and rain.

6. Laser in other fields:

- Used as laser scanner in super market to scan bar code.
- Used in storage technique of CD player to increase storage capacities.

Formulae:

1. Numerical aperture of Optical Fiber $\sin\theta_{max} = \sqrt{n_1^2 - n_2^2} = n1\sqrt{2\Delta}$
2. Acceptance Angle $\theta_{max} = Sin^{-1}(\frac{{n^2}_{core} \; -{n^2}_{cladding}}{{n_0}^2})$
3. Fractional index change is$\Delta = \frac{n_1 - n_2}{n_1}$
4. Divergence of Laser

$$\Theta = \frac{d_2 - d_1}{z_2 - z_1}$$

Sample multiple choice questions:

1. Which of the following is an example of optical pumping?

a) Ruby laser

b) Helium-Neon laser

c) Semiconductor laser

d) Dye laser

Answer: a

2. What is the need to achieve population inversion?

a) Stimulated emission

b) To bring most of the atoms to ground state

c) To achieve stable condition

d) To reduce the time of production of laser

Answer:a

3. Which of the following is a unique property of laser?

a) velocity

b) Speed

c) Coherence

d) Wavelength

Answer: c

4. Sine of the maximum launching angle of an optical fibre is called -----

a) Acceptance angle

b) Acceptance cone

c) Numerical aperture

d) Fractional index change

Answer: c

5. If n1, n2 are refractive indices of core and cladding respectively of an optical fibre, then

a) n1=n2

b) n1 > n2

c) n1 < n2

d) No relation between n1 and n2

Answer: b

Problems

1. An optical fiber has a numerical aperture of 0.20.Find the acceptance angle for the fiber.

   Solution: Numerical Aperture=Sin(Acceptance angle)

   Acceptance angle = Sin^{-1}(Numerical Aperture)

   $= \text{Sin}^{-1}(0.20)$

   $= 11.54^0$

2. Calculate the fractional index change for a given optical fiber if the refractive indices of the core and cladding are 1.563 and 1.498 respectively.

   Solution: Fractional index change is $\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.563 - 1.498}{1.563} = 0.0416$

3. Calculate the numerical aperture and acceptance angle for an optical fiber with core and cladding refractive indices 1.48 and 1.45 respectively.

   Solution = $\text{Numerical Aperture} = \sqrt{n_1^2 - n_2^2} = \sqrt{1.48^2 - 1.45^2 = 0.296}$

4. For a He-Ne laser, the output beam spot diameters are 4mm and 6mm at 1m and 2m distances Respectively. Calculate its divergence.

   Solution:

$$\Theta = \frac{d_2 - d_1}{z_2 - z_1}$$

$$= \frac{0.006 - 0.004}{2 - 1} = 0.002$$